Grade Level 9 - NMT 2023 **Questions**

1. If $f(x) = 2x^2 - 3x + 5$, compute $|f(-3)|$.
2. Compute the sum of the coefficients of the product of $(2x - 3)$ and $(4x^2 - 5x - 6)$ when the product is expanded and simplified.
3. The area of a triangle is 30. If the length of one of its sides is $3\sqrt{5}$, and the length of its corresponding altitude is $a\sqrt{b}$, compute $a + b$.
4. As a class fundraiser, students are selling slices of pizza for \$2 each and cans of soda for \$1.50 each. Suresh has \$20 and he needs to buy 5 slices of pizza. Compute the maximum number of cans of soda he can buy with the 5 slices of pizza.
5. The lengths of the diagonals of a rectangle are represented by $\frac{4}{5}x - 1$ and $\frac{3}{4}x + 2$. Compute the value of x.
6. The graph of a line contains the points whose coordinates are (3, 11) and (-2, 1). Compute the value of y if the point whose coordinates are $(2, y)$ is also on this line.
7. The coordinates of the vertex of the graph of $f(x) = x^2 - 14x + 20$ are expressed as (a, b). Compute the value of $|b|$.
8. Line segments $\overline{AB}$ and $\overline{CD}$ intersect at point E. If $m\angle AED = 8x + 4y$, $m\angle AEC = 9x + 7y$, and $m\angle BEC = 12x - 4y$, compute $m\angle AEC$, in degrees.
9. Jia plants 2 different trees in her yard. Tree A is $3\frac{3}{4}$ feet tall and grows 16 inches per year. Tree B is $5\frac{5}{6}$ feet tall and grows 6 inches per year. Compute the height of the trees, in inches, when they are the same height.
10. The number of cat owners in a city increased by 100 and then decreased by 15%, at which time the city had 56 fewer cat owners than it did before the increase by 100. Compute the number of cat owners in the city before the increase by 100.
11. If $\frac{a}{b} = \frac{5}{4}$ and $\frac{c}{d} = \frac{12}{25}$, compute the value of $|p + q|$ when the value of $\frac{5ac-bd}{7bd-4ac}$ is expressed in simplest $\frac{p}{q}$ form where p and q are integers.
12. If $2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}}$ is expressed in simplest $a^{\frac{p}{q}}$ form, where a, p and q are positive integers and a is as small as possible, compute the value of p.
13. Compute the units digit of 23^{2023}.
14. A box contains 11 balls which are numbered 23, 24, 25, … 33. If 6 balls are drawn at random without replacement, the probability that the sum of the numbers on the 6 balls is an even number is expressed in simplest $\frac{a}{b}$ form where a and b are integers. Compute the value of a.
15. In trapezoid $TRAP$ with bases $\overline{TR}$ and $\overline{AP}$, $TR = 52$, $RA = 12$, $AP = 39$ and $PT = 5$. Compute the area of trapezoid $TRAP$.

Grade Level 10 - NMT 2023 **Questions**

1. A palindrome is a word, sentence, or number that reads the same whether it is read forwards or backwards. Examples of number palindromes are 121, 25752, and 44444. Compute the positive difference between 9999 and the greatest palindrome number that is less than 9999.

2. A bicycle has two different sized wheels. The diameter of the front wheel is 30 inches and the diameter of the rear wheel is 20 inches. Compute the number of complete rotations the rear wheel makes while the front wheel makes exactly 10 complete rotations.

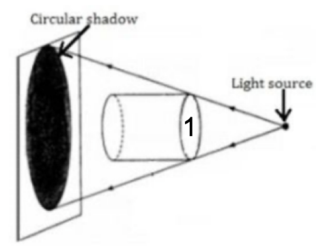
3. Compute the positive difference between the sum of the positive even integers less than or equal to 406 and the sum of the positive odd integers less than 406: $(2+4+6+\cdots+406)-(1+3+5+\cdots+405)$.

4. The number 2023 has two distinct prime factors, p and q. Compute pq.

5. Linear functions $2x-7y=5$ and $ax+63y=c$ share more than 2 points when graphed. Compute $|a+c|$.

6. Let r_1 and r_2 be the roots of $x^2-3=1022x-1023$. Compute $\frac{r_1+r_2}{2}$.

7. A cylinder is placed between a light source and wall as shown. Centers of the face of the cylinder (labeled "1"), the circular shadow, and the light source are colinear. The light source is 3 feet away from circular face (labeled "1") and the light source is 10 feet from the wall. If the area of circular face 1 is 18 square feet, compute the area of the circular shadow in square feet.



