

Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM #

Mathematics Tournament 2019

No calculators may be used on this part.
 All answers will be integers from 0 to 999 inclusive.
 One (1) point for each correct answer.

Name _____ School _____ Score _____

Time Limit: 45 minutes

Lower Division

Answer Column

1. The area of a circle is 36π . Compute the length of the diameter of the circle.	1.
2. A line passes through the points whose coordinates are $(31, -2)$ and $(79, -10)$. Compute the x -intercept of this line.	2.
3. The sum of $\sqrt{72}$ and $3\sqrt{162}$ is expressed as $a\sqrt{b}$ in simplest radical form. Compute $a + b$.	3.
4. If $abcd = 0$ and $abde = 1$, compute $c(a + b + c + d + e)$.	4.
5. The mean of Jim's first 7 tests this marking period is 72. There are 3 tests remaining for the marking period. If the maximum score on a test is 100, and all scores are integers, compute the minimum score he can receive on any of the next 3 tests if he wants to achieve a mean marking period score of at least 80.	5.
6. The fraction $\frac{5}{13}$ is written in decimal form. Compute the digit in the 2019 th place to the right of the decimal point.	6.
7. One leg of a right triangle has a length of 20. The other two sides have lengths that are consecutive odd integers. Compute the length of the hypotenuse.	7.
8. On the same set of rectangular coordinate axes, the lines $y = 1$, $y = x + 5$, and $x + y = 5$ are drawn. Compute the area of the triangular region bounded by these lines.	8.

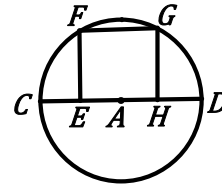
Turn Over

Time Limit: 45 minutes

Lower Division

Answer Column

9. If $f(x) = x^2 + 5x - 36$ and $f(2a - 1) = 0$, compute the positive value of $2a$.	9.
10. At the same time, two trains, 1000 miles apart, start traveling toward each other on a straight track. During successive hours, one train travels 30 miles, 31 miles, 32 miles... , increasing one mile each hour. During the same hours, the other train travels 30 miles, 29 miles, 28 miles... , decreasing one mile each hour. Throughout each hour, each train travels at a constant rate. How many miles apart are the trains 23 minutes before they pass each other?	10.
11. Square $EFGH$ is inscribed in a semicircle whose center is point A and whose diameter is \overline{CD} . If $CD = 10$, compute the area of square $EFGH$.	11.
12. A positive integer, n , has a remainder of 9 when divided by 11, a remainder of 7 when divided by 8, a remainder of 5 when divided by 6, a remainder of 3 when divided by 4, and a remainder of 1 when divided by 2. Compute the smallest possible positive value of n that meets these conditions.	12.
13. Given the integers x, y , and z . The arithmetic mean of x and y is 3 less than z . The arithmetic mean of x and z is 6 less than y . The arithmetic mean of y and z is x less than 3. Compute z .	13.
14. A box contains 20 slips of paper numbered 1 through 20. Six slips of paper are selected at random, without replacement, and placed on a table in the order they were selected. If the probability that the papers were selected in ascending numerical order is expressed as a fraction $\frac{a}{b}$ in simplest form, compute $a + b$.	14.
15. The length of each side of a square-shaped pond of constant depth is 10 meters. The bottom of the pond and the surface of the pond have the same shape and area. The height of a reed growing from the bottom center of the pond is 1 meter higher than the pond's surface. When pulled to a corner of the pond, the top of the reed just touches the surface of the pond. If the depth of the pond is d , compute $10d$.	15.



Mathematics Tournament 2019

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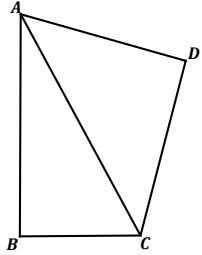
Name _____ School _____ Score _____

