9

Nassau County Interscholastic Mathematics League

Grade 9

TEAM # ☐

Mathematics Tournament 2020

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** __________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. The circumference of a circle is 28π, and its area is expressed as $k\pi$. Compute k.	1.
2. Compute the sum of three positive consecutive odd integers such that the square of the least integer is 9 more than 5 times the sum of the other two integers.	2.
3. Lines l and m are represented, respectively, by the equations $6x - 3y = -2$ and $ax - by = 5$. If $l \parallel m$, and the y-intercept of line m is -10, compute a.	3.
4. If the expression $5\sqrt{147} - 4\sqrt{108} + 2\sqrt{27}$ is simplified and written in $a\sqrt{b}$ form, where b does not contain an integral perfect square factor greater than 1, compute $a + b$.	4.
5. If x is a real number and $16^{0.5x+1.5} = 2^{3x}$, compute the value of x.	5.
6. In right triangle ABC, with right angle B, $AB = 8\sqrt{2}$ and $AC = 12$. When expressed in simplest form, $\sin A = \frac{1}{k}$. Compute k.	6.
7. Compute the greater integer that is a zero of the function, $f(x) = 2x^2 - 26x + 60$.	7.
8. Points $A(6, 9)$ and $C(10, 3)$ are the coordinates of two opposite vertices of square $ABCD$. Compute the sum of the coordinates of the vertex of the square that is closest to the origin.	8.

Turn Over

9

Mathematics Tournament 2020

Grade 9

Time Limit: 45 minutes

Lower Division

	Answer Column
9. If the average of x and y is z, the average of x and z is one more than y, and the average of y and z is two more than twice x, compute $\lvert x\rvert$.	9.
10. The product of n distinct integers is 18. Compute the maximum value of n.	10.
11. Compute the value of the expression, $\frac{(100^2-99^2+98^2-97^2+\cdots+4^2-3^2+2^2-1^2)}{10}$.	11.
12. If $x+y=3$ and $x^3+y^3=819$, compute the value of $\lvert xy\rvert$.	12.
13. The product $2^7\times 3^4\times 5\times 7^2\times 11^3$ is divisible by n distinct whole number perfect squares. Compute the maximum value of n.	13.
14. Compute the number of lattice points in the first quadrant which lie on the graph of $20x+3y=2003$. [Note: A lattice point is a point whose coordinates are integers.]	14.
15. Four people toss the same fair, standard, six-sided die, and the four results are recorded in the order in which the die was tossed. If the probability that the four results are strictly increasing is expressed as $\frac{a}{b}$ in lowest terms, compute $a+b$.	15.

10

Nassau County Interscholastic Mathematics League

Grade 10

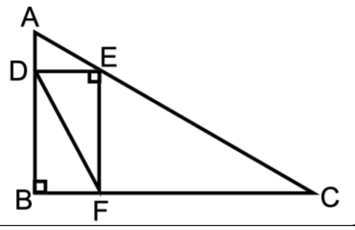
TEAM # ☐

Mathematics Tournament 2020

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ________________________ **School** ________________________ **Score** __________

