

1. **14** $x^2(x + 2) - 2(x^2 - 3)^2 = x^3 + 2x^2 - 2(x^4 - 6x^2 + 9) = x^3 + 2x^2 - 2x^4 + 12x^2 - 18 = -2x^4 + x^3 + 14x^2 - 18$. The required coefficient is 14.
2. **11** $3x^2 - 28x = 55 \rightarrow 3x^2 - 28x - 55 = 0 \rightarrow$ The sum of the roots of this equation is $\frac{28}{3}$. Therefore, the second root is $\frac{28}{3} - \left(\frac{5}{3}\right) = 11$.
3. **8** Call the length of the legs of the original triangle x . Then $\frac{1}{2}x^2 = 32 \rightarrow x = 8$. The smaller triangle is also an isosceles right triangle with a leg length of 4. Its area is $\frac{1}{2} \cdot 4^2 = 8$.
4. **109** $\frac{\sqrt{117} \cdot \sqrt{20}}{\sqrt{26}} \rightarrow \frac{3\sqrt{13} \cdot 2\sqrt{5}}{\sqrt{2} \cdot \sqrt{13}} \rightarrow \frac{6\sqrt{5}}{\sqrt{2}} \rightarrow \frac{6\sqrt{10}}{2} \rightarrow 3\sqrt{10}$. Thus $3^2 + 10^2 = 109$.
5. **76** If x is the degree measure of the larger angle, then $3(180 - x) - 2(x + 10) = 0 \rightarrow 520 = 5x \rightarrow x = 104 \rightarrow 180 - x = 76$.
6. **12** Substitute 0 for y in the given equation: $x^2 - 12x = -20 \rightarrow x^2 - 12x + 20 = 0 \rightarrow (x - 10)(x - 2) = 0 \rightarrow x = 10$ or $x = 2$. The required sum is 12.
7. **9** The units-digit of the powers of 17 form a repeating pattern of 4 digits 7, 9, 3, and 1. Therefore, 17^{2026} will produce the same units-digit as 17^2 , namely 9.
8. **25** Use of the Pythagorean Theorem in $\triangle ADE$ results in $DE = 5$. Therefore, $DG = 5$. Also, $\triangle ADE \cong \triangle ADG$ by Hypotenuse-Leg, so $AG = 12$. Use of the Pythagorean Theorem in $\triangle BDG$ results in $BG = 13$ and $AB = 12 + 13 = 25$.
9. **240** The desired arrangement is of the form $V_1C_1C_2C_3C_4C_5V_2$ where V_k and C_k are vowels and consonants, respectively. The number of ways of choosing the two vowels is $2 \cdot 1$, and the number of ways of choosing the five constants is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Therefore, the total number of 7-letter arrangements is $2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 240$.
10. **460** If the list price is x dollars, then Kevin's commissions are $0.05(x - 20)$ and $0.03(x - 60)$. Then, $0.05(x - 20) = 0.03(x - 60) + 10 \rightarrow x = 460$.
11. **17** Let $x - 2$, x , and $x + 2$ represent the three consecutive positive odd integers. So, $x^2 = 4 \cdot 2x + 105 \rightarrow x^2 - 8x - 105 = 0 \rightarrow (x - 15)(x + 7) = 0 \rightarrow x = 15$. The required integers are 13, 15, and 17. The largest of these integers is 17.
12. **16** The prime factorization of $(20) \cdot (26) = 2^3 \cdot 5^1 \cdot 13^1$. Any factor would then be a form of $2^a 5^b 13^c$ where $0 \leq a \leq 3$, $0 \leq b \leq 1$, and $0 \leq c \leq 1$. Since there are 4 possible positive integers for a , 2 for b , and 2 for c , the number of factors of n that make $\frac{20 \cdot 26}{n}$ a positive integer is $4 \cdot 2 \cdot 2 = 16$.
13. **24** Since the chords connect to a diameter, they form a right angle. The diameter of the largest semicircle is 10. The area of each semicircle is 12.5π , 8π , 4.5π . The area of the triangle is 24. The area of the shaded regions is $8\pi + 4.5\pi - (12.5\pi - 24) = 24$.

14. **156** $252\pi = \pi(6)^2h \rightarrow h = 7$. The surface area of the cylinder is $2\pi rh + 2\pi r^2 \rightarrow 2\pi(6)(7) + 2\pi(6)^2 = 156\pi \rightarrow k = 156$.

15. **348** $P = \frac{\binom{4}{1} \cdot \binom{20}{2}}{\binom{24}{3}} = \frac{95}{253}$. The required sum is 348.