Time Limit: 45 minutes

Lower Division

Answer Column

<p>1. The coordinates of the vertices of $\triangle ABC$ are $A(4, 5)$, $B(2, 1)$, and $C(7, 3)$. If the slope of the altitude from vertex B of $\triangle ABC$ to side \overline{AC}, expressed in simplest form, is $\frac{p}{q}$, compute $p + q$.</p>	<p>1.</p>
<p>2. . If $f(2x - 1) = x^2 - 4x + 1$, compute the value of $f(11)$.</p>	<p>2.</p>
<p>3. In the diagram, $AB = 6\sqrt{3}$, $BC = 6$, $AD = CD = 6\sqrt{2}$, and $AC = 12$. The area of quadrilateral $ABCD$ is $a\sqrt{b} + c$, where b is a prime number, and a and c are integers. Compute $a + b + c$.</p>	<p>3.</p>
<p>4. The roots of $x^2 + ax + b = 0$ can be expressed as $a + 49$ and $a + 50$. Compute the value of b.</p>	<p>4.</p>
<p>5. There are exactly two regular n-sided convex polygons for which the difference in measures of an interior angle and an exterior angle is 60°. Compute the greater value of n.</p>	<p>5.</p>
<p>6. Under a dilation by a factor of 2, centered at the point (a, b), the point $(2, -3)$ is the image of the point $(5, 2)$. Compute the product, ab.</p>	<p>6.</p>
<p>7. The ratio of the surface areas of two spheres is $539 : 44$. If the ratio of the lengths of the radii of the larger sphere to the smaller sphere is $\frac{a}{b}$, where $\frac{a}{b}$ is in simplest form, compute a^b.</p>	<p>7.</p>
<p>8. When Claire runs a particular distance, she runs one quarter of the distance at 12 miles per hour and she runs the rest of the distance at 6 miles per hour. If her average speed for the entire run can be expressed in simplest form as $\frac{p}{q}$, compute $p + q$.</p>	<p>8.</p>

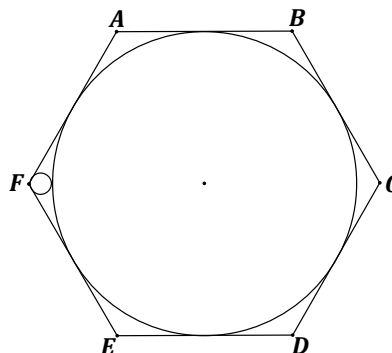


Time Limit: 45 minutes

Lower Division

Answer Column

9. In circle B , the measure of $\sphericalangle ABC$ of sector ABC is 1.5 radians. The perimeter of sector ABC is 35. Compute the <u>length</u> of minor arc AC .	9.
10. A regular octagon is circumscribed about a circle. If the length of each side of the octagon is 20, the length of a radius of the circle is, in simplest form, $a + b\sqrt{c}$. Compute $a + b + c$.	10.
11. In the coordinate plane, a triangle is determined by the origin and the x - and y -intercepts of the line $y = 3x - b$, where $b > 0$. The area of this triangle is 1200. If the line's x -intercept can be written as $(p\sqrt{q}, 0)$, where $p\sqrt{q}$ is in simplest radical form, compute the product, pq .	11.
12. The line $12x + 5y = 60$ is tangent to a circle centered at the origin. If the length of a radius of the circle is $\frac{a}{b}$, where a and b are relatively prime, compute $a + b$. [Note: Two numbers are relatively prime if their only common factor is 1.]	12.
13. When $(x + y + z)^2(2x + 3y - z)$ is expanded and all like terms are combined, the result includes a term of the form axy^2 where a is an integer. Compute the value of a .	13.
14. As shown in the diagram, a circle is inscribed in hexagon $ABCDEF$ and a smaller circle is externally tangent to the larger circle and to sides \overline{AF} and \overline{EF} of the hexagon. [Note: Vertex F of the hexagon is <u>not</u> a point on the smaller circle.] The ratio of the lengths of the radii of the larger circle to the smaller circle is, in simplest form, $a + b\sqrt{c}$. Compute $a + b + c$.	14.
15. Joe has two fair dice. One is blue and the other is red. He rolls the dice simultaneously and the result on the blue die is b and the result on the red die is r . The probability that the expression $br + b + r$ is divisible by 3, in simplest form, is $\frac{p}{q}$. Compute $p + q$.	15.



