9

Nassau County Interscholastic Mathematics League

Grade 9

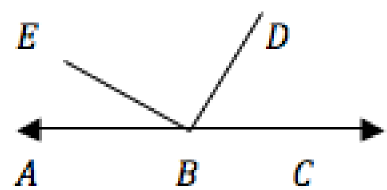
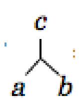
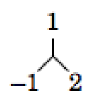
TEAM #

Mathematics Tournament 2022

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. When expanded, $(5x+2)(7x-4)$ has the form ax^2+bx+c. Compute the sum $a+b+c$.	1.
2. Compute the larger zero of the function $f(x)=2x^2-4x-6$.	2.
3. Write $3\sqrt{72}-\frac{2}{5}\sqrt{50}$ in simplest radical form, $a\sqrt{b}$. Compute the sum $a+b$.	3.
4. In the diagram, $\overleftrightarrow{ABC}$ is a line, $\overline{BD}\perp\overline{BE}$, and $m\measuredangle DBC=2m\measuredangle ABE$. Compute $m\measuredangle DBC$. 	4.
5. If  $=a^b-b^c+c^a$, compute  .	5.
6. Compute the number of square units in the area of the triangular region enclosed by the graphs of the lines $x=1, x=-y-6$, and $y=x+6$.	6.
7. Compute the product of the roots of the equation $y^4-4y^2+3=0$.	7.
8. Compute the units digit of the product $(81)(82)(83)\ldots(89)$.	8.

Turn Over

Mathematics Tournament 2022

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. At a mathletes competition, the home team had 2/5 of the participants. Each of the students wore either a sweatshirt or a tee shirt, not both. One-third of the students on the home team wore tee shirts. Two-ninths of the students on the visiting team wore tee shirts. If the fractional part of the total number of participants who wore sweatshirts is written in simplest $\frac{a}{b}$ form, compute the sum $a+b$.	9.
10. Compute the total number of three–digit numbers such that the sum of the digits in each three–digit number is 24.	10.
11. In the data set {17, 15, 10, 17, 11}, the mean is x, the median is y, the mode is z, and the range is r. If $3x+2y-z=4r+k$, compute the value of k.	11.
12. Eight boys and two girls are standing on a ticket line in a random order to purchase concert tickets. The probability that the two girls are next to each other can be expressed as $\frac{a}{b}$ in simplest form. Compute $b-a$.	12.
13. Compute the largest integer for which the expression $x^3 - x$ is divisible by that integer for all possible integral values of x.	13.
14. If a circle is inscribed in a right triangle whose legs have lengths 27 and 36, compute the length of its radius.	14.
15. In a race whose length is d yards, each runner races at his own uniform speed. Art beats Bob by 40 yards, Bob beats Carl by 20 yards and Art beats Carl by 56 yards. Compute d, the length of the race.	15.

10	**Nassau County Interscholastic Mathematics League**	
	Grade 10	**TEAM #**

Mathematics Tournament 2022

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Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. For each wall that a WiFi signal goes through, the signal loses 50% of its strength. There is no other factor that affects WiFi signal strength. If, after the WiFi signal goes through 4 identical walls, the signal strength is $k\%$, compute k to the nearest integer.	1.
2. If $ax^2 - x + c = 2x^2 + bx + a - b$, where a, b, and c are integers, has more than two solutions for x, compute the value of c.	2.
3. A rectangle whose length and width are positive integers has a perimeter of 24. Compute the minimum area of the rectangle.	3.
4. Compute the greatest prime factor of 1798.	4.
5. Each “mathketball” game has two halves of 20 minutes each. Each team must play 5 players throughout the game. If there are 8 players on each team and each player plays the same number of minutes, compute the number of minutes that each player plays.	5.
6. Compute the value of $\frac{S}{10}$, where S is the sum of all positive integers less than 200 whose units digit is 1.	6.
7. Compute the value of c if $(2x-3)^4 = ax^4 + bx^3 + cx^2 + dx + e$ and a, b, c, d, and e are integers.	7.
8. Compute the value of x if $\lvert x+13 \rvert = \lvert x-21 \rvert$, where x is a real number.	8.