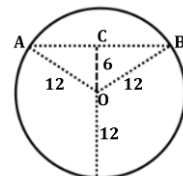
Grade Level 10 – NMT 2026

Solutions

1. **196** The x -intercept is 2. The y -intercept is $-\frac{1}{2}$. The slope is $\frac{1}{4}$. The value of $100a + 10b + 4m = 100(2) + 10\left(-\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) = 200 - 5 + 1 = 196$.
2. **40** In a geometric sequence, $a_5 = a_3r^2 \rightarrow 5r^2 = 10 \rightarrow r^2 = 2 \rightarrow r = \pm\sqrt{2} \rightarrow a_9 = 10(\pm\sqrt{2})^4 = 40$. Alternatively, since a_3, a_5 , and a_9 , are in the problem, the terms a_3, a_5, a_7, a_9 form another geometric sequence. Therefore, the common ratio is 2 and the terms are $a_3 = 5, a_5 = 10, a_7 = 20, a_9 = 40$.
3. **10** To make a triangle requires choosing 3 points out of the 5 points given. So, the number of triangles is $\binom{5}{3} = \binom{5}{2} = 10$.
4. **17** Let $\sin x = \frac{1}{4\cos x} \rightarrow \sin x \cos x = \frac{1}{4} \rightarrow \sin 2x = 2 \sin x \cos x = \frac{1}{2}$. So, $2x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, making $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$. Then, $a = 5$ and $b = 12$ and $a + b = 5 + 12 = 17$.
5. **12** Since $f(x)$ has an x -intercept of 3 and its range is greater than or equal to 0, the function may be re-written as $f(x) = a(x - 3)^2$. Since the y -intercept is 27, $a(0 - 3)^2 = 27 \rightarrow a = 3$. So, $f(1) = 3(1 - 3)^2 = 12$.
6. **7** Compute the number of diagonals of a polygon using $\binom{n}{2} - n = \frac{n(n-1)}{2} - n \rightarrow \frac{n(n-1)}{2} - n = 14 \rightarrow n^2 - n - 2n = 28 \rightarrow n^2 - 3n - 28 = 0 \rightarrow (n - 7)(n + 4) = 0 \rightarrow n = 7$.
7. **216** Since the area of the circle is 36π , its radius is 6. This is also the length of an apothem for the hexagon. The hexagon may be divided into six congruent equilateral triangles. The apothem is also an altitude of any of the equilateral triangles. Using properties of 30-60-90 triangles, it can be determined that the length of a side of each equilateral triangle is $4\sqrt{3}$. Then the area of the hexagon is $6 \times$ area of one equilateral triangle. So, the area of the hexagon as $6\left(\frac{\sqrt{3}}{4}\right)(4\sqrt{3})^2 = 72\sqrt{3}$. The required product is 216.
8. **1** By the method of completing the square, circle M may be re-written as $(x + 3)^2 + (y - 4)^2 = 36$. It has a center at $(-3, 4)$ and a radius of 6. Ellipse N may be re-written as $\frac{(x-7)^2}{9} + \frac{(y-4)^2}{4} = 1$. It has a center at $(7, 4)$ and the length of its major axis is 6. Since the major axis of the ellipse and a diameter of the circle lie along the line $y = 4$, the required minimum distance between points respectively on the circle and the ellipse $(3, 4)$ and $(4, 4)$ is 1.
9. **5** Start with $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 23 + 2 = 25 \rightarrow x + \frac{1}{x} = 5$.

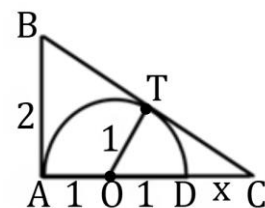
10. **405** Use the distance formula to see that $OO' = 15$. From the given, $OA = 9$. The maximum area of $\Delta AOO'$ must occur when either $\angle AOO'$ or $\angle OAO'$ is a right angle. From the given, $\angle OOA'$ is not a right angle. If $\angle OAO'$ is a right angle, then $AO' = 12$ and the area of right $\Delta AOO' = \frac{1}{2}(12)(9) = 54$. If $\angle AOO'$ is a right angle, then the area of right $\Delta AOO' = \frac{1}{2}(15)(9) = 67.5$. $K = 67.5$ and $6K = 405$.

11. **124** The volume of water in the upright cylinder is $12^2\pi \cdot h$. The volume of the water in the sideways view cylinder is base area times height (24). Separate the base into major sector AOB and ΔAOB . Since $OA = 2 \cdot OC$, $m\angle A = m\angle B = \frac{\pi}{6}$, $m\angle AOB = \frac{2\pi}{3}$, and major sector's angle is $\theta = \frac{4\pi}{3}$. So, the area of the major sector $= \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{2}{3} \cdot \pi 12^2$ and area of $\Delta AOB = \frac{1}{2} \cdot 12\sqrt{3} \cdot 6 = 36\sqrt{3}$. So, the volume of water in the sideways cylinder is $(\frac{2\pi}{3} \cdot 12^2 + 36\sqrt{3})24$. Setting the two volumes equal, $12^2\pi \cdot h = (96\pi + 36\sqrt{3})24 \rightarrow h = \frac{6\sqrt{3}}{\pi} + 16 = \frac{\sqrt{108}}{\pi} + 16$ and the required sum is 124.



12. **37** Since a and b are positive integers, the bigger of the two must be less than or equal to 26 because $26^2 = 676$. For the units-digit to be 9, the units-digits of (a, b) or (b, a) can be $(3, 0)$, $(7, 0)$, $(5, 2)$, and $(5, 8)$. By inspection, $25^2 + 8^2 = 689$ and $20^2 + 17^2 = 689$. Thus, the maximum value of $a + b = 20 + 17 = 37$.
13. **706** Use the Pythagorean theorem in right $\Delta BDC \rightarrow DB = 36$. Then use the theorem that the length of the altitude to the hypotenuse of a right triangle is the mean proportional between the lengths of the segments on the hypotenuse. First, in ΔADC : $36^2 = 27 \cdot AB \rightarrow AB = 48$. Then use the Pythagorean theorem again in right $\Delta ABD \rightarrow AD = 60$. Then $60^2 = 45 \cdot ED \rightarrow ED = 80 \rightarrow \frac{(\Delta AEC)}{(\Delta BDC)} = \frac{\frac{1}{2} \cdot 125 \cdot 60}{\frac{1}{2} \cdot 36 \cdot 27} = \frac{625}{81}$. The required sum is 706.

