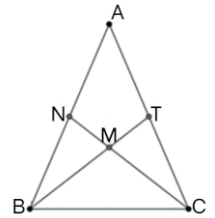
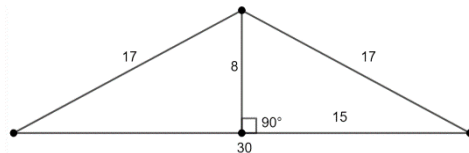


1. **15.** From the given equation, $9x^2 - 2025 = 0 \rightarrow 9x^2 = 2025 \rightarrow x^2 = 225 \rightarrow x = \pm 15$. The positive root is 15.
2. **595.** Let x = the original number of students. It follows that $(x - 75) + 0.2(x - 75) = x + 29 \rightarrow 1.2x - 90 = x + 29 \rightarrow 0.2x = 119 \rightarrow x = 595$.
3. **980.** $\sqrt{n} = \sqrt{20}\sqrt{25} + \sqrt{80} = 2\sqrt{5} \cdot 5 + 4\sqrt{5} = 14\sqrt{5} = \sqrt{196} \cdot \sqrt{5} = \sqrt{196 \cdot 5} = \sqrt{980} \rightarrow n = 980$.
4. **81.** The side lengths of triangle ABC and triangle APQ are in a ratio of 2 to 1. Therefore, the ratio of the areas of these triangles is 4 to 1 and $4 \cdot 20.25 = 81$.
5. **18.** Re-write the given equation as $y = (x + 10)(x - 8)$. Its x -intercepts are $(-10, 0)$ and $(8, 0)$ and $|-10 - 8| = 18$.
6. **16.** $(2x - 1)(x + 4)(2x + 1) = (4x^2 - 1)(x + 4) = 4x^3 + 16x^2 - x - 4$ and the coefficient of the x^2 term is 16.
7. **200.** The three lines intersect at $(0, 0)$, $(8, 200)$, and $(10, 200)$. They form a triangle with a base of 2 and a height of 200. The area of the triangle is $\frac{1}{2}(2)(200) = 200$.

8. **24.** Since triangle ABC is isosceles, $NC = TB$ and point M separates each median in a 1:2 ratio. Then, since $NM = 30 = MT$ and $MB = 60$, $2x + y = 30$ and $4y - 2x = 60 \rightarrow x = 6$ and $y = 18 \rightarrow x + y = 24$.



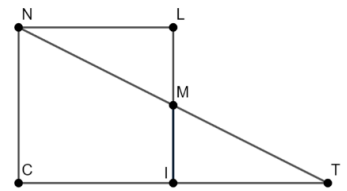
9. **120.** The given triangle is isosceles. Therefore, an altitude separates the triangle into two congruent right triangles, each with a hypotenuse of 17 and a base of 15. The altitude is then 8 (an 8, 15, 17 Pythagorean triple) and the area is $\frac{1}{2}(8)(30) = 120$.



10. **65.** $\frac{25^{20}}{20^{25}} = \frac{(5^2)^{20}}{4^{25}5^{25}} = \frac{5^{40}}{4^{25}5^{25}} = \frac{5^{15}}{(2^2)^{25}} = \frac{5^{15}}{2^{50}} \rightarrow a = 15$ and $b = 50$. The required sum is 65.

11. **15.** The prime factorization of 2025 is $3^4 5^2$. Any factor of 2025 would be of the form $3^a 5^b$ where $0 \leq a \leq 4$ and $0 \leq b \leq 2$. Since there are 5 possible integers for a and 3 for b the total number of factors is 15.

12. **80.** Note that $\triangle LMN \cong \triangle IMT$ by ASA $\rightarrow NM = MT = 10, LM = MI = x \rightarrow NL = IT = 2x \rightarrow x^2 + (2x)^2 = 10^2$ in right $\triangle NLM \rightarrow 5x^2 = 100 \rightarrow x^2 = 20 \rightarrow$ The area of the square $= 4x^2 = 80$.



13. **39.** Note that $\triangle DBC$ is a 30-60-90 triangle whose hypotenuse is 12 and whose legs have lengths: $BC = 6\sqrt{3}$, and $CD = 6$. The area of the rectangle is $6 \cdot 6\sqrt{3} = 36\sqrt{3}$. The required sum is 39.

14. **75.** Note that $CA = \sqrt{(0-3)^2 + (5-2)^2} = 3\sqrt{2}$ and $CA' = \sqrt{(0-9)^2 + (5-(-4))^2} = 9\sqrt{2}$. Thus the scale factor of the dilation is 3. So, $A'B' = 3AB = 3\sqrt{(10-3)^2 + (26-2)^2} = 3\sqrt{7^2 + 24^2} = 3 \cdot 25 = 75$.
15. **161.** Note that for 7 to be a factor of d , n must be a multiple of 7. If $n = 7, 14, 21, 28, 35, 42, 56, 63, 70, 77, 84$, or 91 , then each of these numbers has exactly one half of its factors divisible by 7. For example, if $n = 7$, then its factors are 1 and 7. Exactly one of these is divisible by 7 and the other is not. If $n = 14$, then its factors are 1, 2, 7, and 14. Exactly two of these four are divisible by 7. And so on for the other specified values of n . If $n = 49$, its factors are 1, 7, and 49. Exactly 2 of those 3 are divisible by 7. If $n = 98$, its factors are 1, 2, 7, 14, 49, and 98. Exactly 4 of those 6 are divisible by 7.

The probability we pick $n = 49$ or 98 and then pick d to be a positive divisor of n that is a multiple of 7 is $\frac{2}{3} \left(\frac{2}{100} \right) = \frac{4}{300}$.

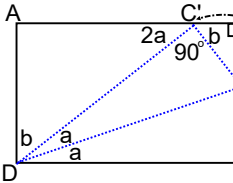
The probability that we pick n to be one of the other 12 multiples of 7 and then pick d to be a positive divisor of n that is a multiple of 7 is $\frac{1}{2} \left(\frac{12}{100} \right) = \frac{6}{100}$.

