

Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM #

Mathematics Tournament 2018

No calculators may be used on this part.
 All answers will be integers from 0 to 999 inclusive.
 One (1) point for each correct answer.

Name _____ School _____ Score _____

Time Limit: 45 minutes

Lower Division

Answer Column

1. Three numbers are in the ratio 3 : 4 : 5. If the arithmetic mean of these three numbers is 24, compute the value of the least of the three numbers.	1.
2. In the xy -plane, the point whose coordinates are $(2, c)$ is on the graph of the line represented by $3x - y = -2$. Compute the value of c .	2.
3. The perimeter of a rectangle is 128. The greatest possible area of this rectangle is represented by A . Compute $\frac{A}{2}$.	3.
4. Compute the sum of the roots of $9x^2 - 9x = 10$.	4.
5. The length of the hypotenuse of a right triangle is 9 and the length of one leg is $3\sqrt{6}$. If the length of the other leg in simplest radical form is $p\sqrt{q}$, compute $p + q$.	5.
6. Amanda took a 60-question test in which each question has exactly one correct answer. Her score on this test is determined by subtracting 20% of the number of her incorrect answers from the number of her correct answers. If she answered every question and earned a score of 42, compute the number of questions she answered correctly.	6.
7. An isosceles triangle has a leg with a length of 10 and a base with a length of 16. The length of the height of a rectangle is 10 less than the length of the base of the rectangle. The area of the triangle is twice the area of the rectangle. Compute the length of the base of the rectangle.	7.
8. The population of Algebra City at 12:01 AM on January 1, 2015 is unknown. At 12:01 AM on January 1, 2016, its population increased by 1200. On January 1, 2017 at 12:01 AM, its population decreased by 11%, leaving 32 people fewer than before the 1200 person increase. If the population of Algebra City at 12:01 AM on January 1, 2015 is represented by x , compute $\frac{x}{20}$.	8.

Turn Over

Time Limit: 45 minutes

Lower Division

Answer Column

9. Eight girls and two boys get into a line in a random order. The probability that the two boys are standing next to each other is calculated and expressed as $p\%$. Compute p .	9.
10. If $N = 10^{101} - 1$ is expressed as an integer, compute the sum of the digits of N .	10.
11. Compute the sum of the integers which satisfy the compound inequality: $x^2 - 2x \leq 8$ and $x^2 - 2x \geq 3$.	11.
12. If $\frac{8^{1,000,000} + 4^{1,000,000}}{8^{400,000} + 4^{1,100,000}}$ is expressed as 256^{4000a} , compute a .	12.
13. It is given that $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$. When $\frac{3mr-nt}{7mr-4nt}$ is expressed in simplest form as $\frac{p}{q}$, compute $p + q$.	13.
14. Five-digit numbers are formed using the digits 1, 2, 3, 4, and 5 without repetition. When all of these five-digit numbers are listed in increasing order, compute the rightmost digit of the 86 th number in the list.	14.
15. Each face of a wooden cube is painted blue. Each edge of the cube has length n , where n is an integer greater than 2. The cube is then cut into n^3 smaller cubes, each with an edge-length of 1. If the number of smaller cubes with exactly one face painted blue is equal to the number of smaller cubes with zero faces painted blue, compute n .	15.

10

Grade 10

TEAM #

Mathematics Tournament 2018

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 One (1) point for each correct answer.