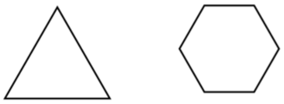
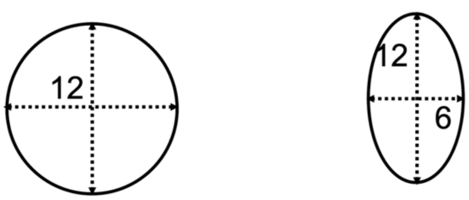
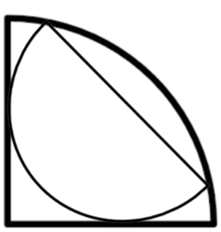
Turn Over

10

Mathematics Tournament 2022

Grade 10

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. The ratio of the area of an equilateral triangle to the area of a regular hexagon is 2:3. If the perimeter of the equilateral triangle is 10, compute the perimeter of the regular hexagon.  (not drawn to scale)	9.
10. Compute the value of the sum $a+b$ if $y=ax+b$ is the equation of the line that is the image of the line $y=x-4$ after it is rotated 45 degrees clockwise about its x–intercept.	10.
11. Compute the number of values of n that have a remainder of 1 when n^4 is divided by 5, if n is a positive integer that is less than 100.	11.
12. Mr. Lee has two fair dice, one green and the other red. The dice are identical except for color. If the probability that the number on the green die is greater than the number on the red die is $\frac{a}{b}$ when expressed in simplest form, compute $a+b$.	12.
13. A circle with a diameter of 12 inches is rotated n degrees about the vertical axis through its center. From your perspective, the circle looks like the image shown below left before it begins to rotate. As it rotates, the image appears to narrow until it looks like the ellipse shown below right. Compute n if $0<n<90$. 	13.
14. If $x>5$, compute the smallest value of the expression $x+\frac{9}{x-5}$.	14.
15. As shown below, a semicircle is inscribed in a quarter circle. If the radius of the semicircle circle is 1, compute the product of 24 and the ratio of the area of the quarter circle to the area of the semicircle. 	15.

Nassau County Interscholastic Mathematics League

11	Grade 11	TEAM #

Mathematics Tournament 2022

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Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

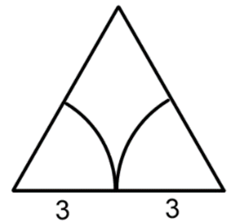
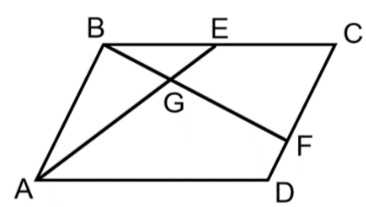
1. Compute the positive root of $\frac{2x^{2021}-99x^{2020}}{x^{2019}}=50$.	1.
2. Compute the number of intersection points of the graphs of $f(x)=10\cos\left(\frac{x}{2}\right)$ and $g(x)=10$ on the interval $[-6\pi,16\pi]$.	2.
3. Kevin has 120 pens (and nothing else) in a bag that are all identical to each other except for color. These 120 pens include 40 blue pens, 40 black pens and 40 red pens. Kevin is blindfolded and removes pens from the bag one at time. Compute the smallest number of pens Kevin must select in order to guarantee that he selects at least one red pen.	3.
4. If $z=\frac{3+bi}{2+2i}$ is a real number and b is a real number, compute the value of b.	4.
5. If $y=f(x)$ is transformed into $y=64f(4x-2)$, compute the product of the horizontal and vertical scale factors.	5.
6. You are given a pyramid with a square base, and with the lengths of the lateral edges not equal to the lengths of the base edges. Compute the number of isosceles triangles that can be formed if each of the vertices of each isosceles triangle is also a vertex of the pyramid.	6.
7. If $f(x)=14-2x$, compute the value of h for which $\lvert f(x+h)\rvert$ is an even function.	7.
8. Compute the reciprocal of the common ratio of the sequence $4^x, 4^{x-3}, 4^{x-6}, 4^{x-9}, \ldots$.	8.