14. **11** Let the center of the semicircle be point O . Draw $\overline{OT}$. The diameter of the semicircle O is $\overline{AD}$. Let $x = DC$. Note that $\Delta COT \sim \Delta CBA \rightarrow \frac{1}{2} = \frac{\sqrt{(x+1)^2 - 1}}{x+2} \rightarrow x + 2 = 2\sqrt{x^2 + 2x} \rightarrow x^2 + 4x + 4 = 4x^2 + 8x \rightarrow 3x^2 + 4x - 4 = 0 \rightarrow (3x - 2)(x + 2) = 0 \rightarrow x = \frac{2}{3}$. So, the area of the triangle is $\frac{1}{2} \left(2 + \frac{2}{3} \right) (2) = \frac{8}{3}$ and $a + b = 8 + 3 = 11$.



15. **24** $x(x+2)(x+4)(x+6) + 16 = [x(x+6)][(x+2)(x+4)] + 16 = [x^2 + 6x][x^2 + 6x + 8] + 16 = (x^2 + 6x)^2 + 8(x^2 + 6x) + 16 = (x^2 + 6x + 4)^2$. This makes $a = 1, b = 6, c = 4$. So, $abc = 24$. Alternatively, let $x = y - 3$. Then the given expression is $(y - 3)(y - 1)(y + 1)(y + 3) + 16 = (y^2 - 9)(y^2 - 1) + 16 = y^4 - 10y^2 + 9 + 16 = y^4 - 10y^2 + 25 = (y^2 - 5)^2 = ((x + 3)^2 - 5)^2 = (x^2 + 6x + 4)^2$ and continue as above.

1. **972** Re-write the given expression using prime number bases and apply exponent rules:

$$\frac{(27)^{5/3}(64)^{5/6}}{(16)^{3/4}} = \frac{(3^3)^{5/3}(2^6)^{5/6}}{(2^4)^{3/4}} = \frac{(3)^5(2)^5}{(2)^3} = (3)^5(2)^2 = (243)(4) = 972.$$
2. **125** Solve the given equation and get $x = \frac{17}{8}$. Substitute into the factored form of the second expression. The result is 125. Alternatively, factor the expression $\frac{x^2+2x+1}{5x^2-20x+20} = \frac{(x+1)^2}{5(x-2)^2}$. Then, since $\frac{x-2}{x+1} = \frac{1}{25} \rightarrow \frac{x+1}{x-2} = 25$ and by substitution, $\frac{1}{5}(25)^2 = \frac{1}{5}(625) = 125$.
3. **44** Square $z = a + bi \rightarrow z^2 = (a + bi)^2 = a^2 + 2abi + b^2i^2 = a^2 + 2abi - b^2 = -105 + 88i$. Set the imaginary parts equal to yield $2ab = 88 \rightarrow ab = 44$.
4. **12** For the system of linear equations to have no solutions, the lines must have equal slopes and unequal y -intercepts. Set the slopes equal: $-\frac{a}{16} = -\frac{9}{a} \rightarrow a^2 = 144 \rightarrow a = \pm 12$, and the positive value is $a = 12$. (The y -intercepts are 40 and 84.)
5. **94** Let $f(x) = ax + b \rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = 9x + 4$. Equating coefficients, $a^2 = 9 \rightarrow a = \pm 3$. But the slope of $f(x)$ must be positive. Therefore, $a = 3$, and $3b + b = 4 \rightarrow b = 1$. So, $f(x) = 3x + 1$ and $f(31) = 94$.
6. **192** Drop the altitude from the vertex angle to the base of the isosceles triangle. This forms two right triangles with a base of 16 and a hypotenuse of 20. Use the Pythagorean Theorem, to find that this altitude of the given triangle is 12. Then the area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(32)(12) = 192$. Alternatively, from Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{36(36-20)(36-20)(36-32)} = \sqrt{36(16)(16)(4)} = 6 \cdot 4 \cdot 4 \cdot 2 = 192$.
7. **26** Set the two given equations equal to each other and re-arrange the terms:
 $\frac{17}{8}x^2 - 34x + 162 - k = 0$. Since the two parabolas have exactly one point in common, the discriminant must equal 0. $b^2 - 4ac = (-34)^2 - 4 \cdot \frac{17}{8}(162 - k) = 0 \rightarrow 1156 - 1377 + \frac{17}{2}k = 0 \rightarrow \frac{17}{2}k = 221 \rightarrow k = 26$. Alternate solution: Using $x = -\frac{b}{2a}$, the axis of symmetry of $y = \frac{1}{8}x^2 - 2x + 34$ is $x = 8$, and the vertex is $(8, 26)$. For $y = -2(x - 8)^2 + k$, the axis of symmetry is also $x = 8$ and the vertex is $(8, k)$. Thus, if the parabolas intersect at just one point, it must be at their vertex and $k = 26$.
8. **339** The given sequence is an arithmetic sequence with a common difference of $\frac{1}{3}$. Then,
 $a_n = a_1 + d(n - 1) = 6 + \frac{1}{3}(n - 1) \rightarrow a_{1000} = 6 + \frac{1}{3}(1000 - 1) = 339$.
9. **9** For any 2-digit number, the sum of its digits is at least 1 and at most 18. Only two perfect cubes are in that interval: 1 and 8. The only two-digit number, the sum of whose digits is 1, is the number 10. The only two-digit numbers, the sum of whose digits is 8, are 17, 71, 26, 62, 53, 35, 44, and 80. So there are a total of 9 possible two-digit numbers whose digits sum to a perfect cube.

