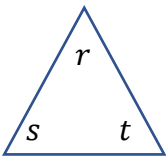
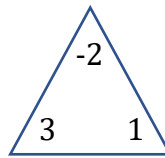


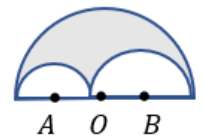
- Four integers are in the ratio 3:5:7:9. If the mean of these integers is 24, compute the largest of these four integers.
- If the sum of $3\sqrt{12}$, $\sqrt{27}$, and $\sqrt{75}$ is expressed as $a\sqrt{b}$ in simplest radical form, compute $a + b$.
- The difference between $5x^2 - 3x - 11$ and $-3x^2 - 4x + 7$ is expressed in simplest $ax^2 + bx + c$ form. Compute $|a + b + c|$.
- Compute the sum of the roots of $3x^2 = 12x - 5$.
- In isosceles triangle ABC , $\angle C$ is the vertex angle. If $m\angle ABC = (2x + 10)^\circ$ and $m\angle C = (x^2)^\circ$, compute the number of degrees in $m\angle C$.
- Two lines whose equations are $2x - 3y = -1$ and $ax + 8y = 4$ are perpendicular to each other. Compute a .
- For all integers, define



$$= |r^s - s^t + t^r|. \quad \text{Compute}$$



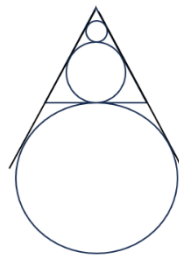
- The score on a standardized test containing exactly 80 questions is determined by subtracting 20% of the number of incorrect answers from the number of correct answers. If Aesha answered every question and scored a 38, compute the number of questions that she answered correctly.
- In the accompanying figure, semicircles A and B are tangent to each other and to circle O . The radius of circle A is 4 centimeters and the radius of circle B is 6 centimeters. If the area of the shaded region is expressed in simplest form as $a\pi$ square centimeters, compute a .



- Compute the units digit of the following product: $23^{97} \cdot 17^{98} \cdot 29^{92}$.
- Compute the perimeter of a rectangle if its area is 18 square inches and the length of its diagonal is 8 inches.
- Compute the number of positive integers that are divisors of 15,015.
- Akash's age is 16 more than the sum of Ben's and Chia's ages. The square of Akash's age is 1,632 more than the square of the sum of Ben's and Chia's ages. Compute the sum of the ages of all three people.
- Compute the area of the region enclosed by the graph of the equation $|x| + |y| = 3\sqrt{5}$.
- Three cubes are glued together on their faces. The edges of the cubes are 3 centimeters, 5 centimeters, and 9 centimeters. Compute the minimum possible number of square centimeters in the exposed surface area of the resulting figure.

1. Compute the number of arrangements of the letters in the word SALE if no repetition of letters is allowed.
2. In circle O , the length of diameter $\overline{AB}$ is 10. If point C is any point on circle O , compute the maximum possible area of $\triangle ABC$.
3. A geometric sequence of positive numbers is denoted by a_n . If $a_3 = 64$ and $a_7 = 4$, then a_{15} may be expressed in simplest form as $\frac{a}{b}$. Compute $a + b$.
4. If $f(x) = x^2 - 4x + 7$, compute the minimum possible value of $f(x)$.
5. Compute the value of k^2 , if x and y are positive integers, $f(x + y) = f(x) \cdot f(y)$, $f(2) = 2$, and $f(5) = k$.
6. Circle O is centered at the origin and contains points $A(\sqrt{3}, -1)$ and $B(-1, \sqrt{3})$. If $m < \angle AOB = \theta$ and $\tan \theta = k$, compute $2025k^2$.
7. Joe walked 1 mile to his bus stop where the bus was waiting. He then rode the bus for 8 miles. If his total travel time was 18 minutes and the bus's average speed was 54 mph faster than Joe's speed of walking, compute the bus's average speed.
8. Claire is trying to determine a passcode. It is a four-digit code and the number that it forms is prime. Each digit is an odd integer from 0 through 9. There is no repeating digit, and the digits increase from left to right. Compute the three-digit integer that forms the first three digits of the passcode.
9. Compute the real component of $(1 - i)^9$.
10. Given $y = f(x)$ is a quadratic function with $f(1) = 8$, $f(2) = 15$, $f(3) = 28$, and $f(4) = 47$. Compute $f(6)$.
11. Given that $P(x)$ is a fifth-degree polynomial function with $P(1) = 4$ and $P(3) = -4$. The remainder when $P(x)$ is divided by $x^2 - 4x + 3$ can be expressed as $ax + b$ where a and b are integers. Compute $a + 10b$.
12. The following equation: $\frac{\sqrt{3}}{\sin x} + \frac{1}{\cos x} = 4$ has two solutions on the interval: $\left[0, \frac{\pi}{2}\right]$. If the sum of the two solutions can be expressed as $\frac{a\pi}{b}$ where a and b are relatively prime, compute $a + b$.
Note that $\sin(A + B) = \sin A \cos B + \sin B \cos A$ and that $\sin(2A) = 2 \sin A \cos A$.
13. Compute $a + b + c + d$ if the solution of the following system of equations is the ordered pairs (a, b) and (c, d) : $|x - 4| + |y - 3| = 2$ and $x = |y - 3| + 4$.
14. In the xy -plane, points A, B and C are located at $(1, 0)$, $(4, 0)$, and (x, y) respectively, with $CB:AC = 2$. The set of all points (x, y) that satisfy the given condition determine a closed region. To the nearest integer, compute the area of the closed region.