Turn Over

Mathematics Tournament 2022

Grade 11

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. Two tangent circles are drawn so that their centers are two vertices of an equilateral triangle. The radius of each circle is 3. The area of the region that is interior to the equilateral triangle and NOT interior to either circle can be expressed, in simplest form, as $a\sqrt{b} - c\pi$. Compute the sum $a + b + c$. 	9.
10. Compute the number of integral values of b for which the graph of $y = 3x^2 + bx + 12$ will not have any x–intercepts.	10.
11. Compute the number of arrangements of the letters in the word COMPASS that are possible if the sequence SS must occur in each arrangement.	11.
12. One of the vertices of the hyperbola $x^2 - y^2 = 1$ is the point whose coordinates are $(1,0)$. There are four points with coordinates (a,b) on the hyperbola whose distance from this vertex is $2\sqrt{3}$. Compute the positive difference of the two possible values of a.	12.
13. One solution of the equation $(\log_{16}x)^2 = \log_4 x$ is $x = 1$. Compute the other solution.	13.
14. Parallelogram $ABCD$ has points E and F on sides $\overline{BC}$ and $\overline{CD}$ respectively. If point E is the midpoint of side $\overline{BC}$, the ratio of $CF : FD$ is 3 and $GE = 12$, compute AG. 	14.
15. If $\cos^4 A - \sin^4 A = \frac{\sqrt{2}}{2}$, compute the sum of all possible degree-values of A in the interval $180° \leq A \leq 360°$.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2022

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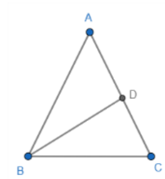
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. Compute the number of integers in the domain of the real-valued function $f(x) = \frac{\sqrt{100-x^2}}{x^2-4}$.	1.
2. Compute $\lim\limits_{x\to 8} \frac{x-8}{\sqrt[3]{x}-2}$.	2.
3. In ΔABC, $\overline{AB} \cong \overline{AC}$, the bisector of $\measuredangle ABC$ intersects $\overline{AC}$ at point D and $\Delta ABC \sim \Delta BCD$. Compute $m\measuredangle C$ in degrees. 	3.
4. For some real number c, the polynomial $x^4 - 10x^2 + c = 0$ has four real zeros that form an arithmetic sequence. Compute the sum of the squares of the zeros.	4.
5. A race car travels a 360-meter track twice. On its second trip, it averages 27 meters per second faster than the first trip and finishes 3 seconds quicker. Compute, in meters per second, the car's average speed for the second trip.	5.
6. If $f(x) = \cos^4 x - \sin^4 x$, compute $f'\left(\frac{3\pi}{4}\right)$.	6.
7. A function $f(x)$ has the property that for all real numbers x: $2f(x) + f(1-x) = x^2$. Compute $30 \cdot f(7)$.	7.
8. Square $ABCD$ has side length 84. Parallel lines drawn through points A and C intersect $\overline{CD}$ and $\overline{AB}$ at points E and F, respectively. If the parallel lines divide square $ABCD$ into three regions of equal area, compute DE.	8.

Turn Over

Mathematics Tournament 2022

Grade 12

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. A rectangle of area 26 is inscribed in a circle of radius 6. Compute the perimeter of the rectangle.	9.
10. Let $z = a + bi$ be a complex number with a and b both real numbers. If z^2 is located exactly 511 units to the left of z in the complex plane, compute $a^2 + b^2$.	10.
11. Let A be the area of a regular hexagon of side length x and let P be its perimeter. If x changes with time t, then $\frac{dA/dt}{dx/dt} = kP$. Compute $360k^2$.	11.
12. Let N be the number of ways the letters of MATH TEAM can be rearranged into two-"word" phrases. Examples of such phrases include "THE MATAM", "THEM ATAM", "A MMTHETA", and "TAMTHEM A". Note that both words must contain at least one letter, but do not need to be real words. Compute the remainder when N is divided by 1000.	12.
13. The line $y = (\tan 15°)x$ intersects the circle $x^2 + y^2 = 2512$ at the point (a, b) in the first quadrant. Compute the area of the triangle with vertices whose coordinates are $(0,0)$, $(a, 0)$, and (a, b).	13.
14. The graph of $x^2 + y^2 - 14\|x\| - 14\|y\| + 49 = 0$ divides the plane into 5 bounded regions and 1 unbounded region. Compute, to the nearest integer, the area of the bounded region containing the origin.	14.
15. Let $f(x) = x^2 + 2021$. Let L be the line tangent to the graph of $y = f(x)$ at the point whose coordinates are $\left(a, f(a)\right)$, where a is a positive real number. Compute the nearest integer to the minimum possible value of a such that line L intersects the graph of $y = -f(x)$.	15.