The total probability in simplest form is $\frac{4}{300} + \frac{6}{100} = \frac{22}{300} = \frac{11}{150}$. The required sum is $11 + 150 = 161$.

Grade Level 10 - NMT 2025

Solutions

- 41.** The area of the vertical or the horizontal parts of letters can be found by counting the boxes contained in those parts. The area contained in the slanted parts of the letters are parallelograms. The area is calculated as follows: $N = 6 + 1(3) + 6 = 15$, $M = 6 + 1(2) + 1(2) + 6 = 16$, $T = 5 + 5 = 10$. So, the total area enclosed by the three letters is 41.
- 60.** Note that 40% of 100 grams plus 70% of 200 grams is 180 grams of cocoa out of 300 grams of liquid chocolate. Then 60% of the mixture is cocoa. Thus $k = 60$.
- 8.** Perform the long division of $100 \div 98 = 1.02040816 \dots$. Thus $b = 8$.
- 6.** Note that $2025 = 3^4 \cdot 5^2$. Each perfect square divisor of 2025 has any of $3^2, 3^4$, or 5^2 as a factor. The divisors are $1, 3^2, 3^4, 3^2 5^2, 3^4 5^2$, and 5^2 . There are 6 of them.
- 15.** During the time that Isabelle runs 10 complete laps, Noah runs 5 complete laps. Thus, while Isabella runs 2 laps, Noah runs 1. During the time that Isabelle runs 2 laps, they meet three times. During the time that Isabelle runs 10 laps, they meet 15 times. Note that this problem can be modeled by a parametric function in the form of $(\cos at, \sin at)$ and $(\cos bt, -\sin bt)$. The total number of intersections for $(0, 2\pi]$ will be $a + b$.
- 19.** In the given system, substitute $x = 1 \rightarrow y + z = 1, -2y - z = 0$, and $4y + 3z = 2$. Add the first two equations to get $y = -1 \rightarrow z = 2 \rightarrow 10c + b = 10 \cdot 2 - 1 = 19$.

7. **72.** Raise the given equation to the sixth power. This result is $n = ((\sqrt{2})(\sqrt[3]{3}))^6 = 2^3 3^2 = 72 \rightarrow n = 72$.
8. **62.** The middle term is the geometric mean between the first and the third term. Therefore, $(x - 4)^2 = 2(x^2 + 8) \rightarrow x^2 - 8x + 16 = 2x^2 + 16 \rightarrow x^2 + 8x = 0 \rightarrow x = 0$, and -8 . Since $x < 0$, $x = -8$, and the terms of the sequence are 2, -12 , and 72. The sum is 62.
9. **539.** Bob wins on the first roll if Abe rolls a 1. The probability of that occurrence is $\frac{1}{6}$. Bob wins on the second roll if Abe rolls a 2 on the first roll and then Abe rolls a number less than Bob. The probability of that occurrence is $\frac{1}{6} \cdot \frac{36-6}{36} \cdot \frac{1}{2} = \frac{5}{72}$. Bob wins on the third roll if Abe rolls a 2 on the first roll, both roll the same number on the second roll, and then Abe rolls a number less than Bob on the third roll. The probability of that occurrence is $\frac{1}{6} \cdot \frac{1}{6} \cdot \left(\frac{36-6}{36} \cdot \frac{1}{2}\right) = \frac{5}{432}$. The required probability is $\frac{1}{6} + \frac{5}{72} + \frac{5}{432} = \frac{107}{432}$. The required sum is 539.
10. **32.** If m and n are the roots of the equation, then $m = \frac{1}{n}$ and $m = 2n \rightarrow 1 = 2n^2 \rightarrow n = \frac{1}{\sqrt{2}}$ and $m = \sqrt{2}$. Since $\frac{b}{\sqrt{c}}$ is the sum of the roots, $\frac{b}{\sqrt{c}} = \frac{1}{\sqrt{2}} + \sqrt{2} = \frac{3}{\sqrt{2}}$. Therefore, $10b + c = 32$.
11. **23.** Let $M(5,5)$ be the midpoint of $\overline{AC}$. Then, $\triangle AMB$ and $\triangle CMB$ are isosceles right triangles. The slope of $\overline{AM} = -\frac{3}{2}$ and the slope of $\overline{BM} = \frac{2}{3}$. Thus, point B could have coordinates $(2,3)$ or $(8,7)$. Since the rotation is given counterclockwise, the coordinates of point B are $(2,3)$.
12. **36.** Label the point on the base, directly below point P as point R . $\overline{QR}$ is the diameter of the circle circumscribed about the base of the hexagonal prism and $QR = 10$ because in a regular hexagon, the length of any side is equal to the length of the circumradius (5). So, the diameter is 10. In right $\triangle PRQ$, $(PQ)^2 = (QR)^2 + (PR)^2 \rightarrow (PQ)^2 = 10^2 + 6^2 \rightarrow (PQ)^2 = 136 \rightarrow PQ = \sqrt{136} \rightarrow PQ = 2\sqrt{34}$ and the required sum is 36.
13. **5.** Since the paper was folded, $DC = DC' = 5$. Then, in right $\triangle ADC'$, $AC' = 4 \rightarrow BC' = 1$. If $BE = x$, then $EC = EC' = 3 - x \rightarrow x^2 + 1 = (3 - x)^2 \rightarrow x^2 + 1 = 9 + x^2 - 6x \rightarrow 6x = 8 \rightarrow x = \frac{4}{3} \rightarrow \text{Area of } \triangle BC'E = \frac{1}{2} \cdot \frac{4}{3} \cdot 1 = \frac{2}{3}$. The required sum is 5.
14. **6.** The first two graphs are $||x| - |y|| \leq 1$, and $||x| + |y|| \leq 2$ respectively. The third graph shows that the intersection of the two regions is a square with vertices $(2,0)$, $(0,2)$, $(-2,0)$, and $(0,-2)$ with four squares removed from the corners.
- 

These can be graphed by graphing the first quadrant only and then reflecting over the x -axis and then the y -axis. The area is $\frac{4 \cdot 4}{2} - 4 \left(\frac{1 \cdot 1}{2} \right) = 6$.