Time Limit: 45 minutes **Lower Division** *Answer Column*

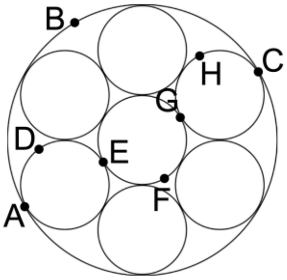
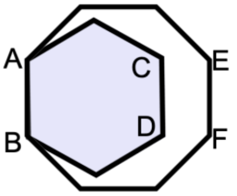
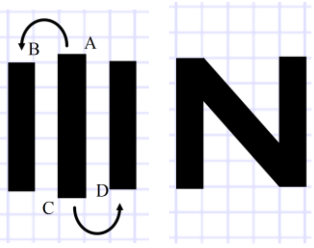
1. Compute the sum of all of the distinct prime factors of 2020.	1.
2. If $\frac{-\frac{4}{3^2}+\frac{1}{3}+3}{1-\frac{1}{3}} = \frac{p}{q}$, where p and q are relatively prime, compute $p+q$. [Note: Two numbers are relatively prime if their greatest common factor is 1.]	2.
3. Noah had a full container of 20% juice drink by volume. (This means that 20% of the contents was juice and the rest was water.) Noah drank half of the container's contents, filled the remainder of the container with water, and mixed the contents thoroughly. Later, he drank half of the contents of the container and again filled the remainder of the container with water. The contents of the container is now p% juice. Compute p.	3.
4. If 0.2020202020… = $\frac{p}{q}$, where p and q are relatively prime, compute $p+q$.	4.
5. If r and s are the roots of $x^2-5x-6=0$, and $2r$ and $2s$ are the roots of $x^2-ax+b=0$, compute $a-b$.	5.
6. The vertices of ΔABC are $A(3,1)$, $B(1,3)$, and $C(-3,-2)$. When the triangle is rotated $90°$ counterclockwise about point C, the image of point A is (h,k). Compute k.	6.
7. If $f\left(\frac{x}{2}\right)=x^2-4x+15$, compute $f(2)$.	7.
8. Right ΔDEF is inscribed in ΔABC with $\overline{DE} \parallel \overline{BC}$ as shown. If $m\measuredangle BAC = m\measuredangle EDF = 60°$, and $DE=\sqrt{3}$, the area of ΔABC is, in simplest radical form, $a\sqrt{b}$. Compute $a+b$. 	8.

Turn Over

Mathematics Tournament 2020

Grade 10

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. Three prime numbers are roots of $x^3 - 20x^2 + 101x - 130 = 0$. Compute the greatest root of the equation.	9.
10. Seven circles, each with a radius of length 1, are externally tangent to each other, and there is a larger circle circumscribing the six outer circles as shown. Points A, E, G, and C are points of tangency, point B is on the circumscribing circle, and points D, F, and H are on the smaller circles. Let b be the length of arcABC, and let s be the sum of the lengths of arcADE, arcEFG, and arcGHC. Compute $b - s$.  	10.
11. In order to play the "Power of 2" game, Noah tosses a standard die 6 times. If exactly three of the six tosses are either 2 or 4, Noah wins. If the probability that Noah wins is expressed as $\frac{p}{q}$, where p and q are relatively prime, compute $p + q$.	11.
12. One can denote 146 in base 7 by $146_7 = 1 \cdot 7^2 + 4 \cdot 7^1 + 6 \cdot 7^0 = 49 + 28 + 6 = 83$. Compute the greatest common integer factor of 45_7 and 302_5. Express your answer in base 10.	12.
13. Three lines, $y = -2$, $y = -2x - 1$, and $y = 2x - 5$ are each tangent to $y = a(x + b)^2 + c$. If the point $(0, -1)$ is one of the points of tangency, compute the product, abc.	13.
14. In the diagram, $\overline{AB}$, with $AB = 2$, is a common side of the regular hexagon and the regular octagon. Points C and D are vertices of the hexagon, and points E and F are vertices of the octagon. The distance between $\overline{CD}$ and $\overline{EF}$ is $a + \sqrt{b} - \sqrt{c}$, where a, b, and c are positive integers. Compute $a + b +$ c.  	14.
15. Two diagrams are provided. In the diagram to the left, there are three rectangles in which the base and height of the first and third rectangles are 1 and 5, respectively, and the base of the middle rectangle is 1. The centers of these three rectangles are collinear. When the middle rectangle is rotated so that point A, the top right vertex of the middle rectangle maps to point B, the top right vertex of the first rectangle, and point C, the bottom left vertex of the middle rectangle maps to point D, the bottom left vertex of the third rectangle, the letter "N" is formed as shown in the diagram to the right. The area of the region contained in "N" is $\frac{a+b\sqrt{c}}{d}$, where a, b, and d are relatively prime, and c does not have an integral perfect square factor greater than 1. Compute $a + b + c + d$.  	15.

Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM #

Mathematics Tournament 2020

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Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. If $f(x)$ is an odd polynomial function such that $f(-5) = -22$ and if $g(x) = f(x-4)$, compute $g(9)$.	1.
2. Compute $(7+5i)(3-2i)+\frac{2-2i}{2+2i}$.	2.
3. Compute the number of even divisors of 14^{10}.	3.
4. Compute the only solution of $\sqrt{2x-4}-\sqrt{x+5}=\sqrt{x-25}$.	4.
5. If $3\cos^2 x - 8\cos x + 4 = 0$ on the interval $\left[0, \frac{\pi}{2}\right]$, the value of $\tan x$ can be expressed in simplest form as $\frac{\sqrt{p}}{q}$. Compute pq.	5.
6. Compute the number of integer values of k, for which it is true that the product of any pair of these powers of 2: $2^7, 2^{15}$, and 2^k is greater than the remaining power of 2.	6.
7. Sophia leaves her house in the morning to go to school. If she rides her bike at an average rate of 10 miles per hour, she will arrive 10 minutes late to first period. If her mother drives her along the same route at an average rate of 25 miles per hour, she will arrive 8 minutes early. Compute the distance, in miles, along the route between Sophia's house and her school.	7.
8. Equilateral ΔABC is inscribed in circle O. Equilateral ΔDEF is inscribed in ΔABC with points *D, E*, and *F* the midpoints of $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$ respectively. Compute k if $DE = 10\sqrt{3}$ and the area of circle O is $k\pi$.	8.

Turn Over

11

Mathematics Tournament 2020

Grade 11

Time Limit: 45 minutes

Upper Division

Answer Column

9. Compute the number of integral solutions of $(x^2+3x+2)(x^2-x-6) \leq 0$.	9.
10. Compute the absolute value of the difference of the solutions to the equation $\log_8 x^3 = 7 - \frac{2}{\log_8 x}$.	10.
11. Compute the value of n that satisfies the equation $(n+2)! - 2n! = 180n!$.	11.
12. The 3-term sequence: $1,\ \sin x,\ \sin(x)\cos(2x)$ is a geometric sequence for several values of x in the interval $[0, 2\pi]$. The sum of all such values of x can be expressed as $\frac{p}{q}\pi$, where p and q are relatively prime. Compute $p+q$. [Note: None of the terms of the given sequence is 0].	12.
13. In a middle school of 200 students, consisting of only seventh and eighth graders, 20% of the students who do not like math are in the seventh grade and 30% of the students who like math are in the eighth grade. The seventh grade consists of 57.5% of the student population. Each middle school student either likes math or does not like math. Compute the number of seventh grade students who like math.	13.
14. In parallelogram $ABCD$, points E and F are on $\overline{BC}$ and $\overline{CD}$, respectively so that point E is the midpoint of $\overline{BC}$ and $CF : FD = 3$. If point G is the intersection of $\overline{BF}$ and $\overline{AE}$, and $GE = 12$, compute AG.	14.
15. Your math teacher is creating a test consisting of 3 true-false questions, followed by 4 multiple-choice questions. The multiple-choice questions each have three choices. Compute the number of versions of the test that the teacher can create if the 3 true-false questions must not all have the same answer and the 4 multiple choice questions can have the questions arranged in any order and the choices in each question arranged in any order.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

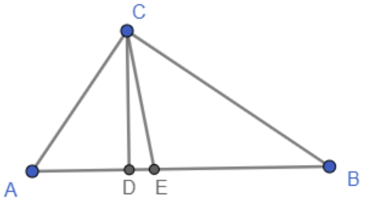
TEAM # ☐

Mathematics Tournament 2020

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. A mathematics textbook was priced originally at \$120. Alice bought a copy with discounts of 25% and 10%, applied in that order. Brendon bought another copy of the same book with the same two discounts, applied in the opposite order. Compute the absolute value of the difference, in dollars, between the prices that Alice and Brendon paid.	1.
2. Compute the sum of the solutions of $(x-17)^2-22(x-17)+57=0$.	2.
3. Consider the equation $(3-i)z-(15+16i)=17+10i$. If $z=a+bi$, where a and b are real, compute $100a+b$.	3.
4. Let $f(x)=\ln\left(\frac{7e^{x^3}}{5}\right)$. Compute $f'(9)$.	4.
5. Compute the value of b such that the graphs of $y=-\frac{1}{2}x+b$ and $y=\lvert x-130\rvert+141$ intersect in exactly one point.	5.
6. If $g(x)=\cos^4 x-\sin^4 x$, compute $16\left[g'\left(\frac{\pi}{6}\right)\right]^4$.	6.
7. Compute the number of arrangements of the letters in the word "COMPUTE" in which no two vowels are adjacent and no two consonants are adjacent.	7.
8. In ΔABC, $AC=3$, $BC=4$, and $AB=5$. Segment $\overline{CD}$ is an altitude and $\overline{CE}$ is an angle bisector of ΔABC. The length of $\overline{DE}$, in lowest terms, is $\frac{p}{q}$. Compute $p+q$. 	8.
9. If $x^3-y^3=133$ and $x-y=7$, compute x^2y^2.	9.