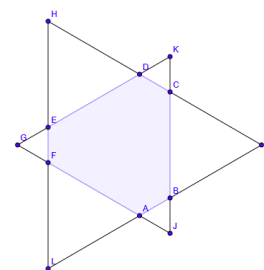
10. **106** Let $x = m\angle ACD$. Then, $m\angle ACB = 2x$. From the given similar triangles, $m\angle ABC = m\angle ACD = x$. In $\triangle ABC$, the sum of the measures of the angles is
 $21 + x + 2x = 180 \rightarrow 21 + 3x = 180 \rightarrow 3x = 159 \rightarrow x = 53 \rightarrow m\angle ACB = 2(53) = 106$.
11. **785** The given expression $\frac{(n+3)!}{(n+2)!} = \frac{(n+3)(n+2)(n+1)n \cdots 3 \cdot 2 \cdot 1}{(n+2)(n+1)n \cdots 3 \cdot 2 \cdot 1} = n + 3 < 789 \rightarrow n < 786$. So, there are 785 positive integer solutions to the given inequality.
12. **72** Re-write the given as $5 \cos x + 7 = \frac{6}{\cos x} \rightarrow 5(\cos x)^2 + 7 \cos x = 6 \rightarrow 5(\cos x)^2 + 7 \cos x - 6 = 0 \rightarrow (5 \cos x - 3)(\cos x + 2) = 0 \rightarrow \cos x = \frac{3}{5}$ or -2 (reject) $\rightarrow \sin x = \frac{4}{5}$
 $75 \sin(2x) = 150 \sin x \cos x = 150 \cdot \frac{4}{5} \cdot \frac{3}{5} = 72$.
13. **8** Multiply the given equation by $\sqrt{2}$ to simplify two of the terms, $2x - \sqrt{2x} = 12 \rightarrow 2x - 12 = \sqrt{2x} \rightarrow (2x - 12)^2 = (\sqrt{2x})^2 \rightarrow 4x^2 - 48x + 144 = 2x \rightarrow 4x^2 - 50x + 144 = 0 \rightarrow 2x^2 - 25x + 72 = 0 \rightarrow (2x - 9)(x - 8) = 0 \rightarrow x = \frac{9}{2}$ or $x = 8$. Reject $x = \frac{9}{2}$. The only solution is $x = 8$. Alternatively, set $x = 2n^2 \rightarrow 2n^2\sqrt{2} - n\sqrt{2} = 6\sqrt{2} \rightarrow 2n^2 - n = 6 \rightarrow 2n^2 - n - 6 = 0$. The only integer solution of this equation is 2 and then $x = 8$.
14. **81** Re-write the given equation using the base change law: $\frac{\log(3x)}{\log 9} + 3 \frac{\log x}{\log 27} = \frac{13}{2} \rightarrow \frac{\log 3}{2 \log 3} + \frac{\log x}{2 \log 3} + \frac{3 \log x}{3 \log 3} = \frac{13}{2} \rightarrow \frac{\log x}{2 \log 3} + \frac{2 \log x}{2 \log 3} = 6 \rightarrow \frac{3 \log x}{2 \log 3} = 6 \rightarrow 3 \log x = 12 \log 3 \rightarrow \log x = 4 \log 3 \rightarrow \log x = \log 81 \rightarrow x = 81$.
15. **148** In order for the sum to be even, both pencils must either be labeled with an odd number or both pencils must be labeled with an even number. The required probability is $\frac{\binom{50}{2} + \binom{50}{2}}{\binom{100}{2}} = \frac{2 \cdot \frac{50 \cdot 49}{2}}{\frac{100 \cdot 99}{2}} = \frac{49}{99}$. Alternatively, whether the first pencil chosen is odd or even, the second pencil must have the same parity as the first. Out of the remaining 99 pencils, 49 have the same parity.

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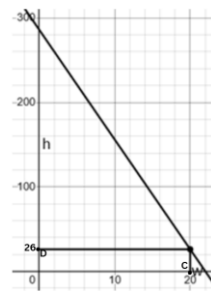
Solutions

1. **5** A classic number trick states that A% of B equals B% of A. So, 2026% of 50 is 50% of 2026 = 1013. The sum of the digits is then $1 + 0 + 1 + 3 = 5$.
2. **360** $f'(x) = \frac{1+2x+3x^2+\cdots+720x^{719}}{1+x+x^2+\cdots+x^{720}}$, so $f'(1) = \frac{1+2+3+\cdots+720}{1+1+1+\cdots+1} = \frac{\frac{(720)(721)}{2}}{721} = \frac{720}{2} = 360$.
3. **75** The surface area of a hemispheric solid is $\frac{1}{2}(4\pi R^2) + \pi R^2 = 3\pi R^2$, remembering the circular base. The surface area of the spherical solid is $4\pi R^2$. So, the ratio is $\frac{3\pi R^2}{4\pi R^2} = \frac{3}{4} = 75\%$.