15. **15.** Method 1. Apply the formula: $\sin(A + B) = \sin A \cos B + \cos A \sin B$ to the two non-zero terms in the given equation. The result is $-\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + \cos k \sin x + \cos x \sin k = \sin x \left(\cos k - \frac{1}{2}\right) + \cos x \left(\sin k + \frac{\sqrt{3}}{2}\right) = 0$. The given equation will equal zero only when $\cos k = \frac{1}{2}$ and $\sin k = -\frac{\sqrt{3}}{2}$. Then, $k = \frac{5\pi}{3}$ and $ab = 5 \cdot 3 = 15$.
Method 2. When the difference in phase shifts is π for two sine expressions, the sum is zero. Therefore, $k = \frac{5\pi}{3}$.

Grade Level 11 - NMT 2025

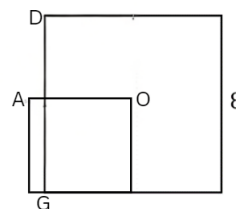
Solutions

1. **32.** Since $x^{2/5}$ is an integer, the value of x will need to be a power of 5. The value of x also needs to be greater than 1, therefore $x = 2^5 = 32$.
2. **648.** From the problem, $6^{x-2} = 3 \rightarrow \frac{6^x}{6^2} = 3 \rightarrow 6^x = 108 \rightarrow 6^{x+1} = 6^x \cdot 6^1 = 108 \cdot 6 = 648$.
3. **3.** The difference between the sum of the first four terms and the first five terms is equal to the fifth term of the sequence a_5 . So $a_5 = 1694 - 569 = 1134$. The difference between the sum of the first five terms and the first six terms is equal to the sixth term of the sequence a_6 . So $a_6 = 5096 - 1694 = 3402$. The common ratio is the ratio of consecutive terms of the sequence. Therefore $r = \frac{a_6}{a_5} = \frac{3402}{1134} = 3$.
4. **57.** Since $\frac{1}{z-2i} = 3 - 4i \rightarrow z - 2i = \frac{1}{3-4i} \rightarrow z - 2i = \frac{1}{3-4i} \left(\frac{3+4i}{3+4i}\right) \rightarrow z - 2i = \frac{3+4i}{25} \rightarrow z = \frac{3+54i}{25}$.
Therefore, the required sum is $25 \left(\frac{3}{25} + \frac{54}{25}i\right) = 25 \left(\frac{57}{25}\right) = 57$.
5. **108.** Using either long division or synthetic division, the equation of the slant asymptote is $y = x + 6$. $\rightarrow 3x + 2(x + 6) = 522 \rightarrow 3x + 2x + 12 = 522 \rightarrow 5x = 510 \rightarrow x = 102$. $\rightarrow y = 102 + 6 \rightarrow y = 108$.
6. **12.** The second equation is equivalent to $\sin(x) = \sin(y)$, $\rightarrow y = x$ or $y = \pi - x$. If $y = x$, $2 \cos(x) = 1 \rightarrow \cos(x) = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$. The two solutions are $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ or $\left(\frac{5\pi}{3}, \frac{5\pi}{3}\right)$. The positive difference is $\frac{5\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$ and the desired product is $4 \cdot 3 = 12$.
If $y = \pi - x$, $\cos(x) + \cos(\pi - x) = 1$. This is false for all values of x because $\cos(x) + \cos(\pi - x) = 0$ is an identity. So, there are no additional solutions.
7. **42.** Let $x = 2024$ and $y = \frac{1}{2024} \rightarrow f\left(2024 \cdot \frac{1}{2024}\right) = f(2024) + f\left(\frac{1}{2024}\right) \rightarrow f(1) = f(2024) + f\left(\frac{1}{2024}\right)$. Let $x = 1$ and $y = \frac{1}{2024} \rightarrow f\left(1 \cdot \frac{1}{2024}\right) = f(1) + f\left(\frac{1}{2024}\right) \rightarrow f\left(\frac{1}{2024}\right) = f(1) + f\left(\frac{1}{2024}\right) \rightarrow f(1) = 0 \rightarrow 0 = f(2024) - 42 \rightarrow f(2024) = 42$. Alternatively, use the property of the given function to determine that the given function is a log function $\rightarrow \log\left(\frac{1}{2024}\right) = \log 1 - \log 1024 \rightarrow -42 = 0 - \log 2024 \rightarrow \log 2024 = 42$.

8.240. Since $\overline{CD}$ is a diameter of the circle with midpoint E , $CE = ED = 4$. Since $\angle CAD$ is inscribed in a semicircle, $m\angle CAD = 90^\circ$, and $m\angle ADC = 60^\circ$. Since $\angle C$ is an inscribed angle and intercepts $\widehat{AD}$, $m\widehat{AD} = 60^\circ$. Then $m\angle AED = 60^\circ$ since it is a central angle that intercepts $\widehat{AD}$. Therefore, $\triangle AED$ is an equilateral triangle and $AD = AE = 4$. Then $m\widehat{AC} = 180^\circ - m\widehat{AD} = 180^\circ - 60^\circ = 120^\circ$. Then, $\angle B$ has a vertex exterior to the circle and is a tangent-secant angle, so $m\angle B = \frac{120^\circ - 60^\circ}{2} = 30^\circ$. $\angle ADB$ is supplementary to $\angle ADC$. So, $m\angle ADB = 120^\circ$ and $\triangle ADB$ is an isosceles triangle. This means $BD = 4$. Drop an altitude from vertex D in $\triangle ADB$ to form two $30^\circ - 60^\circ - 90^\circ$ triangles $\rightarrow AB = 4\sqrt{3}$. In $30^\circ - 60^\circ - 90^\circ \triangle ADC$, $AC = 4\sqrt{3}$. Therefore, the required sum of $(AB)^2 + (AC)^2 + (BC)^2 = (4\sqrt{3})^2 + (4\sqrt{3})^2 + (12)^2 = 48 + 48 + 144 = 240$.