Turn Over

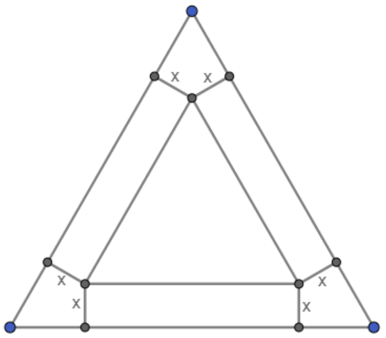
12

Mathematics Tournament 2020

Grade 12

Time Limit: 45 minutes

Upper Division

	Answer Column
10. A fair 6-sided die is rolled 3 times. If p is the probability that neither the second roll nor the third roll is greater than the first roll, compute $216p$.	10.
11. A sequence $\{a_n\}_{n=1}^{\infty}$ has the property that, for all integers $n \geq 2$, $a_n = \frac{a_{n+1}+a_{n-1}}{2}$. If $a_1 = 1$ and $a_{2020} = 10096$, compute a_{123}.	11.
12. Let $k = xy$, where (x, y) is any point on the graph of $2x^2 + 2y^2 = 2020$. Compute the maximum possible value of k.	12.
13. A cone is inscribed in a sphere of radius 10. Cross-sections of the cone taken through its vertex, perpendicular to its base, are equilateral triangles. If the volume of the cone is $n\pi$, compute n.	13.
14. Let $f(x) = x^2 - 6$. The values of k for which $f(k) \neq k$, but $f\big(f(k)\big) = k$, may be written as $k = \frac{-a\pm\sqrt{b}}{c}$, where a, b, and c are positive integers and the largest perfect square factor of b is 1. Compute the product, abc.	14.
15. A piece of cardboard of negligible thickness in the shape of an equilateral triangle, with side length $144\sqrt{3}$, has three congruent kites cut off each corner as shown. The three rectangular "tabs" along the outside are then bent up along their edges to form an open-top box in the shape of an equilateral triangular prism. Compute the value of x such that the filled box contains its maximum possible volume. 	15.

M

Nassau County Interscholastic Mathematics League

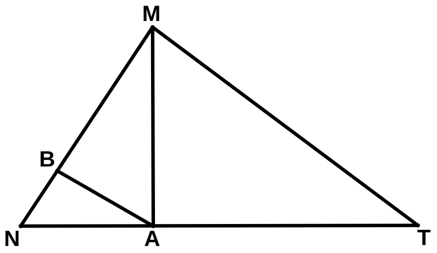
Mathletics

TEAM # ☐

Mathematics Tournament 2020

Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** __________

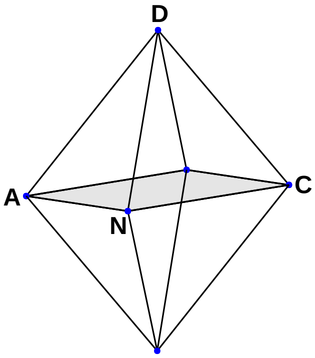
Time Limit: 30 minutes	*Answer Column*
1. The sum of four consecutive even integers is 2020. Compute the greatest of these integers.	1.
2. The positive solution of $(7x-5)^2-(10x+3)^2=0$ can be expressed as $\frac{p}{q}$ in lowest terms. Compute $p+q$.	2.
3. Points A, P, and B are collinear with point P between points A and B such that $AP : PB = 3 : 5$. The coordinates of points A, P, and B are, respectively, $(20,20)$, $(50,44)$, and (x,y). Compute $x+y$.	3.
4. Compute the least positive integer n such that the product $2020 \cdot n$ is a perfect square.	4.
5. In the diagram, $\overline{MA}$ is an altitude of right ΔNMT with hypotenuse $\overline{NT}$, and $\overline{AB}$ is an altitude of ΔNAM. If $NA=500$ and $AT=1000$, compute the length of $\overline{AB}$ to the nearest integer. 	5.