4. **45** Since arctangent is an increasing function, we can maximize f by maximizing e^{-x^2} . Since the exponential function is an increasing function, we can maximize f by maximizing $-x^2$. Since $-x^2 \leq 0$, its maximum value is 0, when $x = 0$. So, the maximum value of f is $f(0) = 45^\circ$.
5. **10** Let $y = 3^x$, so $3y^2 - 82y + 27 = 0$, and $(3y - 1)(y - 27) = 0$, so $y = \frac{1}{3}$ or $y = 27$. Since $y = 3^x$, $x = -1$ or $x = 3$ and therefore $a^2 + b^2 = 1 + 9 = 10$.
6. **711** There are 11 30ths in 2026 (February excluded). Of the 11 months that have at least 30 days, 4 of them do not have a 31st (April, June, September, November). So, there is a $\frac{7}{11}$ chance that the following day is the 31st and $100(7) + 11 = 711$.
7. **0** Since $y = -x^4 + 6x^2 - 2$ is an even function, the tangent lines must be reflections of each other in the y -axis. So, the tangent lines must intersect at a point on the y -axis. This means that $a = 0$ and so $ab = 0$. Note: the tangent lines are $y = 22 - 8x$ and $y = 22 + 8x$.
8. **74** Since the 6 exterior triangles are all equilateral, the sides of the large equilateral triangle all have length 7 and the sides of the smaller equilateral triangle all have length 5. Thus, the ratio of their areas must be $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$ and $49 + 25 = 74$.
9. **55** Completing the square gives us the equation of the circle: $(x - 10)^2 + (y + 5)^2 = 2026$. This is a circle of radius $\sqrt{2026}$, centered at $(10, -5)$, so the set of possible real values of x is $[10 - \sqrt{2026}, 10 + \sqrt{2026}]$. Since $10 - \sqrt{2026} < 0$, we just need to place $10 + \sqrt{2026}$ in the integers. Noting that $45^2 = 2025$, the value of $10 + \sqrt{2026}$ is a tiny bit larger than 55, yielding 55 possible positive integer values of x .
10. **973** Let $a = 2026$, so the point of normalcy is $(a, \frac{a^2}{1946})$. Since the derivative of the curve is $\frac{x}{973}$, the slope of the normal line is $-\frac{973}{a}$. This makes the equation of the normal line $y - \frac{a^2}{1946} = -\frac{973}{a}(x - a)$, which simplifies to $y = -\frac{973}{a}x + \frac{a^2}{1946} + 973$. The y -intercept of this line is $\frac{a^2}{1946} + 973$, which is 973 units above the point of normalcy. Note that this answer does not depend on where the normal line is drawn. Alternatively, since the slope of the normal line is $-\frac{973}{2026}$, and the abscissa of the normal point is 2026, the intersection on the y -axis is 973.
11. **168** Let $z = a + bi$, so $z^2 = (a + bi)^2 = a^2 - b^2 + 2abi = x + 2026i$. So, $a^2 - b^2 = x$ and $2abi = 2026i$. This gives us $ab = 1013$. Since 1013 is a prime number, a and b must be ± 1 and ± 1013 . For x to be a positive real, $a^2 = 1013^2$ and $b^2 = 1$, so $x = 1013^2 - 1 = 1026168$. The remainder when this number is divided by 1000 is its final three digits, 168.
12. **50** Let M be the midpoint of segment AB . Since angle AEB is a right angle, point E must lie on the circle for which segment AB is a diameter. Thus, $ME = 29$. Drop perpendicular MN from M to side DC . Now $MN = 20$ and by Pythagorean theorem, $EN = 21$. So, $EC = EN + NC = 21 + 29 = 50$.



13. **910** Draw perpendiculars and define h and w as shown at right. Note that, since points C and D are fixed, $\frac{dx}{dt} = \frac{dw}{dt}$ and $\frac{dy}{dt} = \frac{dh}{dt}$, $\frac{dw}{dt} = -7$ and $\frac{dh}{dt}$ is our desired answer. By similar triangles, $\frac{h}{20} = \frac{26}{w}$, so $h = 520w^{-1}$. Thus, $\frac{dh}{dt} = -520w^{-2} \frac{dw}{dt}$. At the instant we want, $w = 2$ and so $\frac{dh}{dt} = -\frac{520}{4}(-7) = -130(-7) = 910$.

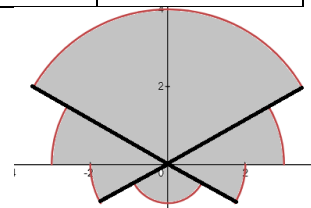


14. **53** Looking at places where $3 + 2 \sin \theta$ is an integer, we obtain the following values:

θ	$[0,30]$	$[30,150]$	$(150,180]$	$(180,210]$	$(210,330)$	$[330,360)$
$[3 + 2\sin\theta]$	3	4	3	2	1	2

So, we can construct that graph as a union of circular sectors. Looking at what fraction of a circle has each radius, the total area is equal to

$$\frac{2}{6}(16\pi) + \frac{1}{6}(9\pi) + \frac{1}{6}(4\pi) + \frac{2}{6}(\pi) = \frac{47}{6}\pi \text{ and } 47 + 6 = 53.$$



15. **343** Proceed by cases based on first roll:

$$n = 1, \text{ then roll a 4: } P = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36} = \frac{1}{6^2}.$$

$$n = 2, \text{ then roll } (1,3), (2,2), \text{ or } (3,1): P = \left(\frac{1}{6}\right)\left(\frac{3}{6^2}\right) = \frac{3}{6^3}.$$

$$n = 3, \text{ then roll } (1,1,2), (1,2,1), \text{ or } (2,1,1): P = \left(\frac{1}{6}\right)\left(\frac{3}{6^3}\right) = \frac{3}{6^4}.$$

$$n = 4, \text{ then roll } (1,1,1,1): P = \left(\frac{1}{6}\right)\left(\frac{1}{6^4}\right) = \frac{1}{6^5}.$$

$$\text{The total probability is } \frac{1}{6^2} + \frac{3}{6^3} + \frac{3}{6^4} + \frac{1}{6^5} = \frac{6^3 + (3)6^2 + (3)6 + 1}{6^5} = \frac{343}{6^5}.$$