9. 55. Let x represent the number of minutes it takes them to mow 1 lawn. Then $\frac{x-15}{120} + \frac{x}{180} = 1 \rightarrow 180(x-15) + 120x = 21600 \rightarrow 300x - 2700 = 21600 \rightarrow 300x = 24300 \rightarrow x = 81$ minutes. Mr. Wiles work rate is $\frac{1}{120}$ lawns per minute, and it took him $81 - 15 = 66$ minutes to mow the lawn. So, $\frac{1}{120}(66) = .55 = 55\%$. Then $k = 55$.

10. 5. Rotate the smaller square about the center of the larger square until its sides are parallel to the sides of the larger square. Then the overlapping area will be a 4 by 4 square with area of 16. The overlapping area is a square because two of its pairs of sides are parallel to the sides of the square and each other and angles G and O are right angles. Further, because point O is at the center of the larger square, $\overline{OA}$ goes through the midpoint of side $\overline{DG}$ of the larger square. Then, $0.64x^2 = 16 \rightarrow x = 5$.



11.252. Note that $f(f(x)) = (((x+16)^2 - 16) + 16)^2 - 16 = ((x+16)^2)^2 - 16 = (x+16)^4 - 16 \rightarrow (x+16)^4 - 16 = 0 \rightarrow (x+16)^4 = 16 \rightarrow x = -14$ or $x = -18$. The desired product is $-14 \cdot -18 = 252$.

12. 11. The function $g(x) = f(x+3)$ translates $y = f(x)$ three units to the left, so $f(-5) = 8 \rightarrow g(-8) = 8$. For an even function $g(x) = g(-x)$, so $g(-8) = g(8) = 8$. Now translate $y = g(x)$ back to the original function $y = f(x)$ by translating three units to the right. So $g(8) = 8 \rightarrow f(11) = 8$. The required value of x is 11.

13. 1. The given equation may be re-written as: $\frac{\log(x+2)}{\log 3} = \frac{\log(x+8)}{\log 9} \rightarrow \frac{\log(x+2)}{\log 3} = \frac{\log(x+8)}{\log 3^2} \rightarrow \frac{\log(x+2)}{\log 3} = \frac{\log(x+8)}{2 \log 3} \rightarrow 2 \log(x+2) = \log(x+8) \rightarrow \log(x+2)^2 = \log(x+8) \rightarrow (x+2)^2 = x+8$. Expanding on the left side of the equation, $x^2 + 4x + 4 = x + 8 \rightarrow x^2 + 3x - 4 = 0 \rightarrow (x-1)(x+4) = 0$. Therefore, the solutions are $x = 1$ or $x = -4$. But since the domain of the original equation is $x > -2$, only $x = 1$ is a valid solution. Alternatively, use the fact that $\log_b a = \log_{b^2} a^2 \rightarrow (x+2)^2 = x+8 \dots$

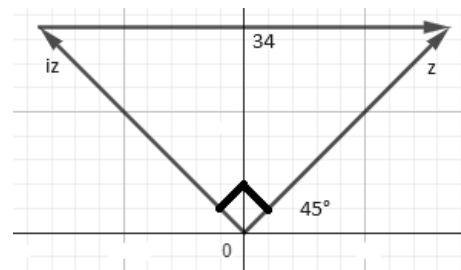
14.126. Since a, b , and c are roots of the equation, $a^3 - 24a - 42 = 0$, $b^3 - 24b - 42 = 0$ and $c^3 - 24c - 42 = 0 \rightarrow a^3 + b^3 + c^3 - 24(a+b+c) - 126 = 0$. The sum of the roots of the given cubic equation is 0 $\rightarrow a + b + c = 0$. Therefore, the required sum is $a^3 + b^3 + c^3 = 126$.

- 15.469. There are $5! = 120$ arrangements that start with each of the letters B, E , and M for a total of 360. There are $4! = 24$ arrangements that start with each of the pairs of first two letters NB, NE, NM , and NR for a total of 96. There are $3! = 6$ arrangements that start with each of the triples of the first three letters NUB and NUE , for a total of 12. The word $NUMBER$ is next in the list. It is in the position number $x = 360 + 96 + 12 + 1 = 469$.