Mathematics Tournament 2019

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Name _____ School _____ Score _____

Time Limit: 45 minutes

Upper Division

Answer Column

1. When $\left(\frac{(-3)^3}{2^6}\right)^{-2/3}$ is expressed in simplest $\frac{p}{q}$ form, where $\frac{p}{q}$ is a real number, compute $2p + q$.	1.
2. If $f(x) = 9x + 7$ and $g(x) = \sqrt{x}$, compute $g(f(2)) \cdot g^{-1}(3)$, where $g^{-1}(x)$ is the inverse of $g(x)$ under composition.	2.
3. If x , $2x$, and $3x$ are the degree-measures of the interior angles of a triangle, compute $\sin^2 x + \sin^2(2x) + \sin^3(3x)$.	3.
4. Square $MATH$ has sides of length 10. When \overline{AT} is extended through point T to point E , $AT = TE$. Then, \overline{ME} intersects side \overline{TH} in point R . Compute the area of quadrilateral $MATR$.	4.
5. Compute the value of k which makes the average rate of change of $f(x) = 2^x$ on the interval $[k, k + 2]$ equal to 48.	5.
6. Two circles whose equations are $(x - 5)^2 + (y + 1)^2 = 45$ and $(x + 5)^2 + (y - 4)^2 = 80$ intersect in points P and Q . Compute the slope of \overline{PQ} .	6.
7. Todd and Sam each mow a lawn at a steady rate. Todd can mow the lawn 20 minutes faster than Sam can. Working together, they can mow the lawn in $\frac{2}{5}$ hour. Compute the number of minutes it takes Sam to mow the lawn by herself.	7.
8. Compute the remainder when the product of all the positive integer values of x that satisfy the equation $(x^2 - 11x + 29)^{(x^2 - 9x + 18)} = 1$ is divided by 1000.	8.

Time Limit: 45 minutes

Upper Division

Answer Column

9. A rectangular prism whose side lengths are $5\sqrt{2}$, 12, and 13 is inscribed in a sphere. Compute the length of a side of the cube that can be inscribed in the same sphere.	9.
10. Compute the smallest positive integer which has exactly 10 positive integer factors.	10.
11. The value of x that satisfies the equation $3 + 6x + 12x^2 + 24x^3 + \dots = 25$ can be written in simplest form as $\frac{p}{q}$. Compute $p + q$. [Note: The given series is a geometric series.]	11.
12. The sum of the roots of the equation $\log_3 x + \log_x 3 = \frac{10}{3}$ can be expressed in the form $p + \sqrt[3]{q}$. Compute $\frac{p}{q}$.	12.
13. The coordinates of a minimum point of the graph of the function $f(x)$ are $(-4, -1)$. Compute the x -coordinate of the corresponding minimum point on the graph of the function $g(x) = 5f\left(-\frac{x}{3} + 10\right)$.	13.
14. If $f(x) = 8x^4 + ax^3 + bx^2 + ax + 8$, where a and b are positive integers and $f(x)$ has four distinct rational roots, compute $f(1)$.	14.
15. Mrs. Noether has five distinct name tags for five different students in her class. Compute the number of ways she can distribute the five name tags to the five students in her class so that no student has his/her own tag.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2019

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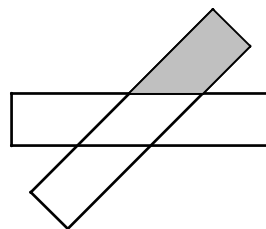
Name _____ School _____ Score _____

Time Limit: 45 minutes

Upper Division

Answer Column

1. Let $f(x) = 144 - 8x$. If $f(f(a)) = f(a)$, compute a .	1.
2. Let $f(x) = x^{1000} + x^{999} + x^{998} + \dots + x + 1$. Compute $ f'(-1) $.	2.
3. Compute the absolute value of the product of the least and greatest roots of $3x^3 - x^2 - 33x + 11 = 0$.	3.
4. Let $\{a_n\}$ be a geometric sequence such that $a_1 = 17$ and $a_{2019} = 34$. Compute the sum of all possible values of a_{1234} .	4.
5. Let b be the y -coordinate of the y -intercept of a tangent line to the graph of $y = x^2 + 4x + 5$. Considering every possible tangent line that can be drawn to the parabola, compute the maximum possible value of b .	5.
6. An integer k yields a remainder of 3 when divided by 6 and an integer h yields a remainder of 4 when divided by 8. Compute the number of possible remainders when kh is divided by 24.	6.
7. Compute $\lim_{x \rightarrow 1^-} (1 - x^2)(1 + x + x^2 + x^3 + \dots)$.	7.
8. A 2-by-10 rectangle is rotated 45° about its center, as shown. If the area of the shaded quadrilateral, in simplest form, is $a + b\sqrt{c}$, compute $100a + 10b + c$.	8.
9. Let $\triangle ABC$ be drawn in the complex plane with vertex A at the origin, vertex B at $6 + 0i$, and vertex C at $6 + 7i$. Compute the area of the triangle formed if each vertex of $\triangle ABC$ is multiplied by $3 - 4i$.	9.