8. Let $x^3+18x^2+kx+210=(x+a)(x+b)(x+c)$, where a, b, and c are consecutive integers. Compute the value of k.

9. The probability of a number from 1 to 6 appearing on a "special" six-faced die is proportional to the value of the number. That is, the number 2 is twice as likely to appear as the number 1 and the number 6 is three times as likely to appear as the number 2, etc. The probability of getting 11 as the sum of the values when this die is rolled twice is $\frac{m}{n}$, where m and n are relatively prime. Compute the value of $m+n$.

10. In the xy–plane, point P whose coordinates are $(3,-2)$ is dilated by a factor of 100 about point C whose coordinates are $(1,-5)$. If the coordinates of the image of point P are (a, b), compute the value of $a+b$.

11. Compute the number of even integer factors of 840.

12. A triangle whose side lengths are 4, 5, and 6 is inscribed in a circle. If area of the circle is $\frac{a\pi}{b}$, where a and b are relatively prime, compute the value of $a+b$. (If ΔABC is inscribed in a circle with radius R, then $\frac{c}{\sin C}=2R$.)

13. Two parallel lines are tangent to a circle with center O. A third line, also tangent to the circle, meets the two parallel lines at points A and B. Compute the degree-measure of $\angle AOB$.

14. Let $f(x)$ be a cubic polynomial function with $f(0) = 7, f(1) = 18, f(2) = 61$, and $f(3) = 148$. Find the value of $f(5)$.

15. Compute x if $\log_x 2 + \log_x 3 + \log_x 6 = 2$.

Grade Level 11 - NMT 2023 **Questions**

1. Sam tossed a two-sided fair coin 100 times each day for 5 consecutive days. On the first day the coin landed on heads 72 times, on the second day 43 times, on the third day 61 times and on the fourth day 40 times. For the fifth day, she forgot to record any data. Compute the number of times the coin could have landed on heads the fifth day if, on average, it landed on heads 50 times each day.

2. If $f(x)$ is a linear function and $f(10) - f(4) = 24$, compute $f(8) - f(1)$.

3. If $81^{\sqrt{x}} = \sqrt{3}$, compute the value of $\frac{9}{\sqrt{x}}$.

4. If $f(x) = 2x + 4$ and $g(x) = x^2 + 3x$, compute the value of $f^{-1}(g(5))$.

5. Compute the sum of the solutions of $\sqrt{25 - x} + \sqrt{x} = 7$.

6. Compute the value of $100\left(\frac{\cos^2(35^o)+\cos^2(55^o)}{\sin^2(30^o)}\right)$.

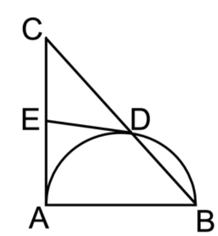
7. The quadratic equation $x^2 + 5x + m = 0$ has roots twice those of $x^2 + nx + 5 = 0$, where neither m nor n are zero. Compute the value of $\frac{m}{n}$.

8. If the equation $\frac{1}{2}x^2 + (5 + 3i)x + c = 0$ has only one solution, compute the product of c and its complex conjugate.

9. Given $x + y = 9$ and $3xy = 42$, compute the value of $x^3 + y^3$.

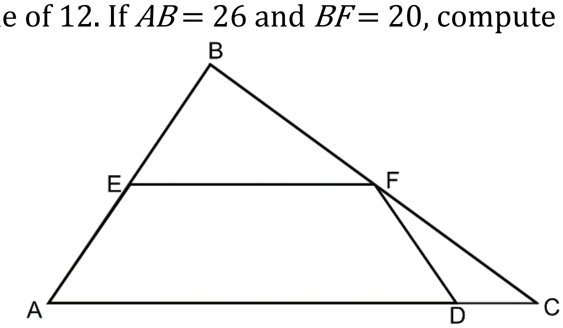
10. Points whose coordinates are $(3, 24)$and $(-2, k)$ are on a parabola that opens upward and has a vertex of $(1, 4)$. Compute the value of k.

11. Triangle ABC has a right angle at vertex A. A circle with diameter $\overleftrightarrow{AB}$ intersects side $\overline{BC}$ at point D. Also, a line tangent to the circle at point D intersects side $\overline{AC}$ at point E. If $CD = 4$, $DE = 3$, and the area of triangle ABC is k, compute the value of k^2.



12. The sum of 25 consecutive even integers is 900. Compute the value of the smallest of these 25 consecutive even integers.

13. In the given diagram $\overline{EF}$ is the midsegment of triangle ABC and quadrilateral $AEFD$ is an isosceles trapezoid with an altitude of 12. If $AB = 26$ and $BF = 20$, compute FE.