Mathletics - NMT 2026

Solutions

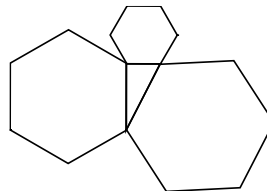
- 30** The altitude and median from the vertex B has a length of 8 because the altitude divides $\triangle ABC$ into two 8-15-17 right triangles. Since $\triangle BPQ \sim \triangle BAC$ with a ratio of similitude of $\frac{1}{2}$, the area of $\triangle BPQ$ is one-fourth the area of $\triangle ABC = \frac{1}{4}\left(\frac{1}{2} \cdot 30 \cdot 8\right) = 30$.
- 484** Square the given equation: $4 + \sqrt{3 + \sqrt{x}} = 9 \Rightarrow \sqrt{3 + \sqrt{x}} = 5 \Rightarrow 3 + \sqrt{x} = 25 \Rightarrow \sqrt{x} = 22 \rightarrow x = 484$.
- 26** If $m > 0, m^3 < 1826 \Rightarrow m \leq 12$. If $m < 0, m^3 > -2226 \Rightarrow m \geq -13$. Thus, $-13 \leq m \leq 12$. In that interval, there are 26 integers.
- 64** Point P is $\frac{1}{4}$ of the way from point A to point B . Therefore, $(x, y) = \left(2 + \frac{1}{4}(20 - 2), 4 + \frac{1}{4}(26 - 4)\right) = \left(\frac{13}{2}, \frac{19}{2}\right)$ and $4x + 4y = 64$.
- 24** Use the formula $a_n = a_1 + d(n - 1) = 20 + 6(n - 1) = 6n + 14$ to get the n th term. To get the sum of n terms, use $S_n = \frac{n(a_1 + a_n)}{2} = \frac{n(20 + 6n + 14)}{2} = n(3n + 17) > 2026$. From a table of values, determine the least integer n to satisfy the given inequality is 24.

6. **88** Let L , G , and M represent the speeds of the three participants and let D represent the track distance. From the given, $\frac{D}{M} - \frac{D}{L} = 100 \Rightarrow \frac{D}{M} - \frac{D}{1.2M} = 100 \Rightarrow \frac{D}{M} = 600$. The required number of seconds is $\frac{D}{G} - \frac{D}{L} = \frac{D}{1.02M} - \frac{D}{1.2M} = \frac{D}{M} \left(\frac{1}{1.02} - \frac{1}{1.2} \right) = 600 \left(\frac{1}{1.02} - \frac{1}{1.2} \right) \approx 88.235$.
7. **46** Suppose $x = \sqrt[3]{10 - \sqrt{74}}$, then $x^3 = 10 - \sqrt{74} \Rightarrow (x^3 - 10)^2 = (-\sqrt{74})^2 \Rightarrow x^6 - 20x^3 + 26 = 0$. A graph of this function verifies that it has the two desired roots. Alternatively, let $y = x^2$, then $y^2 - ay + b = 0$. If $y = 10 - \sqrt{74}$, $(10 - \sqrt{74})^2 - a(10 - \sqrt{74}) + b = 0 \rightarrow 174 - 20\sqrt{74} - 10a + a\sqrt{74} + b = 0$. Similarly, if $y = 10 + \sqrt{74}$, $(10 + \sqrt{74})^2 - a(10 + \sqrt{74}) + b = 0 \rightarrow 174 + 20\sqrt{74} - 10a - a\sqrt{74} + b = 0$. Then, by subtraction, $40\sqrt{74} - 2a\sqrt{74} = 0 \rightarrow a = 20$. By addition, $348 - 40a + 2b = 0 \rightarrow b = 26$. The required sum is 46.
8. **16** A line through the point and its image must pass through the center of the dilation. That point is on the line $y = x$. The equation of the line connecting the point and its image is $y - 26 = \frac{5}{2}(x - 20)$. Replace both variables with a : $a - 26 = \frac{5}{2}(a - 20) \rightarrow a - 26 = \frac{5}{2}a - 50 \rightarrow 24 = \frac{3}{2}a \rightarrow a = 16$. **Alternatively**, let d be the dilation factor. Then, $d(20 - a) + a = 28$ and $d(26 - a) + a = 46 \rightarrow$ (by subtraction) $6d = 18 \Rightarrow d = 3$. Then, $a = 16$. **Another alternative**: Translate the center of dilation from (a, a) to $(0, 0)$. The given points are now $(20 - a, 26 - a)$ and $(28 - a, 46 - a)$. So, the constant of dilation is the ratio of either the x -coordinates or of the y -coordinates. Thus, $\frac{28 - a}{20 - a} = \frac{46 - a}{26 - a} \rightarrow 728 - 54a + a^2 = 920 - 66a + a^2 \rightarrow 12a = 192 \rightarrow a = 16$. **Yet another alternative** requires a diagram using similar triangles: $\frac{8}{20 - a} = \frac{20}{26 - a} \rightarrow a = 16$.
9. **174** In order to calculate the height, h , of the pyramid, consider a right triangle, one of whose legs is a segment from the upper vertex of the pyramid to the center of the base. The other leg is a segment from the center of the base to a vertex of the base. The hypotenuse is an edge length of an isosceles triangle face of the pyramid. So, $h^2 + (4\sqrt{2})^2 = 12^2 \rightarrow h = \sqrt{112}$. The volume of the pyramid is given by $V = \frac{1}{3}Bh = \frac{1}{3}(64)\sqrt{112}$. The required number of minutes is the volume divided by 1.3. Thus, to the nearest integer, the answer is 174.
10. **643** The period of $f(x) = \sin(2026x)$ is $\frac{2\pi}{2026} = \frac{\pi}{1013}$. In the interval $[0, 0.5]$ there are $\frac{0.5}{\left(\frac{\pi}{1013}\right)} \approx 161.22$ cycles of the sinusoid. Each complete cycle will include two points of intersection. Note that $f(0.5) < g(0.5)$. So, the sinusoid will be below the line at $x = 0.5$. Thus, there will be 322 intersection points on $[0, 0.5]$. In the interval $[-0.5, 0]$, there will be another 322 intersections. Since this method counts the origin twice, the number of points of intersection is $2 \cdot 322 - 1 = 643$. Since $0.22 < \frac{1}{4}$, there will be less than $\frac{1}{4}$ of a sine graph cycle at the end of each interval. This can be confirmed by graphing the two functions on the window $[0.495, 0.5]$

1. **13** The ratio of the areas of similar polygons is the square of the ratio of a pair of corresponding sides. The square of the side we seek is the sum of the squares of the sides of the other 2 hexagons or $s^2 = 5^2 + 12^2 = 169$ so $s = 13$.