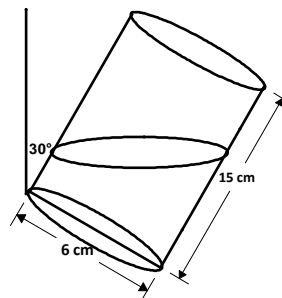
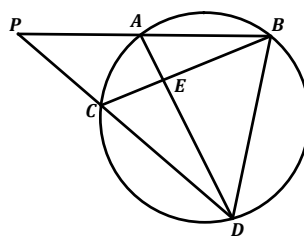
Turn Over

Time Limit: 45 minutes

Upper Division

Answer Column

<p>10. Two large containers each initially held 10 liters of liquid. The first held a solution which was half water, half alcohol. The second held only water. Two liters of solution are removed from the first container, added to the second container, and stirred thoroughly. Then, 2 liters of solution are removed from the second container, added to the first container, and stirred thoroughly, so that both containers once again hold 10 liters of liquid. After these actions, the fraction of the first container which is alcohol is, in simplest form, $\frac{p}{q}$. Compute $100p + q$.</p>	10.
<p>11. Nine points P_1, P_2, \dots, P_9 are placed, evenly spaced, in order about a circle. Then, every fourth point is connected by line segments to form the 9-pointed star $P_1P_5P_9P_4P_8P_3P_7P_2P_6$. Note that the final point is connected back to the initial point to make a closed, self-intersecting circuit of 9 line segments called an enneagram. Given that no three segments of the star are concurrent, compute the number of points inside the circle where two segments of the star intersect.</p>	11.
<p>12. The integers from 1 to 18 are randomly sequenced. The probability that the 1st number is greater than the 2nd number, the 3rd is greater than the 4th, the 5th is greater than the 6th, and so on for every pair of numbers is $\frac{1}{p}$. Compute p.</p>	12.
<p>13. Compute the total length of the polar curve $r = [\theta]$, where θ is measured in radians, and $0 \leq \theta < 6$. (Note: $[x]$ represents the greatest integer less than or equal to x).</p>	13.
<p>14. In the diagram (not drawn to scale), secants \overleftrightarrow{AB} and \overleftrightarrow{CD} to a circle intersect at point P. Chords \overline{AD} and \overline{BC} intersect at point E. If points $P, A, E,$ and C are concyclic, and arc BD (not arc BCD) has length 55, compute the length of \overline{BD} to the nearest integer. (Note: Points are concyclic if they lie on the same circle).</p>	14.
<p>15. A restaurant server is filling a cylindrical glass of water. The glass has a diameter of 6 cm and a height of 15 cm. The water is poured into the glass at a constant rate of $360 \text{ cm}^3/\text{min}$ with the glass held at a 30° tilt. The rate, in cm/min, at which the surface of the water is rising at the moment when the glass is half full is, in simplest form, $\frac{a\sqrt{b}}{\pi}$. Compute $a + b$.</p>	15.



Nassau County Interscholastic Mathematics League

M

Mathletics

TEAM #

Mathematics Tournament 2019

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 One (1) point for each correct answer.