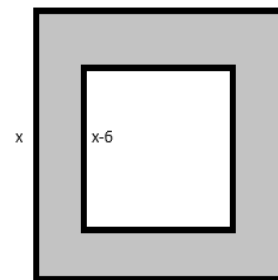
Grade Level 12 - NMT 2025

Solutions

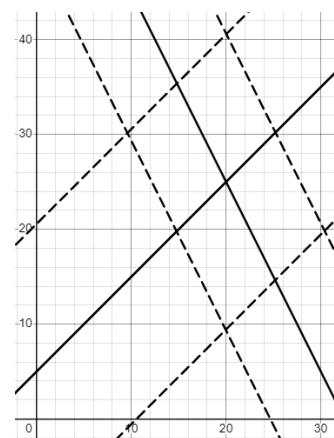
1. 50. Abby does $\frac{1}{10}$ of the job per hour. Together they do the job in $\frac{25}{3}$ hours. So, they do $\frac{3}{25}$ of the job per hour. Bobby alone does $\frac{3}{25} - \frac{1}{10} = \frac{1}{50}$ of the job per hour. So, Bobby would take 50 hours to finish alone.
2. 147. Since $f(x) = x^3 \cos x \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = x^3$, $f'(x) = 3x^2$ and so $f'(7) = 3(7)^2 = 147$.
3. 91. Since $|y| = y$ if and only if $y \geq 0$, the equation reduces to $2025 - x^2 \geq 0 \rightarrow x^2 \leq 2025 \rightarrow -45 \leq x \leq 45$, which has 91 integer solutions.
4. 13. Complete the square: $x^2 - 26x + 169 + y^2 + 12y + 36 = 0$
 $(x - 13)^2 + (y + 6)^2 = 0$
 Since both squares are nonnegative, each of the squares must be zero. Alternatively, this is a circle of radius 0, i.e., a point. Either way, the only solution is $(x, y) = (13, -6)$. Thus, $x = 13$.
5. 25. Let $f(x) = \sqrt{x} - x$, represent the increase in a student's grade using the curve, with $0 \leq x \leq 1$. Then $f'(x) = \frac{1}{2\sqrt{x}} - 1 = 0 \rightarrow \sqrt{x} = \frac{1}{2} \rightarrow x = \frac{1}{4} = 25\% \rightarrow k = 25$.
6. 187. Method 1 (graphing): $f(z) = iz + 34$ will rotate z by 90° counterclockwise about the origin and then translate it 34 units to the right. If $f(z) = z$, z must be configured as shown at the right. Since z and iz are equal in magnitude, the vector triangle is an isosceles right triangle. Therefore, the angle between vector z and the x -axis is a 45° angle, and $a = b$. Since the magnitude of the hypotenuse of the triangle is 34, $a = 17$, and $b = 17$. Therefore, $10a + b = 187$.
 Method 2 (brute-force algebra): From the given: $f(a + bi) = i(a + bi) + 34 = 34 - b + ai = a + bi$. Then, $34 - b = a$ and $ai = bi$. The latter equation implies that $a = b$ and so $34 - a = a \rightarrow a = 17 = b$.
7. 91. $f'(x) = xe^x + e^x = (x + 1)e^x$
 $f''(x) = (x + 1)e^x + e^x = (x + 2)e^x$
 In general (proof omitted), $f^{(n)}(x) = (x + n)e^x$. So, $f^{(47)}(x) - f^{(34)}(x) = (x + 47)e^x - (x + 34)e^x = 13e^x$. Therefore, $f^{(47)}(\ln 7) - f^{(34)}(\ln 7) = 13e^{(\ln 7)} = 13 \cdot 7 = 91$.
8. 255. Re-write the given using a log property as $\log_2(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_k(k + 1)) = 3 \rightarrow \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_k(k + 1) = 8 \rightarrow \frac{\ln 3}{\ln 2} \cdot \frac{\ln 4}{\ln 3} \cdot \frac{\ln 5}{\ln 4} \cdots \frac{\ln(k+1)}{\ln k} = 8 \rightarrow \frac{\ln(k+1)}{\ln 2} = 8 \rightarrow \log_2(k + 1) = 8 \rightarrow k + 1 = 256 \rightarrow k = 255$.



9. **336.** Region R is the region shaded at right, within 3 cm of S, which is the area between a square of side length x and a square of side length $x - 6$. Let A be the area of R, so $A = x^2 - (x - 6)^2 = x^2 - (x^2 - 12x + 36) = 12x - 36$
 $\frac{dA}{dt} = 12 \frac{dx}{dt} = 12(28) = 336$.
 Note: the fact that $x = 45$ is irrelevant.

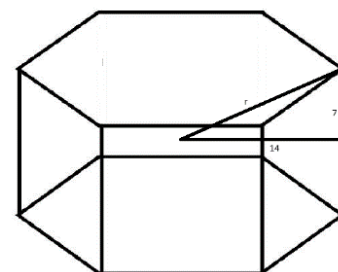


10. **180.** The graph at right shows the two given lines (as solid lines) and the sets of points the specified distances from the given lines (as dashed lines). The four desired points are the points of intersection of the dashed lines. Since the dashed lines are parallel to the given lines, the four points form the vertices of a parallelogram. The center of this parallelogram is the intersection of the two given lines, which is the point (20,25). Since the diagonals of the parallelogram bisect each other, the coordinates of the center are the average of the coordinates of the vertices, and so the x -coordinates of each diagonal's endpoints add to 40 and the y -coordinates of each diagonal's endpoints add to 50. Because there are 2 diagonals, the coordinates of all 4 vertices add to $2(40 + 50) = 180$.

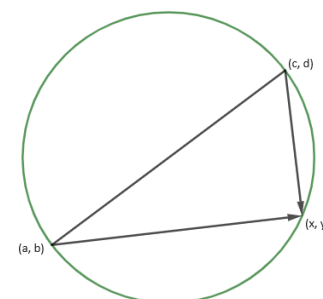


11. **17.** Method 1 (direct): There are $6^3 = 216$ possible outcomes. To have one pair match, we must first pick which 2 dice match, (in $\binom{3}{2} = 3$ ways), then pick which number is the pair (6 ways), and finally which number is the odd one out (5 ways). This gives us a total of $3 \cdot 6 \cdot 5 = 90$ successful outcomes. So, the answer is $\frac{90}{216} = \frac{5}{12}$, and $5 + 12 = 17$.
 Method 2 (indirect): Start from 216 and subtract combinations that don't work: 6 ways for all three numbers to match, and $6 \cdot 5 \cdot 4 = 120$ ways for all three numbers to be different. Then $216 - 6 - 120 = 90$, and proceed as above.

12. **980.** In the figure at the right, the center of the prism is connected to a vertex of the prism forming a radius r of the sphere. Since a regular hexagon can be broken into 6 equilateral triangles, the horizontal leg of the right triangle shown has length 14. Since the height of the prism is 14, the vertical leg of the right triangle has length 7. By the Pythagorean Theorem, $r^2 = 14^2 + 7^2 = 245$. The surface area of the sphere is then $4\pi r^2 = 4\pi(245) = 980\pi$.