15. A circle is inscribed in a triangle with side lengths of 13, 13, and 10. A second smaller circle is inserted tangent to the congruent sides of the triangle and tangent to the first circle. A third and bigger circle is tangent to the first circle and tangent to the extensions of the congruent sides of the triangle. The ratio of the radius of the smallest of the three circles to the radius of the largest of the three circles may be expressed in simplest form as $\frac{a}{b}$. Compute $a + b$.



Grade Level 11 - NMT 2024

Questions

1. The math club surveyed 300 students who own a calculator about whether they own a scientific calculator, a graphing calculator or both. They found that four out of every nine students who own a graphing calculator also own a scientific calculator. If 225 students own a graphing calculator, compute the number of students that own a scientific calculator.

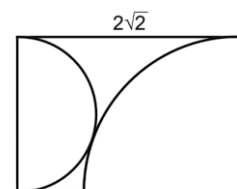
2. Compute the smallest positive integer x such that $\frac{4^x}{8^{12}}$ is the square of an integer.

3. The area of an equilateral triangle can be expressed as $a\sqrt{b}$, where b has no perfect square factors. If the line segment connecting the midpoints of two sides of the triangle has a length of 7, compute the product ab .

4. A sequence of 123 consecutive even integers has a median of 6. Compute the sum of the sequence.

5. Consider the rational function $f(x) = \frac{3x^2 + 5x - 1}{2x - 1}$. Compute the y-coordinate of the intersection of the oblique and vertical asymptotes.

6. A half-circle and a quarter-circle are inscribed in a rectangle and share one point of tangency as shown in the diagram. If the longer side of the rectangle has a length of $2\sqrt{2}$, the perimeter of the rectangle can be expressed as $a + b\sqrt{c}$. Compute the sum $a + b + c$.



7. If $1011x + 1013y = 5061$, $1013x + 1011y = 5059$ and $xy = 6$, compute the value of $x^2 + y^2$.

8. Given $f(x) = \frac{x-63}{x-15}$. Compute the product of the x values such that $f(x) = f^{-1}(x)$.

9. Compute the value of $\sqrt{300} \left(\cos^4 \left(\frac{\pi}{12} \right) - \sin^4 \left(\frac{\pi}{12} \right) \right)$.

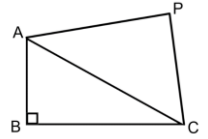
10. Compute the integer value of x that satisfies the inequality: $\log(10x + 50) - \log(x - 4) \geq 2$.

11. Compute the number of real values of b that satisfies $bi = (a - bi)^2$ where $i = \sqrt{-1}$.

12. During the summer, two mathematics teachers must grade 6400 statewide mathematics contests submitted by students. They each work at a steady rate. Ms. Lovelace grades 100 contests every 3 hours and Mr. Pascal grades 50 contests every hour and a quarter. In total, Mr. Pascal grades 5 hours less than Ms. Lovelace. Together, they complete grading all of the 6400 submitted contests. Compute the number of hours that Ms. Lovelace spent grading the contests.

13. Compute the smallest integer value of x , with $x > 2$ that satisfies the equation $\cos(x^2) = \cos(2x)$, where x is measured in degrees.

14. In right $\triangle ABC$, $AB = 4$ and $BC = 2\sqrt{2x}$. Point P is chosen so that $m\angle APC = 60^\circ$. If $CP = x$ and $AP = 2x$, compute x .



15. Emmy was assigned 10 homework problems last night but did not have enough time to complete them all. She decided to randomly do half the assigned problems. Her teacher decided to check only 5 of the problems. The probability that the teacher checked the exact 5 problems that Emmy did is $\frac{a}{b}$, where a and b are relatively prime. Compute the sum $a + b$.

Grade Level 12 - NMT 2024

Questions

1. Compute $2024 \div 44$. (Hint: $2025 = 45^2$.)

2. Compute the greater root of $x^2 - 20x + 99 = 2024$.

3. Curt needs to buy 140 cakes and can hire three different bakers:

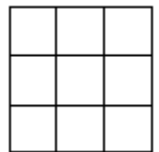
- Art can bake 5 cakes an hour and charges \$22 per hour.
- Helen can bake 7 cakes an hour and charges \$30 per hour.
- Pete can bake 8 cakes an hour and charges \$35 per hour.