Name _____ School _____ Score _____

Time Limit: 30 minutes

Answer Column

<p>1. Beth is selling a bunch of her books to the college bookstore. The cost of a geography book and biology book combined is equal to the cost of a mathematics book. The cost of a geography book and an economics book combined is equal to the cost of a biology book. Additionally, the cost of three economics books is equal to the cost of two mathematics books. If the cost of a biology book is n times the cost of a geography book, compute n.</p>	<p>1.</p>
<p>2. When the fraction $\frac{673}{3333}$ is expressed as a decimal, compute the 200th digit after the decimal point.</p>	<p>2.</p>
<p>3. A group of people went out to dinner and had to split equally a \$612 bill. One person forgot his wallet, so each of the remaining diners contributed an additional \$2 to cover his share of the bill. Compute the number of people that went out to dinner.</p>	<p>3.</p>
<p>4. Given $\frac{7x-4y}{2x+y} = 7$. Compute the value of $\frac{13x-11y}{x+y}$.</p>	<p>4.</p>
<p>5. A circle whose radius has length 4, has one diameter on each of the lines: $20x + 19y + 162 = 0$ and $y - x = 12$. If an equation of this circle is $x^2 + y^2 + ax + by + c = 0$, compute $a + b + c$.</p>	<p>5.</p>

Time Limit: 30 minutes

Answer Column

<p>6. Given the two series: $R = 3 + 1 + \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^{n-2}} + \cdots$, and $T = -50 + (-47) + (-44) + \cdots + 97 + 100$. Compute $T - 100R$.</p>	6.
<p>7. Emma and Ray start running at the same moment from opposite ends of the same straight road that is 1800 feet long. They each run the full length of the road, passing each other on their way to the other side. Without losing any time, they then turn around and run back to their starting positions, passing each other a second time. Emma runs at a constant rate of 12 feet per second, and Ray runs at a constant rate of 15 feet per second. Compute the total number of seconds it takes for Emma and Ray to pass each other the <i>second</i> time.</p>	7.
<p>8. A positive integer divisor of the number $20!$ is chosen at random. The probability that this chosen divisor is odd can be expressed in lowest terms as $\frac{p}{q}$. Compute $p + q$.</p>	8.
<p>9. An almost-square pyramid has a rectangular base with dimensions of lengths 19 and 20, and a height of length 30. A plane parallel to the base slices the pyramid into two solids with equal volume. Compute the area of the cross-section at the intersection of the given pyramid and the intersecting plane. Round your answer to the nearest integer.</p>	9.
<p>10. An entire firm of lawyers had to sort through two equally dense boxes of documents to prepare for trial. One box was three times larger than the other. The entire firm spent the first day and half of the second day sorting the documents in the larger box. During the second half of the second day, half of the firm started working on documents in the smaller box and the other half of the firm continued working on the documents in the larger box. At the end of the second day, the documents in the larger box were completely sorted, but the documents in the smaller box were not. On the third day, only eight of the lawyers worked on this case and finished the job after working the entire third day. Assuming that all of the lawyers worked at the same steady speed and that each working day had the same number of hours, compute the number of lawyers in the firm.</p>	10.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2019

HAND IN ONLY ONE ANSWER SHEET PER TEAM
 Calculators may be used on this part.
 All answers will be integers from 0 to 999 inclusive.
 Three (3) points per correct answer.

Team Copy School _____ Score _____

Time Limit: 60 minutes

Answer Column

1. Given $f(x) = x^2 - 7x + a$ and that $f(a) = -9$, compute $f(12)$.	1.
2. Compute the greatest integer less than 200 that has exactly 3 positive integer divisors.	2.
3. Three positive prime integers, a, b , and c form an arithmetic sequence. Compute the number of ordered triples (a, b, c) such that $a + b + c = 51$ and (a, b, c) is an arithmetic sequence.	3.
4. A circle is circumscribed about a square and another circle is inscribed in the same square. Compute the ratio of the area of the circumscribed circle to the area of the inscribed circle.	4.
5. Given that $x(x + y) = 7$ and $y(x + y) = 18$, compute the positive value of $x + y$.	5.
6. Compute the sum of all real roots of the equation $5(5^x + 5^{-x}) = 5^0 + 5^2$.	6.
7. A line with an equation in the form $Ax + By + C = 0$, where the only common factor of A, B , and C is 1, has a slope of $-\frac{2}{5}$ and the same y -intercept as the line with equation $3y + 8 = 5x$. Compute $A + B + C$.	7.
8. Three edges of a rectangular prism have lengths 5, 84, and 12. Compute the length of a diagonal of the rectangular prism.	8.
9. Compute x such that $\sqrt{x + \sqrt{x + 26}} + \sqrt{x - \sqrt{x + 26}} = 6$.	9.
10. Each of three circles is tangent externally to the other two circles. The triangle formed by connecting the three centers of the circles is a right triangle. If the radii of two of the circles are 5 and 6, compute the only possible integer radius of the third circle.	10.