Nassau County Interscholastic Mathematics League

M

Mathletics

TEAM #

Mathematics Tournament 2022

Calculators may be used on this part.
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One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes *Answer Column*

1. The set $\boldsymbol{S} = \{1, 3, 5, 7, .., 51\}$. $\boldsymbol{S}$ is separated into two disjoint subsets, $\boldsymbol{A}$ and $\boldsymbol{B}$, such that the sum of the elements of $\boldsymbol{A}$ and the sum of the elements of $\boldsymbol{B}$ are both equal to the same number. Compute the number.	1.
2. Given that $f(x) = 7x + a$, $g(x) = \sqrt{x - 22}$ and $g\big(f(10)\big) = 12$, compute the value of a.	2.
3. When David works alone and at a steady rate, he can shovel all of the snow off of his driveway in 2 hours. When his brother William works alone and at his steady rate, he can shovel all of the snow off of that same driveway in 3 hours. If they work together, and each works at the same steady rate as when shoveling alone, compute the number of **minutes** it will take them to clear the driveway.	3.
4. A circle is inscribed in a rhombus that has diagonals with lengths 18 and 80. Compute the area of the inscribed circle and round your answer to the nearest integer.	4.
5. The sequence $\{a_n\}$ is defined by $a_1 = 2, a_{10} = 21$ and a_n is the average of the first n – 1 terms for all $n > 2$. Compute a_2.	5.
6. Given that $x^3 + y^3 = 10$ and $x + y = 12$, $x^2 + y^2$ can be expressed as $\frac{a}{b}$ in lowest terms. Compute $a + b$.	6.
7. A polynomial $x^3 - ax^2 + 2111x - 2021$ has three positive integer roots. Compute the value of the coefficient a.	7.

Turn Over

M

Mathematics Tournament 2022

Mathletics

Time Limit: 30 minutes *Answer Column*

8. John, Alex, and Davesh play a game of chance where each has an equal probability of winning each round of the game. If all three of these people play for 5 rounds, the probability that Davesh wins more rounds than the other two players combined can be expressed in lowest terms as $\frac{p}{q}$. Compute the sum $p+q$.	8.
9. One of the roots of the equation $20x^2+bx+c=0$ is r and $$r=\cfrac{1}{20+\cfrac{1}{21+\cfrac{1}{20+\cfrac{1}{21+\cdots}}}}$$ Compute the coefficient b, if both b and c are integers.	9.
10. Region R is bounded by the y–axis, and the lines $y=\frac{x}{5}+10$, $y=\frac{x}{5}$ and $y=20-x$. Compute the area of quadrilateral region R.	10.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

TEAM #

Mathematics Tournament 2022

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________________ **Score**__________

Time Limit: 60 minutes *Answer Column*

1. In a magic square, all the rows, columns and major diagonals must have the same sum. Fill in the accompanying 4 by 4 square with the four digits in the number 2021 so that the digits in every row, column, and major diagonal sum to 5. That is, each row, each column and each major diagonal must contain exactly two 2's, exactly one 1, and exactly one zero.

1.

2. We define a "gap" between two consecutive prime numbers as the number of composites between them. For example, the gap between 7 and 11 is 3. Given the set of all 2-digit positive integers, compute the largest gap.

2.

3. Given $\frac{2}{3} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ where $a + b + c$ are 3 distinct whole numbers, compute the product abc in which the sum $a + b + c$ is as small as possible.

3.

4. Compute the number of integers that can be formed using exactly seven 1's and four 0's if none of the integers start or end with a 0 and none of the integers have two 0's next to each other.

4.