Turn Over

M

Mathematics Tournament 2020

Mathletics

Time Limit: 30 minutes	*Answer Column*
6. An equation of circle M is $(x-12)^2+(y-4)^2=9$ and an equation of circle N is $x^2+y^2+12x+20y+120=0$. The minimum distance between circle M and circle N is $\sqrt{p}-q$. Compute $p+q$.	6.
7. A closed triangular region of the plane is bounded by the lines n, m, and t. Lines n and m pass through the point $(20,20)$ and have slopes of 4 and $\frac{1}{4}$, respectively. The equation of line t is $x+y=80$. Compute the area of the triangular region.	7.
8. Cindy and Kevin play the following card game of chance: Cindy is holding 5 red cards and Kevin is holding 5 black cards. On each of five turns a fair coin is tossed. If it lands heads up, Cindy places a red card on the table. If it lands tails up, then Kevin places a black card on the table. After five turns, there are five cards on the table arranged in a row in the order in which they were placed. Each card, of course, is either black or red. Cindy is the winner if and only if all of the red cards are adjacent to each other and all of the black cards are adjacent to each other. As an illustration, these are some possible game outcomes: RRRBB: Cindy wins; BRRRB: Kevin wins; BBBBB: Cindy wins; RRBBR: Kevin wins The probability that Cindy wins, in simplest form, is $\frac{p}{q}$. Compute $p+q$.	8.
9. Define a sequence recursively as follows: $a_1=1, a_2=\frac{2}{3}$, and $a_n=\frac{(a_{n-2})(a_{n-1})}{2a_{n-2}-a_{n-1}}$, where n is an integer greater than 2. If $a_{800}=\frac{p}{q}$ in lowest terms, compute $p+q$.	9.
10. The volume of a regular octahedron is 2020. To the nearest integer, compute the area of the planar cross-section that includes points A, N, and C.  	10.

T

Nassau County Interscholastic Mathematics League

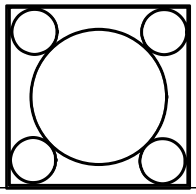
Team Problem Solving

TEAM # ☐

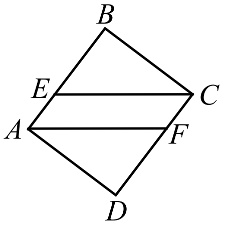
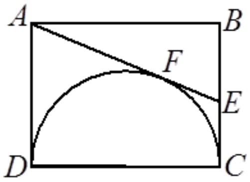
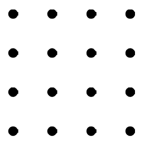
Mathematics Tournament 2020

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________________ **Score** ________