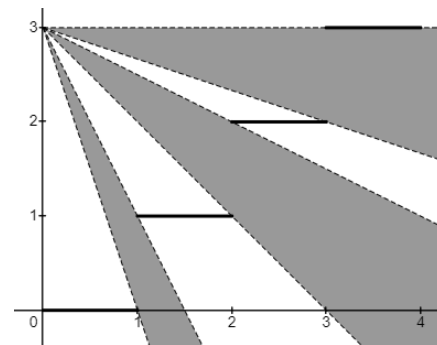
13. **260.** Method 1 (algebraic): Multiply out (without a calculator!) and complete the square:
 $x^2 + 50x - 1875 + y^2 + 160y - 8000 = 0$
 $x^2 + 50x + y^2 + 160y = 9875$
 $x^2 + 50x + 625 + y^2 + 160y + 6400 = 16900$
 $(x + 25)^2 + (y + 80)^2 = 130^2$
 So, $r = 130$ and the diameter is 260.



Method 2 (vector): Let (a, b) and (c, d) be the endpoints of a diameter of a circle and let (x, y) be any point on the circle. The two vectors shown at right have components $\langle x - a, y - b \rangle$ and $\langle x - c, y - d \rangle$. Since the angle formed by the vectors is inscribed in a semicircle, the vectors are perpendicular, and their dot product is 0. So, the equation $(x - a)(x - c) + (y - b)(y - d) = 0$ describes the circle. Comparing this to

the problem shows that $(-75, -200)$ and $(25, 40)$ are endpoints of the diameter. The length of the diameter is then $\sqrt{100^2 + 240^2}$, which is 20 times a 5 – 12 – 13 triangle. So, the length of the diameter is $13 \cdot 20 = 260$.
(Note: endpoints $(-75, 40)$ and $(25, -200)$ also work and give the same answer.)

14. **550.** The graphs of the two functions must not intersect. The graph of $y = \lfloor x \rfloor$ is shown and is discontinuous. For the graphs not to cross, the line, with a y – intercept of 3, must go between the segments of the floor graph. That means that the lines must lie in one of the shaded regions shown. For that to happen, b must be in the set $(0, \frac{1}{3}] \cup (\frac{1}{2}, 1] \cup (2, 3]$. Drawn on a number line, this set has total length $\frac{1}{3} + \frac{1}{2} + 1 = \frac{11}{6}$, and $300k = 300(\frac{11}{6}) = 550$.



15. **288.** Work indirectly by computing $1 - p(n)$, the probability that n is skipped in the sequence. In order to skip n , either:

- $n - 1$ is in the sequence and $k = 2, 3$, or 4
- $n - 2$ is in the sequence and $k = 3$ or 4
- $n - 3$ is in the sequence and $k = 4$

So, $1 - p(n) = \frac{3}{4}p(n - 1) + \frac{2}{4}p(n - 2) + \frac{1}{4}p(n - 3)$

Let $p = \lim_{n \rightarrow \infty} p(n)$, which also equals $\lim_{n \rightarrow \infty} p(n - 1)$, $\lim_{n \rightarrow \infty} p(n - 2)$, and $\lim_{n \rightarrow \infty} p(n - 3)$. So, taking the limit of both sides of the above equation,

$$1 - p = \frac{3}{4}p + \frac{2}{4}p + \frac{1}{4}p \rightarrow 1 = \frac{10}{4}p \rightarrow p = \frac{2}{5} \text{ and so } 720 \left(\frac{2}{5} \right) = 288.$$

Note: We can also get this by considering that, on average, any 4 choices of k will produce a 1, 2, 3, and 4, meaning that, on the average, every time n increases by 4, a_n will increase by 10 and so, in the long run, 4 out of every 10 numbers will be in the sequence.

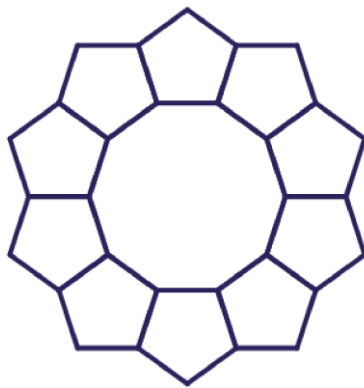
Mathletics - NMT 2025

Solutions

- 780.** The function defines an arithmetic sequence whose first term is 2025 and whose common difference is -5. Then, $f(n) = f(1) + d(n - 1) \rightarrow f(250) = 2025 - 5 \cdot 249 = 780$.
- 78.** Compute the energy supplied in kWh: $\left(\frac{1}{6}\text{h}\right)(300\text{kW}) + \left(\frac{2}{60}\text{h}\right)(60\text{kW}) = 52\text{kWh}$. This 52 kWh of energy added to the battery represents 67% of the battery capacity. If x represents the car's battery capacity, then $0.67x = 52 \rightarrow x \approx 78$.
- 30.** Note that $2025 - 1804 = 221 = 13 \cdot 17$. So, the two cycles must be 13 and 17 years. The required sum is 30.
- 30.** If the three integers are $x - 1$, x , and $x + 1$, then $\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1} = \frac{229}{990}$. This can be solved graphically and $x = 10$.
Alternatively, we know that x is an integer and the given fraction is in lowest terms. Note that $990 = 9 \cdot 10 \cdot 11$, the product of three consecutive integers! Try $x = 10$ and verify that $\frac{1}{9} + \frac{1}{10} + \frac{1}{11} = \frac{299}{990}$.