14. Let $y = \log_{16} x + \log_x 16$ in the first quadrant. The minimum value of y occurs at the point whose coordinates are (a, b). Compute ab.

15. Compute the number of ways to distribute 12 chocolate chip cookies to 6 friends if each friend is to get at least one chocolate chip cookie.

Grade Level 12 - NMT 2023 **Questions**

1. If 150% of 48 is 72, compute 48% of 150.

2. Bobby has a set of n identical square tiles $(n > 1)$, arranged to form a square. Bobby then separates the set into two groups and arranges each group to form a square. Compute the smallest possible value of n.

3. Let M be the maximum value of $f(n) = \cos(17n)°$, where n can be any integer value. Compute $(12M)^2$.

4. An object travels a distance of d meters in t seconds at a constant velocity of v meters per second. If the object traveled 12 meters per second faster, it would cover a distance of d meters in 5 fewer seconds. If d, v, and t are all positive integers, find the smallest possible value of t.

5. Define a sequence $\{a_n\}_1^\infty$ as follows: $a_1 = 2$ and for $n \geq 2$, a_n is the smallest positive integer not divisible by any of $\{a_1, a_2, \ldots, a_{n-1}\}$. Compute a_{15}.

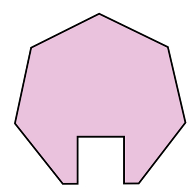
6. If $\dfrac{16}{(1+i)^3} = a + bi$ in complex numbers, compute $|a+b|$.

7. Compute the value of h such that the system of equations

$$y = |x| + 2 \text{ and } y = -|x - h| + (36 - h)$$

has an infinite number of solutions.

8. From the edge of a large regular heptagonal region, a square region is removed, as shown. If the increase in the heptagonal region's perimeter is numerically equal to the decrease in the heptagonal region's area, compute the side length of the removed square region.



9. A right triangle has a hypotenuse of length 14 and an area of 42. If the volume of the solid of revolution formed by revolving the interior of the triangle through 360° about its hypotenuse is $k\pi$, compute k.

10. If $0° \leq \theta < 360°$, compute k if the total length of the polar graph of $r = |91 \sin\theta|$ is $k\pi$.

11. The lengths of the edges of a cube are increasing at a constant rate of 29 cm/min. At what rate (in cm^3/min) is the volume of the cube increasing at the moment when the surface area of the cube is 62 cm^2?

12. The graph of $y = x^4 + bx^2 + 2023$ is tangent to the line $y = 87$ in two places. Compute $|b|$.

13. Let $f(x)$ be a function such that $f(x+a)=f(x)+3x^2a+3xa^2+a^3+9a$ for all real values of x and a. Compute $f'(16)$.

14. Compute the number of permutations of $\{1, 2, 3, 4, 5\}$ where each element of the set is used exactly once and in which no adjacent pair of numbers differ by 1.

15. Compute $\lim_{h \to 0} \frac{(6+h)^4-(6-h)^4}{2h}$.

Mathletics - NMT 2023 **Questions**

1. Ethan has three dogs: a Yorkie, a Beagle, and a Labradoodle. The Beagle weighs two pounds more than twice the Yorkie and the Labradoodle weighs 11 pounds more than the Beagle. If the average weight of the three dogs is 20 pounds, compute the Beagle's weight in pounds.

2. If the perimeter of a square is increased by 40%, then the area of this square is increased by k%. Compute k.

3. When $16^{\frac{23}{20}}$ is expressed in the form $a\sqrt[5]{b}$, where a and b are positive integers, compute the minimum possible value of $a + b$.

4. Square $ABCD$ has side length 40 and the midpoints of sides AB, BC, CD, and DA are labeled P, Q, R, and S respectively. Compute the distance between the parallel segments PQ and RS and round your answer to the nearest integer.

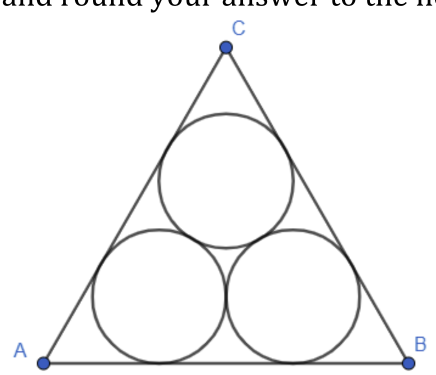
5. The piecewise function $f(x)$ is given by $f(x) = \begin{cases} 2x - 12, x \geq 4 \\ |x+3|, -4 < x < 4 \\ 10 - x, x \leq -4 \end{cases}$

   Compute $f(f(f(f(2))))$

6. Compute the least integer, N, such that 2023N is a perfect cube.

7. The first term of a geometric sequence is 2 and the 10th term is 2023. Compute the 9th term of the geometric sequence rounded to the nearest integer.