<table>
<tr><th>Time Limit: 60 minutes</th><th>Answer Column</th></tr>
<tr><td>1. The sum of 20 consecutive prime numbers is odd. Compute the least of these primes.</td><td>1.</td></tr>
<tr><td>2. The cube of the surface area of a cube is n times the square of the volume of the cube. Compute n.</td><td>2.</td></tr>
<tr><td>3. The rows in the grid at the right are partially filled with three 4-digit numbers, one digit per box. The two greatest prime factors of the 4-digit number in the top row of the grid are 17 and 31, in the middle row are 11 and 19, and in the bottom row are 11 and 43. Compute the sum of the 5 missing digits.
<table>
<tr><td>4</td><td>2</td><td></td><td>6</td></tr>
<tr><td></td><td></td><td>2</td><td>4</td></tr>
<tr><td></td><td></td><td>6</td><td>8</td></tr>
</table></td><td>3.</td></tr>
<tr><td>4. A positive integer is 343 times the square root of its reciprocal. Compute the integer.</td><td>4.</td></tr>
<tr><td>5. If n, m, and t are positive integers such that $nm = 48$, $mt = 54$, and $nt = 72$, compute the value of $n + m + t$.</td><td>5.</td></tr>
<tr><td>6. Given rectangle $ABCD$ with shorter side $AB = 6$. Point P is on $\overline{BC}$ such that $\measuredangle BAP \cong \measuredangle PAC \cong \measuredangle CAD$. If the perimeter of ΔPAC can be written as $a + b\sqrt{3}$, compute the product, ab.</td><td>6.</td></tr>
<tr><td>7. Use each of the 10 different digits exactly once to compute the least possible difference of two 5-digit numbers.</td><td>7.</td></tr>
<tr><td>8. In the diagram, the four congruent circles each have a radius of 1 and they are each tangent to two sides of the square and the center circle whose radius is 3. The area of the square can be written as $a + b\sqrt{2}$. Compute $a + b$.
</td><td>8.</td></tr>
<tr><td>9. If $a + \frac{1}{a} = 3$, compute $a^4 + \frac{1}{a^4}$.</td><td>9.</td></tr>
<tr><td>10. Given that d is a single digit, the numbers $d.5$ and $0.d5$ are decimal numbers. If $\frac{(d.5)^4}{(0.d5)^2} = 5625$, compute the value of d.</td><td>10.</td></tr>
</table>

Turn Over

11. In rectangle $ABCD$, $BC = 120$. Point E is on $\overline{AB}$ such that $AE = 45$ and $EC = 150$. If F is on $\overline{CD}$ such that $\overline{AF} \parallel \overline{EC}$, compute the distance between the parallel lines $\overleftrightarrow{EC}$ and $\overleftrightarrow{AF}$. 	11.
12. The three numbers, 12, b, c, in this order, form an arithmetic sequence. If the numbers, 12, $b + 3$, $c + 18$, in this order, form a geometric sequence, compute the positive value of c.	12.
13. The mean and the median of the five numbers $22, 37, 56, 89$, and x are the same number. There are three possible values of x. Compute the sum of these three values.	13.
14. At a local animal shelter, the ratio of dogs to cats is 5 : 3. The average weight of the dogs is 28 pounds, and the average weight of the cats is 20 pounds. Compute the average weight of all of the dogs and cats at the shelter.	14.
15. Let $f\left(\frac{x}{2}\right) = x^2 - x + 12$. Let k be the sum of the solutions of $f(2w) = 14$. Compute $68k$.	15.
16. The sum of all of the roots of the equation $\sqrt{3}\cot(5x) = 1$ on the interval $[0, 2\pi]$ is, in simplest form, $\frac{a\pi}{b}$, where a and b are integers. Compute $a + b$.	16.
17. In rectangle $ABCD$, $AB = 84$ and $AD = 63$. The diameter of the semicircle shown is $\overline{DC}$ and $\overline{AFE}$ is tangent to the semicircle at point F with point E on $\overline{BC}$. Compute AE. 	17.
18. Compute the units digit of the product $2^{2019} \times 3^{2020} \times 7^{2021}$.	18.
19. Compute the value of the expression $\frac{1}{\log_2 10!} + \frac{1}{\log_3 10!} + \frac{1}{\log_4 10!} + \cdots + \frac{1}{\log_{10} 10!}$.	19.
20. Four points are to be randomly chosen from the 16 points in the diagram. All points are lattice points on a square grid and equally likely to be selected. Compute the reciprocal of the probability that the 4 points all lie on the same line. 	20.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

TEAM #

Mathematics Tournament 2020

DO <u>NOT</u> HAND THIS COPY IN. HAND IN THE ONE TEAM COPY.
Calculators may be used on this part.
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Three (3) points per correct answer.