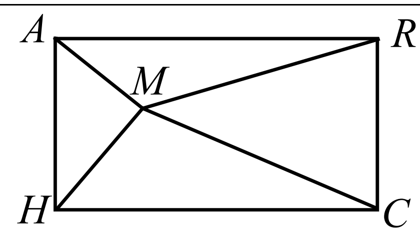
5. A mother bought her three children a total of 100 puzzles for them to do over the summer. The total cost for the puzzles was \$87.70. There were three different types of puzzles. Type A puzzles cost 40¢ each, type B puzzles cost 70¢ each, and type C puzzles cost \$1 each. If she bought k more type C puzzles than type A puzzles, compute k.

5.

6. Compute the base ten equivalent of the binary (base 2) number 1010101010_2.

6.

7. Given rectangle $CHAR$ with point M in its interior, compute $(MA)^2$ given that $MH = 15$, $MR = 20$, and $MC = 24$.



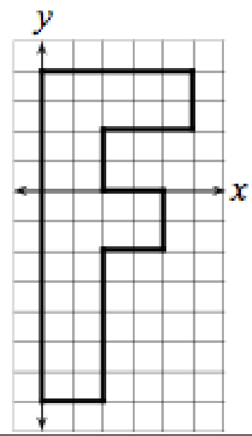
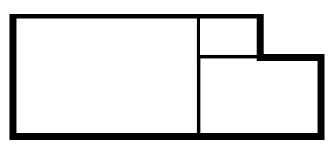
7.

8. Given that $\log_2(\log_4(\log_5(p))) = \log_4(\log_5(\log_2(q))) = \log_5(\log_2(\log_4(r))) = 0$, Compute the sum $p + q + r$.

8.

Turn Over

Time Limit: 60 minutes *Answer Column*

9. A unit fraction is a fraction whose numerator is 1 and whose denominator is a counting number greater than 1. Compute the number of unit fractions that exist between 100 and its reciprocal.	9.
10. A square pyramid is inscribed in a cube and the volume of the cube is 56π cm³. The volume of a sphere is equal to the volume of the square pyramid. Compute the volume of a second cube whose edge is equal in length to the radius of the sphere.	10.
11. Compute the integer solution of x: $2^{x^2-8x-9} = 3^{2x^2-19x+9}$.	11.
12. The capital letter F is placed on a grid so that a portion of the letter lies on the y-axis and has its endpoints at (0, 4) and (0, –7) as seen in the diagram. A line passing through the origin can divide the letter into 2 regions of equal area. Compute the opposite reciprocal of the slope of that line. 	12.
13. Compute: $\sqrt{\dfrac{27^8+9^8}{27^2+9^7}}$.	13.
14. The product of three consecutive odd integers, reduced by 24, is 140 less than the cube of the sum of the smallest of the three consecutive odd integers and 2. Compute the mean of the three integers.	14.
15. Three similar rectangles, each with integer side lengths in the ratio 2:3, form a non-convex hexagon as shown. The shorter side of the largest rectangle is 6. Compute the perimeter of the hexagon. 	15.
16. Compute the value of $58 \sin x \cos x$, if $2 \sin x = 5 \cos x$.	16.

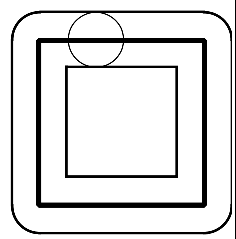
T

Mathematics Tournament 2022

Team Problems

Time Limit: 60 minutes

The last four questions form a mini relay where the answer to question 17 (A_{17}) is used in question 18. Similarly, the answer to question 18 (A_{18}) is used in question 19 and the answer to question 19 (A_{19}) is used in question 20.

	Answer Column
17. Positive integers r, s, and t have the following two properties: (1) $rst = 144$ and (2) $r^2 + s^2 + t^2 = 149$. Compute $r + s + t$. [Label your answer A_{17}]	17.
18. Compute the sum of the two solutions to the equation: $\|x - 2\| = A_{17}$. [Label your answer A_{18}]	18.
19. The lengths of each of the three sides of two given triangles are positive integers. If the two triangles have a common side length and a second side of each triangle has a length of A_{18}, where A_{18}] is less than the length of the common side, compute the greatest possible difference between the lengths of the third sides of the two triangles? [Label your answer A_{19}]	19.
20. A circular disc of radius 1 is rolled so that its center is always kept on the perimeter of a square whose side length is A_{19}. The area of the region covered by the disc when it has completed one circuit about the square can be written in the form $a + b\pi$. Compute the sum $a + b$. The diagram shows a picture of the region covered. 	20.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2022