Alternative: Proof without words.

An application of the Pythagorean theorem.

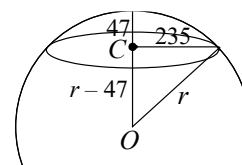


2. **953** If n is the position in the sequence, the number of unshaded squares is $4n + 4$ and the number of shaded squares is n^2 . The difference is $n^2 - 4n - 4$. Create a table or sketch the graph to determine the greatest possible 3-digit value for $n^2 - 4n - 4$. It occurs at $n = 33$.

3. **18** There are 3 different shaped trapezoids. Count the number of each shape and add. There are 12 of the smallest trapezoids, 3 of the medium trapezoids, and 3 of the largest trapezoids. The total number is $12 + 3 + 3 = 18$.



4. **611** Apply the Pythagorean theorem to the right triangle seen in the diagram: $(r - 47)^2 + 235^2 = r^2$ or simplify the process by realizing that $235 = 5 \times 47$. Let $r = 47R$. Then solve for r using $(R - 1)^2 + 5^2 = R^2 \rightarrow R = 13$. Since we divided the numbers by 47, to find r multiply $47 \times R = 47 \times 13 = 611$.



5. **5** Show that $k!$ can have 1, 2, 3, 4, and 6 terminal zeroes. Terminal zeroes occur when factors of 5 appear. The first factor of 5 occurs in $5! = 120$. The next factor of 5 will occur in $10! = 3628800$ which has 2 terminal zeroes. Similarly, $15!$ will have 3 terminal zeroes, $20!$ will have 4 terminal zeroes, but $25!$ will have 6 terminal zeroes since 25 adds two factors of 5. There is no value of k such that $k!$ will have 5 terminal zeroes.

6. **18** Since b must be an integer and lattice points are points whose coordinates are integers, we need consider only multiples of 3 for x . The lattice points for $f(x) = -2x + 16$, located in quadrant I have the domain $1 \leq x \leq 7$. Since 3 and 6 are the only multiples of 3 on that domain: when $x = 3$, the coordinates of the lattice point are $(3, 10)$ and $b = 9$ and when $x = 6$, the coordinates of the lattice point are $(6, 4)$ and $b = 2$. The product of these two values of b is $9 \times 2 = 18$.

7. **55** Let $n = 2$, then if $f(6) = 2 + f(3) = 2 + 1 = 3$. When $n = 3$, $f(9) = 3 + f(6) = 3 + 3 = 6$. Increasing n by 1 generates the sequences of triangular numbers: 1, 3, 6, 10, 15, A formula to generate the n th triangular number is $T(n) = \frac{n(n+1)}{2}$, $n > 0$. Note that $f(30) = T(10) = 55$.

8. **17** Let T, G, and A be the number of Tetras, Guppies, and Angelfish. Then, $G + A = 7$, $T + A = 12$, and $T + G = 15$. Sum the equations and divide by 2 to get $T + G + A = 17$.
Alternative: Let the number of fish = F, then $F - 7$ is the number of Tetras, $F - 12$ is the number of Guppies, and $F - 15$ is the number of Angelfish. Set up and solve the equation: $F = F - 7 + F - 12 + F - 15 = 3F - 34$. Thus, $F = 3F - 34$ and $F = 17$.

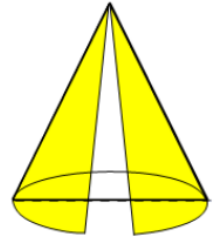
9. **18** Determine the number of 4-digit whole numbers that have no duplicate digits and subtract that number from the number of 4-digit whole numbers. There are $9 \times 9 \times 8 \times 7 = 4536$ four-digit numbers without repeats and there are $9 \times 10 \times 10 \times 10 = 9000$ four-digit numbers.

There are $9000 - 4536 = 4464$ four-digit numbers with at least 2 digits the same. The sum of the digits of this number is $4 + 4 + 6 + 4 = 18$.

10. **11** Rewrite $\frac{35}{12} = 2 + \frac{11}{12} = 2 + \frac{1}{\frac{12}{11}} = 2 + \frac{1}{1+\frac{1}{11}}$. Thus, $x = 11$.

Alternative: Simplify the fraction containing x : $\frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = \frac{x}{x+1}$. Since $\frac{35}{12} = 2 + \frac{11}{12}$, $\frac{x}{x+1} = \frac{11}{12}$, and $x = 11$.

11. **60** The radius of the semicircle is 1 and it becomes the slant height of the cone and the length of each of the congruent sides of the isosceles triangle. The semicircular arc of the semicircle becomes the circumference of the base of the cone. The semicircular arc has length equal to $\frac{1}{2} \cdot 2\pi = \pi$. Since the circumference of the base of the cone is also π , the diameter of the base is 1. The isosceles triangle is also equilateral, so its vertex angle is 60° .



12. **83** Note that $\frac{c}{a} = \frac{2d/7}{6b/5} = \frac{2d/7}{6 \cdot \frac{3d}{4}/5} = \frac{2d}{7} \cdot \frac{20}{18d} = \frac{20}{63}$. The required sum is 83.