Individual Copy

Time Limit: 60 minutes *Answer Column*

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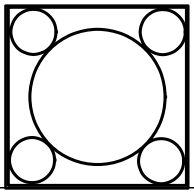
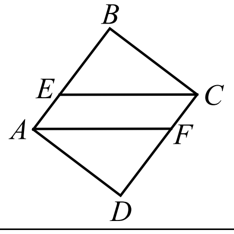
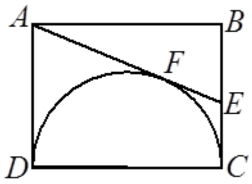
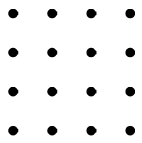
<tr><td>4. A positive integer is 343 times the square root of its reciprocal. Compute the integer.</td><td>4.</td></tr>
<tr><td>5. If n, m, and t are positive integers such that $nm = 48$, $mt = 54$, and $nt = 72$, compute the value of $n + m + t$.</td><td>5.</td></tr>
<tr><td>6. Given rectangle $ABCD$ with shorter side $AB = 6$. Point P is on $\overline{BC}$ such that $\measuredangle BAP \cong \measuredangle PAC \cong \measuredangle CAD$. If the perimeter of ΔPAC can be written as $a + b\sqrt{3}$, compute the product ab.</td><td>6.</td></tr>
<tr><td>7. Use each of the 10 different digits exactly once to compute the least possible difference of two 5-digit numbers.</td><td>7.</td></tr>
<tr><td>8. In the diagram, the four congruent circles each have a radius of 1 and they are each tangent to two sides of the square and the center circle whose radius is 3. The area of the square can be written as $a + b\sqrt{2}$. Compute $a + b$.
</td><td>8.</td></tr>
<tr><td>9. If $a + \frac{1}{a} = 3$, compute $a^4 + \frac{1}{a^4}$.</td><td>9.</td></tr>
<tr><td>10. Given that d is a single digit, the numbers $d.5$ and $0.d5$ are decimal numbers. If $\frac{(d.5)^4}{(0.d5)^2} = 5625$, compute the value of d.</td><td>10.</td></tr>
</table>

Turn Over

T

Mathematics Tournament 2020

Team Problems

Time Limit: 60 minutes	*Answer Column*
11. In rectangle $ABCD$, $BC = 120$. Point E is on $\overline{AB}$ such that $AE = 45$ and $EC = 150$. If F is on $\overline{CD}$ such that $\overline{AF} \parallel \overline{EC}$, compute the distance between the parallel lines $\overleftrightarrow{EC}$ and $\overleftrightarrow{AF}$. 	11.
12. The three numbers, 12, b, c, in this order, form an arithmetic sequence. If the numbers, 12, $b+3$, $c+18$, in this order, form a geometric sequence, compute the positive value of c.	12.
13. The mean and the median of the five numbers $22, 37, 56, 89$, and x are the same number. There are three possible values of x. Compute the sum of these three values.	13.
14. At a local animal shelter, the ratio of dogs to cats is 5 : 3. The average weight of the dogs is 28 pounds, and the average weight of the cats is 20 pounds. Compute the average weight of all of the dogs and cats at the shelter.	14.
15. Let $f\left(\frac{x}{2}\right) = x^2 - x + 12$. Let k be the sum of the solutions of $f(2w) = 14$. Compute $68k$.	15.
16. The sum of all of the roots of the equation $\sqrt{3}\cot(5x) = 1$ on the interval $[0, 2\pi]$ is, in simplest form, $\frac{a\pi}{b}$, where a and b are integers. Compute $a + b$.	16.
17. In rectangle $ABCD$, $AB = 84$ and $AD = 63$. The diameter of the semicircle shown is $\overline{DC}$ and $\overline{AFE}$ is tangent to the semicircle at point F with point E on $\overline{BC}$. Compute AE. 	17.
18. Compute the units digit of the product $2^{2019} \times 3^{2020} \times 7^{2021}$.	18.
19. Compute the value of the expression $\frac{1}{\log_2 10!} + \frac{1}{\log_3 10!} + \frac{1}{\log_4 10!} + \cdots + \frac{1}{\log_{10} 10!}$.	19.
20. Four points are to be randomly chosen from the 16 points in the diagram. All points are lattice points on a square grid and equally likely to be selected. Compute the reciprocal of the probability that the 4 points all lie on the same line. 	20.

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2020

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: *Answer Column*

1.	1.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: *Answer Column*

2.	2.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: *Answer Column*

3.	3.