Name _____ School _____ Score _____

Time Limit: 45 minutes

Lower Division

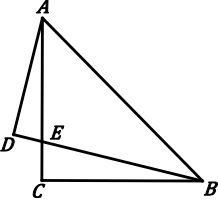
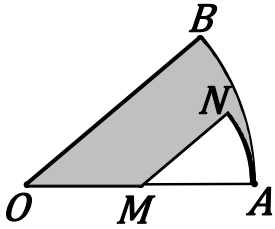
Answer Column

1. Compute the positive value of x that satisfies $\frac{1}{1-\frac{1}{x}} = x$.	1.
2. The least common multiple of n positive integers is 100. Compute the greatest value of n .	2.
3. The difference between the measures of an interior angle and an exterior angle of a regular n -sided convex polygon is 132° . Compute n , the number of sides of the polygon.	3.
4. Four cards labeled M, A, T, and H are arranged in a row from left to right in any order. All arrangements are equally likely. The probability that card "A" is the third card from the left is $\frac{a}{b}$, where a and b are positive integers that are relatively prime. Compute a^b . [Note: Two numbers are relatively prime if their only common factor is 1.]	4.
5. When $\sqrt[4]{9} - \sqrt{\frac{1}{3}}$ is expressed in simplest radical form as $\frac{a}{b}\sqrt{c}$, compute $a + b + c$.	5.
6. The lengths of the sides of a triangle are 40, 50, and 70. A second triangle is formed by joining the midpoints of the sides of the first triangle, and a third triangle is formed by joining the midpoints of the sides of the second triangle. Compute the perimeter of the third triangle.	6.
7. An isosceles triangle, the lengths of whose sides are 10, 10, and 12 is inscribed in a circle whose radius has length r . Compute $100r$.	7.
8. The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is 6 and the product of the roots is 10. If the roots are represented by p and q , compute $p^3 + q^3$.	8.

Time Limit: 45 minutes

Lower Division

Answer Column

<p>9. An equilateral triangle and a regular hexagon have the same perimeter. The ratio of the area of the triangle to the area of the hexagon is $a : b$, where a and b have no common factor other than 1. Compute $a + b$.</p>	<p>9.</p>
<p>10. An integer triangle is a triangle whose sides have lengths that are integers. Triangle ABC is an obtuse integer triangle with $AB = 5$ and $BC = 7$. Compute the sum of all possible values of AC.</p>	<p>10.</p>
<p>11. Triangle ABC and triangle ABD are right triangles with $m\angle C = m\angle D = 90^\circ$, $m\angle CBA = 45^\circ$, and $m\angle DBA = 30^\circ$. The ratio of the area of $\triangle ADE$ to the area of $\triangle BCE$ is $\frac{a}{b}$, where $\frac{a}{b}$ is reduced to lowest terms. Compute $a + b$.</p>	 <p>11.</p>
<p>12. Let $f(2a) = 2(f(a))^2 - 1$, where $f(a) \leq 0$ for all $a \in \mathbb{R}$. For example, if $f(b) = -\frac{1}{3}$, then $f(2b) = -\frac{7}{9}$. If $f(4c) = -\frac{1}{2}$, then $f(c) = -\frac{m}{n}$, where m and n are positive integers that are relatively prime. Compute $m + n$.</p>	<p>12.</p>
<p>13. In the diagram, AOB and AMN are sectors of circles O and M, respectively. Point M is the midpoint of OA and $m\angle AOB = m\angle AMN = \frac{2}{3}$ radians. If the perimeter of the shaded region $ABOMN$ is 18, compute the area of the shaded region $ABOMN$.</p>	 <p>13.</p>
<p>14. Four groups of students are to be seated in 8 chairs about a round table. Each group consists of 2 students. Compute the number of ways that each student will be seated next to his or her group member. [Note: If one particular seating arrangement is rotated either clockwise or counter-clockwise, it counts as one arrangement.]</p>	<p>14.</p>
<p>15. Operation C is defined as follows : If an inputted number is even, operation C will divide the number by 2, and if an inputted number is odd, operation C will add 1 to the number and then divide the result by 2. In either case, operation C continues until the output becomes 1. For example, if the input is 4, operation C will make it a 2 and then a 1. Another example: If the input is 5, operation C will make it a 3, followed by 2, and finally a 1. Thus, it takes 3 implementations of operation C to take an input of 5 and produce an output of 1. Compute the maximum number of inputted positive integers that will require 5 implementations of operation C to produce an output of 1.</p>	<p>15.</p>