8. Three congruent circles are interior to equilateral triangle ABC as shown in the figure below. Each circle is tangent to two sides of triangle ABC and is tangent to the other two circles. Compute the area of one of these circles if $AB = 90$ and round your answer to the nearest integer.



9. Ten boys and ten girls go on a weekend school field trip. The group consists of 5 pairs of boys and 5 pairs of girls where each member of a pair considers the other member to be his or her best friend. The school has booked ten hotel rooms with two boys or two girls assigned to each room. There is a unique rooming arrangement, where each student will be paired with his or her best friend. However, the school has decided to assign roommates randomly instead of placing each student with his or her best friend. Compute the value of n such that the probability that each student is paired with his or her choice of roommate is $1/n^2$.

10. Compute the number of ordered pairs of positive integers, (x, y) that satisfy $17x + 119y < 2023$.

Team Problem Solving - NMT 2023 Questions

1. Compute the number of 3-digit numbers of the form htu that are possible if $h > t > u$.

2. Whole numbers a, b, and c are such that $a^2 + b^2 + c^2 = 13^2$. Compute $a + b + c$.

3. An isosceles right triangle has its vertex angle on the center of a square whose sides have a length of 12. The legs of the triangle also have a length of 12. Compute the area of the region inside the triangle but outside the square.

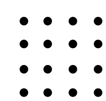
4. Compute the number of positive multiples of 7 that are perfect squares less than 1,000,000.

5. In the cryptarithm at the right, each letter appearing more than once has the same digit value every time. The nine digits from 0 to 8 will appear at least once and the letter E is 8. Compute the number represented by the word S I X.

$$\begin{array}{r} \text{S E V E N} \\ \text{S E V E N} \\ +\ \text{S I X} \\ \hline \text{T W E N T Y} \end{array}$$

6. Compute the 1000th term in the infinite sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, … .

7. Compute the number of squares that can be drawn using the 4 by 4 grid of dots so that each vertex of each of the squares is a dot in the grid.



8. One side of a square is tangent to a circle and the 2 vertices of the square not on the tangent side are located on the circle. If a side of the square is 40 and the circumference of the circle can be represented as $n\pi$, compute n.

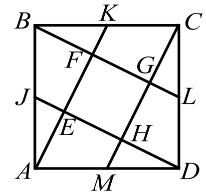
9. A set of five whole numbers has a mode of 17, a mean of 68 and a median of 85. Compute the greatest possible number in the set.

10. Compute the sum of all integer values of n that satisfy the inequality $\frac{5}{4} < \frac{12}{n} < \frac{9}{2}$.

11. Two of the factors of $x^5 + x^4 - 30x^3 - 76x^2 + ax + b$ are $(x + 2)$ and $(x - 1)$. Compute $b - a$.

12. Given the function $f(x) = \frac{3}{2-x}$ determine all the real numbers that are not in the domain of $f(f(f(x)))$ and then compute the product of the squares of those numbers.

13. Midpoints of the sides of square $ABCD$ are labeled J, K, L, and M. These midpoints are joined to the vertices of the square as shown and they form square $EFGH$. Compute the area of $EFGH$ given that $AB = 20$.



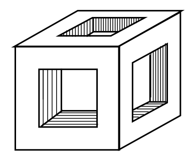
14. The graphs of the functions $y = 5^x$ and $y = 60x - 55$ intersect in two points. If the points of intersection have coordinates (a, b) and (c, d), compute $a + b + c + d$.

15. The number of 4-digit numbers that have at least two digits the same can be represented by $pqrs$. Compute $p + q + r + s$.

16. Compute $4d + 3e + 2f - g$ given that $d + e + f + g = 45$ and $d + 7 = e + 3 = 3f = g/5 + 11$.

17. Compute $\sqrt{\dfrac{-18i}{\dfrac{1}{12+9i}-\dfrac{1}{12-9i}}}$, where $i = \sqrt{-1}$.

18. Each edge of the solid cube shown has length 5. Square holes cut from three of the faces through to the opposite faces have edge lengths equal to 3. Compute the surface area of the resulting shape, including its interior.



The last two questions form a mini relay where the answer to question 19 (A_{19}) is used in question 20.

19. Let the three prime factors of 126 be a, b, and c where $a < b < c$. Compute $\log(a^{10} - 8b) + \log(b + c)$ and label the result A_{19}.

20. Each edge of regular octagon $ABCDEFGH$ has length A_{19}. The area of square $ACEG = p + q\sqrt{r}$, where r has no perfect square factors other than 1. Compute $p + q + r$.