If Curt can use any combination of the three bakers and wants to spend as little as possible, compute the amount of money Curt will have to spend, rounded to the nearest dollar.

4. The y -intercept and x -intercepts of the graph of $y = \frac{1}{12}x^2 - \frac{25}{12}x + 12$ are used as the vertices of a triangle. Compute the perimeter of this triangle.

5. In regular octagon $ABCDEFGH$, $\overline{AF}$ and $\overline{BH}$ intersect at point I . Compute $m\angle FIH$, in degrees.

6. From the 3-by-3 grid below, two distinct 1-by-1 squares are chosen at random. Let the probability in lowest terms, be p/q that the two chosen squares are adjacent either horizontally or vertically but NOT DIAGONALLY. Compute $p + q$.



7. The horizontal and vertical asymptotes of $f(x) = \frac{6x^2 + 42x}{x^2 - 6x - 91}$ intersect in a point whose coordinates are (a, b) . Compute $10a + b$.

8. Four circles are drawn with centers at the vertices of a square and diameters equal in length to the side length of the square. A smaller circle is drawn at the center of the square, externally tangent to all four of the larger circles. If the ratio of the area of one larger circle to the area of the smaller circle is, in simplest form, $a + b\sqrt{c}$, compute $100a + 10b + c$.

9. Let $f(x) = 1234 \sin x \sqrt{1 - \sin^2 x}$. Compute $f'\left(\frac{\pi}{6}\right)$.

10. In $\triangle ABC$, the measure of angle C is a constant 75° and the length of side AC is decreasing at a constant rate of 102 cm/min. The length of side BC changes at a rate such that the area of $\triangle ABC$ remains constant. Compute the rate, in cm/min, at which the length of side BC is increasing at the moment when $BC = 20$ cm and $AC = 24$ cm.

11. Compute the number of integer values of m for which the graphs of $y = x^2 + 2024$ and $y = mx$ do not intersect.
12. In complex numbers, let $z_1 = 24 + 7i$ and $z_2 = 0.6\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$. Compute $|z_1 z_2 - z_1|$.
13. Define a sequence by $a_1 = 0$, $a_2 = 1$, and, for $n \geq 3$, $a_n = \frac{a_{n-1} + a_{n-2}}{2}$. Compute $900 \lim_{n \rightarrow \infty} a_n$.
14. The graphs of $f(x) = ae^x$ and its inverse intersect at exactly one point. Compute the nearest integer to $108a$.
15. Let $R(n)$ be a randomly chosen integer in the interval $[1, n]$. Compute $2024 \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n R(k)}{n^2}$.

Mathletics - NMT 2024

Questions

1. Three positive prime integers, p , q , and r satisfy $pqr^3 = 2024$. Compute $p + q + r$.
2. Katie and Vishrat work in a café which sells coffee and hot chocolate. They each created exactly one signature drink. The *Katie Cocoa* has a sales price of \$4.50 per drink and the *Vishraticino* has a sales price of \$5.25 per drink. One cold morning, total revenue from sales of these two drinks (*Katie Cocoa* and *Vishraticino*) was \$261. There were 6 more *Katie Cocoas* sold than *Vishraticinos*. Compute the number of *Katie Cocoas* that were sold.
3. Point P is on the graph of $y = 36 - (x - 20)^2$ and point Q is on the graph of $y = x^2 - 40x + 500$. Compute the minimum distance between P and Q , rounded to the nearest integer.
4. The length of the longest altitude of a triangle with side lengths 13, 14, and 15 can be expressed in lowest terms as $\frac{p}{q}$. Compute the sum $p + q$.
5. Sam and Jay play a game that takes place in exactly 8 rounds, and it *appears* to be fair. However, it turns out that the Jay has a strategy that lets him win any given round with a probability of $\frac{4}{7}$. Let p represent the probability that Sam wins more rounds than Jay. Compute $1000p$, rounded to the nearest integer.
6. A circle with a diameter of 1000 is divided into three regions of equal area by two parallel chords. Compute the distance between these two chords, rounded to the nearest integer.
7. The solution set to the following inequality: $40|x| + 96|y| \leq 960$ is a region. Compute the area of that region.
8. Let S_n represent the sum of the first n terms of the arithmetic sequence 1, 5, 9, 13, 17, ... and define $f(n) = \sum_{k=1}^n S_k$. Compute the least integer, t , such that $f(t) > 2024$.
9. A sequence of functions is recursively defined by $f_1(x) = |x - 5|$ and $f_{n+1}(x) = |5 - f_n(x)|$. Compute the number of solutions to the equation $f_{23}(x) = f_{24}(x)$.