Mathematics Tournament 2018

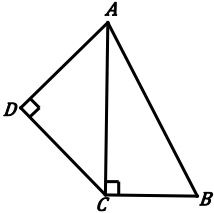
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Name _____ School _____ Score _____

Time Limit: 45 minutes

Upper Division

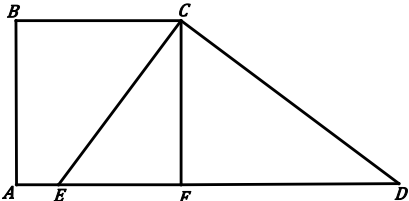
Answer Column

<p>1. Lily's first year's annual salary at Integrals Incorporated was \$2,018,314. After one year, she received a 100% raise in her annual salary. After her second year, she received a 50% cut in her annual salary. Compute the number of dollars by which her third year's annual salary exceeds her first year's annual salary.</p>	<p>1.</p>
<p>2. When the largest possible domain of the function $f(x) = \sqrt{105 - x - 2x^2}$ is expressed in interval form as $[a, b]$, compute the number of integers contained in the interval $[a, b]$.</p>	<p>2.</p>
<p>3. Euler, Newton, and Gauss have decided to write a mathematics textbook together. Euler can write a page of the textbook every 20 minutes. Newton can write a page of the textbook every 60 minutes and Gauss can write a page of the textbook every 30 minutes. Because of their teaching schedules, Euler can write only every third day. Gauss can write only two out of every three days. Newton teaches no classes and can write every day. If, on the days that they write the textbook, they each work for 10 hours, compute the number of days that it will take them to write 500 pages.</p>	<p>3.</p>
<p>4. For the function $f(x) = ax^5 - bx^3 + 2x - 1$, $f(-2) = -5$. Compute $f(2)$.</p>	<p>4.</p>
<p>5. If x and y are positive real numbers such that $xy = 25$ and $x^2 + y^2 = 350$, compute $x + y$.</p>	<p>5.</p>
<p>6. In the diagram, AB is two more than AC and BC is 9 less than AB. If $AD = DC$, then the perimeter of quadrilateral $ABCD$ can be expressed in simplest form as $a + b\sqrt{c}$. Compute $a + b + c$.</p>	 <p>6.</p>
<p>7. Compute the sum of the roots of $5 x - 20 ^2 - 19 x - 20 + 12 = 0$.</p>	<p>7.</p>
<p>8. An equilateral triangle is inscribed in a circle and the circle is inscribed in a square. If the length of one side of the equilateral triangle is 6, compute the area of the square.</p>	<p>8.</p>

Time Limit: 45 minutes

Upper Division

Answer Column

<p>9. If $2 \log_{72} x + 3 \log_{72} y = 1$, and x and y are positive integers, compute xy.</p>	<p>9.</p>
<p>10. Compute the number of points of intersection when the ellipse $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$ and the hyperbola $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$ are graphed in the same coordinate plane.</p>	<p>10.</p>
<p>11. If $2\cos(ax) = \frac{5}{3}$, where a is a real number, compute the value of $6 \sin\left(ax - \frac{3\pi}{2}\right)$.</p>	<p>11.</p>
<p>12. If the sum of the infinite series $\frac{1}{2^x} + \frac{1}{2^{2x}} + \frac{1}{2^{3x}} + \dots$ is $\frac{1}{255}$, compute the value of the positive integer x.</p>	<p>12.</p>
<p>13. A region is bounded by the parabola $y^2 = 36x$, the line $x = 4$, and the x-axis. The area of the circular cross-section with largest radius formed by revolving the bounded region about the x-axis is $a\pi$. Compute a.</p>	<p>13.</p>
<p>14. A trapezoid $ABCD$ has point E on \overline{AD}. When \overline{CE} is drawn, right triangle DCE is formed with right angle DCE. An altitude is drawn from point C. The foot of the altitude is point F on \overline{AD}, and $ABCF$ is a square. If the area of trapezoid $ABCE$ is 120 and $AE = 8$, compute FD.</p>	 <p>The diagram shows a trapezoid ABCD with vertices A, B, C, and D. The top base is BC and the bottom base is AD. Point E is located on the bottom base AD. A line segment CE is drawn, forming a right angle at E with the segment ED. Point F is also on the bottom base AD, to the right of E. A vertical line segment CF is drawn from point C to point F, representing an altitude. The quadrilateral ABCF is a square. The segment AE is labeled with a length of 8.</p>
<p>15. Compute the number of ways to distribute 10 identical pencils among 5 students if each student must receive at least one pencil.</p>	<p>15.</p>