5. **54.** By direct substitution, $f(30,19) = \frac{49}{2} + \frac{11}{2} = 30 \rightarrow f(30,41) = \frac{71}{2} + \frac{11}{2} = 41 \rightarrow f(41,54) = \frac{95}{2} + \frac{13}{2} = 54$. Then, notice that the given $f(a,b)$ is a formula for the maximum of two inputs!
- $$f(a,b) = \begin{cases} a, & a > b \\ b, & b \geq a \end{cases}$$

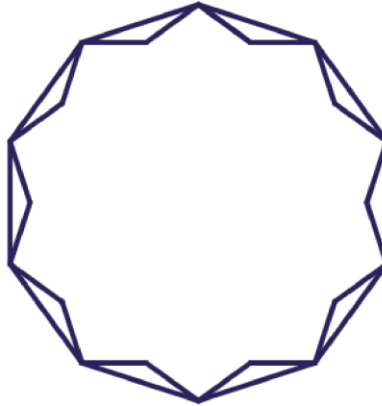
6. **9.** In a right triangle, the length of an altitude to the hypotenuse is the mean proportional between the segments on the hypotenuse. Thus, $EB = 20$. Use the Pythagorean Theorem in right $\triangle AEB \rightarrow AE = 20\sqrt{5}$. Then, $BD = \frac{1}{5}AE = 4\sqrt{5}$ because the ratio of the corresponding sides of $\triangle BDC$ to the corresponding sides of $\triangle AEC$ is 1 to 5. The required sum is 9.
7. **23.** Since the 4th degree polynomial has integer coefficients whose roots have a sum of zero, the solution set must consist of two conjugate pairs: $\sqrt{5} \pm \sqrt{2}$ and $-\sqrt{5} \pm \sqrt{2}$. Using the sum and product of the roots, the two quadratics that yield these roots are $x^2 - 2\sqrt{5}x + 3$ and $x^2 + 2\sqrt{5}x + 3$. The product of these quadratics is $x^4 - 14x^2 + 9 \rightarrow x^4 - 14x^2 + 0x + 9 \rightarrow a = 14, b = 0$, and $c = 9$. The required sum is 23.
8. **55.** There are 9 ways to form a group that places the three friends together. Since there are $\binom{12}{4} = 495$ ways to choose 4 from a group of 12, the required probability is $\frac{9}{495} = \frac{1}{55}$. Thus $n = 55$.
9. **136.** Note that all the terms have the form $\binom{2025}{a} (x^3\sqrt{3})^a (y^5\sqrt{5})^b$ where $a + b = 2025$. For this term to be rational, a must be a multiple of 3 and b must be a multiple of 5. Since both $a + b$ and b are multiples of 5, a must also be a multiple of 5. Thus, a is a multiple of both 3 and 5. Thus, the values of a are 0, 15, 30, ..., 2025. These values form an arithmetic sequence with a common difference of 15, first term of 0, and n th term of 2025. Then, $a_n = a_1 + d(n - 1) \rightarrow 2025 = 0 + 15(n - 1) \rightarrow 135 = n - 1 \rightarrow n = 136$.
10. **180.** Solution 1: Bisect each of the 144° angles. The result is interior angles each measuring 108° . This suggests that ten pentagons can be drawn as follows:



Call the side length x . Compute the area of each pentagon by drawing 5 isosceles triangles each with a vertex at the center of the pentagon. In each triangle, draw an altitude to the base. The length of the altitude is $\frac{1}{2}x \tan 54^\circ$. Then the area of each pentagon is $5 \cdot \frac{1}{2}x \cdot \frac{1}{2}x \tan 54^\circ = \frac{5}{4}x^2 \tan 54^\circ$. Compute the area of the enclosed decagon in a similar way. It is the sum of the areas of ten isosceles triangles each with a base of x and a height of $\frac{x}{2} \tan 72^\circ$. The area of the

decagon is $\frac{5}{2}x^2 \tan 72^\circ$. The area of the 20-sided polygon is $10 \cdot \frac{5}{4}x^2 \tan 54^\circ + \frac{5}{2}x^2 \tan 72^\circ = 2025 \rightarrow x \approx 9.018 \rightarrow 20x \approx 180$.

Solution 2: Connect every other vertex to create a circumscribed regular decagon as depicted here:

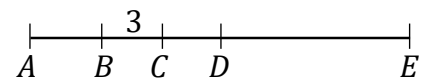


The circumscribed decagon includes ten isosceles triangles that are not included in the area of the original 20-sided polygon. These triangles have side lengths, x, x , and $2x \cos 18^\circ$ and area $x^2 \sin 18^\circ \cos 18^\circ$. The area of the enclosed decagon is $\frac{5}{2}(2x \cos 18^\circ)^2 \tan 72^\circ$. Subtract from that to get the area of the 20-sided polygon: $\frac{5}{2}(2x \cos 18^\circ)^2 \tan 72^\circ - 10x^2 \sin 18^\circ \cos 18^\circ = 2025 \rightarrow x \approx 9.018 \rightarrow 20x \approx 180$.

Team Problem Solving - NMT 2025

Solutions

1. **45.** Evaluate $(3 \Delta 2) = (3 + 2) + (3)(2) - 2 = 5 + 6 - 2 = 9$.
Then evaluate $(9 \Delta 4) = (9 + 4) + (9)(4) - 4 = 13 + 36 - 4 = 45$.
2. **6.** The number of positive divisors of 45 is based on prime factors of 45. Since $45 = 3^2 \times 5$, there will be $(2 + 1) \times (1 + 1) = 6$ positive divisors.
3. **40.** Let C = the original price in dollars, then, solve $(0.75)(0.85)C = 25.50 \rightarrow C = 40$.
4. **100.** Let p, n, d, q , and h be the number of pennies, nickels, dimes, quarters, and half dollars respectively. Then $n = \frac{q}{2}, d = \frac{3q}{4}, h = \frac{q}{2} + \frac{3q}{4} = \frac{5q}{4}$, and $p = \frac{5q}{2}$. The total value of the coins is \$40. Solve $1\left(\frac{5q}{2}\right) + 5\left(\frac{q}{2}\right) + 10\left(\frac{3q}{4}\right) + 25(q) + 50\left(\frac{5q}{4}\right) = 4000$ for $q \rightarrow 5q + 5q + 15q + 50q + 125q = 8000 \rightarrow q = 40$ and $p = 100$.
5. **27.** Expand $(x - 4)^3 = x^3 + 3x^2 \cdot -4 + 3x \cdot (-4)^2 + (-4)^3 = 1x^3 - 12x^2 + 48x - 64$. The sum of the coefficients is -27. The absolute value of that sum is 27. Alternatively, replace x with 1 in the given: $(1 - 4)^3 = (-3)^3 = -27$ and its absolute value is 27.
6. **105.** Draw the diagram. Since $\frac{AB}{BE} = \frac{2}{3}, AB = \left(\frac{2}{5}\right)AE$ and since $\frac{AC}{CE} = \frac{3}{4}, AC = \left(\frac{3}{7}\right)AE$. We know that $BC = \sqrt[3]{A_5} = \sqrt[3]{27} = 3$. So, $AC - AB = 3 \rightarrow \left(\frac{3}{7} - \frac{2}{5}\right)AE = 3 \rightarrow AE = 105$.