10. The diameter of a cone with a volume of 500 is equal to its height. This cone is divided into five pieces of equal volume by slicing it with three planes. The first planar slice is taken parallel to the base (dividing the original cone into a frustum and a smaller cone). The other two slicing planes include the center of the base of the larger object (the frustum) and are perpendicular to its base and to each other. Compute the *total surface area* of the five objects to the nearest integer.

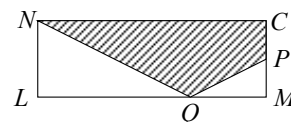
Team Problem Solving - NMT 2024

Questions

1. Compute the greatest positive 3-digit integer such that the product of its digits is 120.
2. Given the recursive formula $f(n + 1) = 2f(n) + 1$ and that $f(0) = 1$. Compute the greatest value of $f(n)$ such that $f(n)$ is a 3-digit prime number.
3. The number 6351 factors into three prime numbers. Compute the greatest of these three prime factors.
4. A piggy bank contains 105 coins consisting of pennies, nickels, and dimes. If the number of pennies is halved, the number of nickels is multiplied by 4, and the number of dimes is doubled, then the total value of the coins would be doubled. Compute the number of nickels in the piggy bank.
5. In a cryptarithm, different letters represent different digits, and no leading digit equals 0. Compute the least prime value of NOT in the given cryptarithm.

$$\begin{array}{r} \text{I} \\ + \text{DID} \\ \hline \text{NOT} \end{array}$$
6. A six-sided, fair die is rolled three times. If the first, second, and third outcomes are denoted by a , b , and c respectively, then the probability that $a < b < c$ is, in simplest terms, $\frac{p}{q}$. Compute $p + q$.
7. Jill jogs from home to work and back home along the same route every day. The round trip takes her a total of 75 minutes. When she jogs on level ground, she jogs at 4 mph. When she jogs uphill, she jogs at only 3 mph. When she jogs downhill, she jogs at 6 mph. Compute the number of miles in her round trip to and from work.
8. The digit 7 is written to the right of a two-digit number forming a three-digit number that is 538 more than the original number. Compute the original two-digit number.
9. The three-digit numbers acb , $a79$, $b0c$, $bb1$ are consecutive terms in an arithmetic sequence. Compute the value of $a + bc$.
10. The date 2/2/2024, is almost a primedate. To be a primedate, the day, the month, and the year must EACH be prime. If the last primedate before 2/2/24 was $d/m/y$, representing the day, the month and the 4-digit year, respectively, compute the remainder when y is divided by the sum $d + m$.

11. Rectangle $NCML$ has $NC = 3CM$, with $CP = MP$ and $LQ = 2QM$. If the area of the rectangle is 1 and the area of the shaded quadrilateral $NCPQ$ is in simplest terms, $\frac{a}{b}$, compute $a + b$.



12. Given $(2x - 3)(x - q) + 9 = 2x^2 - (p + 1)x + 3p$. Compute $p + q$.

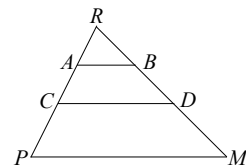
13. The coordinates of point A are $(3, -1)$, the coordinates of point B are $(5, 4)$, and the coordinates of point C are (x, y) . If points A, B , and C are collinear, with point B between points A and C and $\frac{BC}{AC} = \frac{4}{5}$, compute $x + y$.

14. The volume of a right rectangular prism is 864 and the areas of two of its faces are 72 and 96. Compute the length of the diagonal that joins a pair of opposite vertices of the prism.

15. Given the trig formula $(\cos x)(\cos y) = \frac{\cos(x+y) + \cos(x-y)}{2}$, compute the value of the expression $(184)(\cos 20^\circ)(\cos 40^\circ)(\cos 80^\circ)$.

16. The value of a that satisfies the equation $\log_a 512 + \log_a 243 - \log_a 6 = 4$, is a two-digit number. Compute the sum of the two digits.

17 In $\triangle PRM$, points A and C are on $\overline{PR}$, points B and D are on $\overline{MR}$, and $\overline{AB} \parallel \overline{CD} \parallel \overline{PM}$. Let x = the area of trapezoid $ABDC$ and y = the area of trapezoid $CDMP$. If $RA = 4$, $AC = 5$, and $CP = 6$ and the area of $\triangle RAB = 16$, compute $y - x$.



18. If $6x + 5y = 28$, $x - 2z = 19$, and $2y + 9z = 37$, compute x .

19. Compute $S/3$, given:

$$S = \frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \cdots + \left(\frac{1}{13} + \frac{2}{13} + \frac{3}{13} + \cdots + \frac{12}{13}\right).$$

20. The two roots of the equation $49^x - 7^{x+1} + 12 = 0$ have a sum that can be written as $\log_a b$. Compute $a + b$.