13. **165** The probability of choosing 2 brown socks is $P(2b) = \binom{3}{6} \binom{2}{5} = \frac{1}{5}$. The probability of choosing 2 white socks is $P(2w) = \binom{2}{6} \binom{1}{5} = \frac{1}{15}$. The probability of choosing 2 gray socks is $P(2g) = 0$. Therefore, the probability of choosing 2 of the same color is $\frac{1}{5} + \frac{1}{15} = \frac{4}{15}$ and the probability that the 2 socks differ in color is $\frac{p}{q} = 1 - \frac{4}{15} = \frac{11}{15}$. The product $pq = 165$. Alternatively, There are 3 ways to select 2 socks with different colors: brown-white, brown-gray, and white-gray. The respective probabilities in that order are $(3/6)(2/5)$, $(3/6)(1/5)$, and $(2/6)(1/5)$. The sum of these is $6/30 + 3/30 + 2/30 = 11/30$. Since order doesn't matter, we double this to arrive at $\frac{p}{q} = 2(11/30) = 11/15$, so $pq = 11 \cdot 15 = 165$.

14. **64** Excluding all the zeroes, each of the vertices will have 4 of the same x -coordinates, 4 of the same y -coordinates, and 4 of the same z -coordinates. The sum of all the non-zero coordinates will be $4(7 + 5 + 4) = 4(16) = 64$.

15. **25** Multiply the left side of the equation by $\frac{a+bi}{a+bi} : \left(\frac{a+bi}{a+bi}\right) \left(\frac{a+bi}{a-bi}\right) = \frac{a^2-b^2+2abi}{a^2+b^2}$. Let $r = a^2 + b^2$, then $a^2 - b^2 = 7$ and $2ab = 24$. If a and b are integers, and the difference of their squares is 7, then $a = 4$ and $b = 3$ and $r = 4^2 + 3^2 = 25$.

16. **58** Let the four quiz scores be a, b, c , and d . Then $a + b + c = 140$, $a + b + d = 124$, $a + c + d = 116$, and $b + c + d = 103$. Add the 4 equations to get $3(a + b + c + d) = 483 \rightarrow a + b + c + d = 161$. The highest grade on Tom's quizzes is $161 - 103 = 58$.

17. **403** From the given equation: $31x - 31y = 13x + 13y \rightarrow 9x = 22y \rightarrow y = \frac{9}{22}x \rightarrow x^2 - y^2 = x^2 - \left(\frac{9}{22}x\right)^2 = \frac{403x^2}{22^2}$. Then, since the GCD of 22 and 403 is 1, the least integral value of $x^2 - y^2 = 403$ and that occurs when $x = 22$. Alternatively, since the GCD of 13 and 31 is 1, set $x - y = 13$ and $x + y = 31 \rightarrow x^2 - y^2 = 13 \cdot 31 = 403$.

18. **34** The area of rectangle $DBPQ$ = the area of rectangle $ABCD$ since both contain $\triangle DBC$ and the sum of the areas of $\triangle DCQ$ and $\triangle BCP$ equals the area of $\triangle DBC$. The area of rectangle $ABCD$ equals $\sqrt{17} \times 2\sqrt{17} = 2 \times 17 = 34$. Thus, the area of rectangle $DBPQ = 34$.

Alternative: Since $\angle ABD$ and $\angle PBC$ are complements of $\angle DBC$ they are congruent to each other and $\triangle ABD \sim \triangle PBC$. It follows that $\frac{BP}{BC} = \frac{BA}{BD}$ so $\frac{BP}{\sqrt{17}} = \frac{2\sqrt{17}}{\sqrt{85}}$ and $BP = \frac{34}{\sqrt{85}}$.

The area of rectangle $DBPQ = BP \times DB = \frac{34}{\sqrt{85}} \times \sqrt{85} = 34$.

19. **4** Use the rules of logarithms to rewrite the given equations in base 10.

$$(\log_{16} 7)x + (\log_3 5)y = \log_4 49 \rightarrow \left(\frac{\log 7}{4 \log 2}\right)x + \left(\frac{\log 5}{\log 3}\right)y = \frac{2 \log 7}{2 \log 2} \text{ and}$$

$(\log_{25} 3)x - (\log_7 2)y = \log_{25} 81 \rightarrow \left(\frac{\log 3}{2 \log 5}\right)x + \left(\frac{\log 2}{\log 7}\right)y = \frac{4 \log 3}{2 \log 5}$. Multiply the first equation by $\frac{\log 2}{\log 7}$ and the second equation by $\frac{\log 5}{\log 3}$ and then add the two results. The equations become $\frac{x}{4} + \left(\frac{\log 2}{\log 7}\right)\left(\frac{\log 5}{\log 3}\right)y = 1$ and $\frac{x}{2} - \left(\frac{\log 5}{\log 3}\right)\left(\frac{\log 2}{\log 7}\right)y = 2$. The sum of these two equations is $\frac{x}{4} + \frac{x}{2} = 3 \rightarrow \frac{3x}{4} = 3 \rightarrow x = 4$.

20. **6** Note that $k^2 - 6k - 27 = (k - 9)(k + 3)$. The two factors will never be equal. The product of two integers is prime if

(1) both are positive, one factor is 1, and the other factor is prime OR

(2) both are negative, one factor is -1 and the absolute value of the other factor is prime.

In the first case, if $k - 9 = 1 \rightarrow k = 10$ and $k + 3 = 13$, and the required product is a positive prime. If $k + 3 = 1$, then $k = -2$ and $k - 9 = -11$, but the required product is not a positive prime.

In the second case, if $k - 9 = -1 \rightarrow k = 8$ and $k + 3 = 11$, and the required product is not a positive prime. If $k + 3 = -1 \rightarrow k = -4$ and $k - 9 = -13$ and the required product is a positive prime.

The only values of k for which the given expression is a positive prime number are 10 and -4 . Their sum is 6.