DO <u>NOT</u> HAND THIS COPY IN. HAND IN THE ONE TEAM COPY.
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Team Copy **School** ______________________________ **Score**________

Time Limit: 60 minutes *Answer Column*

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5. A mother bought her three children a total of 100 puzzles for them to do over the summer. The total cost for the puzzles was \$87.70. There were three different types of puzzles. Type A puzzles cost 40¢ each, type B puzzles cost 70¢ each, and type C puzzles cost \$1 each. If she bought k more type C puzzles than type A puzzles, compute k. — 5.

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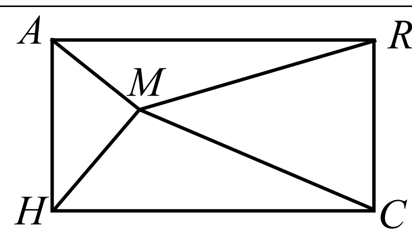

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Turn Over

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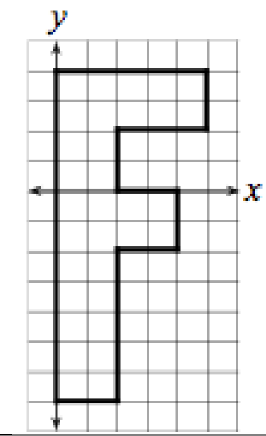
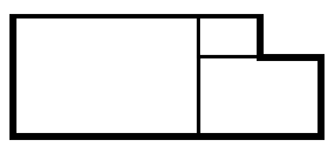
12. The capital letter F is placed on a grid so that a portion of the letter lies on the y-axis and has its endpoints at (0, 4) and (0, –7) as seen in the diagram. A line passing through the origin can divide the letter into 2 regions of equal area. Compute the opposite reciprocal of the slope of that line. 	12.
13. Compute: $\sqrt{\dfrac{27^8+9^8}{27^2+9^7}}$.	13.
14. The product of three consecutive odd integers, reduced by 24, is 140 less than the cube of the sum of the smallest of the three consecutive odd integers and 2. Compute the mean of the three integers.	14.
15. Three similar rectangles, each with integer side lengths in the ratio 2:3, form a non-convex hexagon as shown. The shorter side of the largest rectangle is 6. Compute the perimeter of the hexagon. 	15.
16. Compute the value of $58 \sin x \cos x$, if $2 \sin x = 5 \cos x$.	16.

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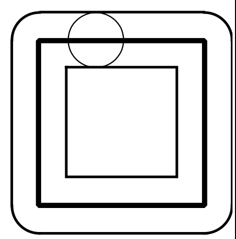
Mathematics Tournament 2022

Team Problems

Time Limit: 60 minutes

The last four questions form a mini relay where the answer to question 17 (A_{17}) is used in question 18. Similarly, the answer to question 18 (A_{18}) is used in question 19 and the answer to question 19 (A_{19}) is used in question 20.

Answer Column

17. Positive integers r, s, and t have the following two properties: (1) $rst = 144$ and (2) $r^2 + s^2 + t^2 = 149$. Compute the sum $r + s + t$. [Label your answer A_{17}.]	17.
18. Compute the sum of the two solutions to the equation: $\lvert x - 2\rvert = A_{17}$. [Label your answer A_{18}.]	18.
19. The lengths of each of the three sides of two given triangles are positive integers. If the two triangles have a common side length and a second side of each triangle has a length of A_{18}, where A_{18}] is less than the length of the common side, compute the greatest possible difference between the lengths of the third sides of the two triangles? [Label your answer A_{19}.]	19.
20. A circular disc of radius 1 is rolled so that its center is always kept on the perimeter of a square whose side length is A_{19}. The area of the region covered by the disc when it has completed one circuit about the square can be written in the form $a + b\pi$. Compute the sum $a + b$. The diagram shows a picture of the region covered. 	20.