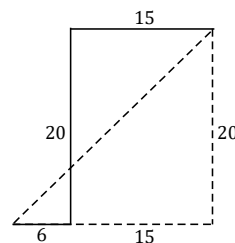


7. 27. Since $4 \times K$ is less than 10, K is either 1 or 2. Since $4 \times S = K$, K must be even, so $K = 2$. Now S can be only either 3 or 8 since 4×3 and 4×8 have a units digit of 2. Since $4 \times K = S$, S must be 8. Consider that $4 \times N$ must be less than 10, so N equals 0 or 1. N cannot be 0 since $4 \times T + 3 = N$ and $4 \times T + 3$ must be odd. Hence $N = 1$, $T = 7$, and $I = 9$. Finally, $8 + 7 + 9 + 1 + 2 = 27$.

$$\begin{array}{r} 2 \text{ N I T S} \\ \times 4 \\ \hline \text{S T I N 2} \end{array} \quad \begin{array}{r} 2 \text{ N I T 8} \\ \times 4 \\ \hline 8 \text{ T I N 2} \end{array} \quad \begin{array}{r} 2 \text{ 1 I T 8} \\ \times 4 \\ \hline 8 \text{ T I 1 2} \end{array} \quad \begin{array}{r} 2 \text{ 1 I 7 8} \\ \times 4 \\ \hline 8 \text{ 7 I 1 2} \end{array} \quad \begin{array}{r} 2 \text{ 1 9 7 8} \\ \times 4 \\ \hline 8 \text{ 7 9 1 2} \end{array}$$

8. 2. Simplify $\log_9(A_7) \times \log_{(A_7)} 3 = \log_9(27) \times \log_{27} 3 = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$ and the reciprocal of $\frac{1}{2}$ is 2.

9. 29. Consider the given diagram. Use the Pythagorean theorem to find the distance from start to end: $\sqrt{21^2 + 20^2} = \sqrt{841} = 29$.

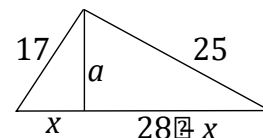


10. 7. Note that $29 = 2 \cdot 12^1 + 5 \cdot 12^0$. So, $29_{10} = 25_{12}$, and $2 + 5 = 7$.

11. 152. Call the numbers x and $y \rightarrow x + y = 8$ & $x^2 + y^2 = 34 \rightarrow y = 8 - x$ & $x^2 + (8 - x)^2 = 34 \rightarrow x^2 + 64 - 16x + x^2 = 34 \rightarrow 2x^2 - 16x + 30 = 0 \rightarrow 2(x - 3)(x - 5) = 0 \rightarrow x = 3$ or $x = 5 \rightarrow (x, y) = (3, 5)$ or $(5, 3)$. Either way, the sum of the cubes of these numbers is 152.

12. 5. Since $152 = 2^3 \cdot 19$. The prime factors of 152 are $p = 2$ and $q = 19$. Since the maximum value of $\sin x$ is 1, the maximum value for $f(x) = \sqrt{(p \sin x)^3 + q - 2}$ is $\sqrt{2^3 + 19 - 2} = \sqrt{25} = 5$.

13. 15. The shortest altitude must be drawn to the longest side since their product determines the area of the triangle. Apply the Pythagorean theorem: $a = \sqrt{17^2 - x^2} = \sqrt{25^2 - (28 - x)^2} \rightarrow 289 - x^2 = 625 - (784 - 56x + x^2) \rightarrow 56x = 448 \rightarrow x = 8 \rightarrow a = 15$. Alternatively, since the answer must be an integer, consider the common Pythagorean triples such as 3-4-5, 5-12-13, and 8-15-17. When $x = 8$ and $a = 15$, $28 - x = 20$. Another method equates the area of the triangle using Heron's formula and one-half base times height.



14. 0. $K = 123456789101112131415$. Since the number ends in 5 and the sum of the digits is a multiple of 3 (using casting out 3's), K is divisible by 15. Thus, the remainder is 0.

15. 7. Since $\sin x = 2 \cos x$, $\tan x = 2$. Consider the right triangle with legs of 1 and 2 so the hypotenuse is $\sqrt{5}$. Then $\sin x \cos x = \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{2}{5}$ and $2 + 5 = 7$.

16. 11. Notice that $\triangle APB \cong \triangle CQD$ and that $\triangle BPC \cong \triangle DQA$. Based on the labels in the diagram $AD = 11$ and $AB = y + 6$, so the area of the rectangle, denoted by $[ABCD]$ is $11(y + 6)$. Then $[APCQ] = [ABCD] - 2\{[APB] + [BPC]\}$. $[APCQ] = 11(y + 6) - 2\left\{\frac{3(y+6)}{2} + \frac{(6)(11)}{2}\right\} = 11y + 66 - \{18 + 3y + 66\} = 8y - 18$. We were given that $[APCQ] = 10 \times A_{15} = 10 \times 7 = 70$ so $8y - 18 = 70 \rightarrow y = 11$. Alternatively, the area of the parallelogram can be determined by what is often called the shoestring method. Then it follows that $y = 11$.