11. The sum of the divisors of 120 is 360. Compute the sum of the reciprocals of the divisors of 120.	11.																														
12. If $a_0 = 5$ and $a_1 = 7$, and for $n \geq 2$, $a_n = \frac{3a_{n-1} + 2a_{n-2}}{3}$, compute a_3 .	12.																														
<p>13. Complete the number puzzle with only one digit in each box and then compute the sum of all the digits. Place the sum in the answer box.</p> <table style="width: 100%; border: none;"> <tr> <td style="text-align: center; width: 50%;"><u>Across</u></td> <td style="text-align: center; width: 50%;"><u>Down</u></td> <td style="border: 1px solid black; width: 30px; height: 30px; text-align: center;">1</td> <td style="border: 1px solid black; width: 30px; height: 30px; text-align: center;">2</td> <td style="border: 1px solid black; width: 30px; height: 30px; text-align: center;">3</td> <td style="border: 1px solid black; width: 30px; height: 30px; text-align: center;">4</td> </tr> <tr> <td>1. A palindromic number.</td> <td>1. A perfect fourth power.</td> <td style="border: 1px solid black; text-align: center;">5</td> <td style="border: 1px solid black;"></td> <td style="border: 1px solid black;"></td> <td style="border: 1px solid black;"></td> </tr> <tr> <td>5. A multiple of 401.</td> <td>2. Another perfect fourth power.</td> <td style="border: 1px solid black; text-align: center;">6</td> <td style="border: 1px solid black;"></td> <td style="border: 1px solid black;"></td> <td style="border: 1px solid black;"></td> </tr> <tr> <td>6. A number divisible by 11.</td> <td>3. An even Fibonacci number.</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td>4. A number with 3 different prime factors.</td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	<u>Across</u>	<u>Down</u>	1	2	3	4	1. A palindromic number.	1. A perfect fourth power.	5				5. A multiple of 401.	2. Another perfect fourth power.	6				6. A number divisible by 11.	3. An even Fibonacci number.						4. A number with 3 different prime factors.					13.
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14. The digit 7 is written to the right of a two-digit number, forming a three-digit number that is 754 more than the original two-digit number. Compute the original number.	14.																														
15. A group of counting numbers, all different, has a sum of 726, and the mean of the numbers is 66. Compute the greatest possible number in the group if one of the numbers is 176.	15.																														
16. Quadrilateral $ABCD$ is a rectangle with point E on side \overline{AD} . If $\angle BEC$ is a right angle, with $BE = 23$ and $CE = 37$, compute the area of the rectangle.	16.																														
17. The value of x that satisfies the equations $(\log_5 16)x + (\log_3 7)y = \log_{25} 8$ and $(\log_7 27)x - (\log_2 5)y = \log_{49} 243$ can be written as the fraction $\frac{a}{b}$, where $\frac{a}{b}$ is in simplest form. Compute ab .	17.																														
18. Compute the sum of all the integer values for x such that $x + 18$, $3x$, and $5x - 9$ can be the lengths of the sides of a triangle.	18.																														
19. For some x , $\sin x = 2 \cos x$. If $\sin x \cos x = \frac{c}{d}$, where $\frac{c}{d}$ is in simplest form, compute $10d + c$.	19.																														
20. Given that $f(f(f(x))) = 64x + 399$, where $f(x) = mx + b$, compute the sum $m + b$.	20.																														

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2019

DO NOT HAND THIS COPY IN. HAND IN THE ONE TEAM COPY.

Calculators may be used on this part.

All answers will be integers from 0 to 999 inclusive.

Three (3) points per correct answer.

Individual Copy

Time Limit: 60 minutes

Answer Column

Turn Over

T

Mathematics Tournament 2019

Team Problems

Time Limit: 60 minutes

Answer Column

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2019

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name _____ School _____ Score _____

Time Limit:

Answer Column

1.	1.
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Name _____ School _____ Score _____

Time Limit:

Answer Column

2.	2.
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Name _____ School _____ Score _____

Time Limit:

Answer Column

3.	3.
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