Mathematics Tournament 2018

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Time Limit: 45 minutes

Upper Division

Answer Column

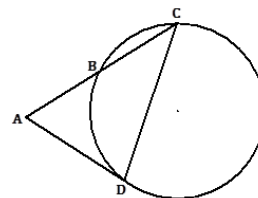
1. Let $f(x) = (x + 5)^2 - (x - 5)^2$. Compute $f'(2018)$.	1.
2. An m -by- n rectangle has a perimeter of 264 and an area of 12. Compute $\frac{1}{m} + \frac{1}{n}$.	2.
3. In the xy -plane, let d be the distance between the x -intercepts of $y = x^2 - 25x + 6$. Compute d^2 .	3.
4. Let a_1, a_2, a_3, \dots be an arithmetic sequence with a first term of 1 and a common difference of 3. Compute $\sum_{n=1}^{10} a_n$.	4.
5. When turned on full force by itself, a faucet will fill an empty pool in 10 hours. When turned on by itself, a pump will empty the same full pool in 12 hours. If both the faucet and the pump are run at the same time, compute the number of hours it will take to fill an initially empty pool. Assume that both the faucet and the pump run at constant rates.	5.
6. Let $f(x)$ be a function such that, for all real x , $f(x + 1) = f(x) + 2x + 1$. If $f(1) = 1$, compute $f(17)$.	6.
7. When a pile of n coins is divided into groups of 3, there is 1 coin left over. When divided into groups of 4, there are 2 left over. With groups of 5, there are 3 left over. With groups of 6, there are 4 left over, and with groups of 7, there are 5 left over. Compute the least possible positive value of n .	7.
8. The letters of the word "PEPPER" are written on six identical cards which are shuffled and dealt randomly in a row. One letter is written on each card. Let $\frac{1}{p}$ be the probability that the cards, from left to right, spell out "PEPPER." Compute p .	8.

Time Limit: 45 minutes

Upper Division

Answer Column

9. Point D is on the circle in the given diagram. Secant \overline{BC} and tangent \overline{AD} intersect outside the circle at point A . If B is the midpoint of \overline{AC} , $m\angle A = 60^\circ$, and $AD = \sqrt[4]{96}$, compute the area of $\triangle ACD$.



10. Let $P(n)$ be the perimeter of a regular n -gon with side-length 1. Let $A(n)$ be the area of the same polygon. Compute the integer closest to $\lim_{n \rightarrow \infty} \frac{7[P(n)]^2}{A(n)}$.

10.

11. Compute the straight-line distance between the points with polar coordinates $(39, 17^\circ)$ and $(65, 137^\circ)$.

11.

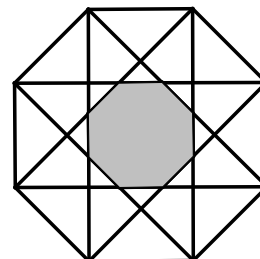
12. In the xy -plane, the line tangent to the graph of $y = x^3 - x$ at the point $(-1, 0)$ intersects the graph again at the point (a, b) . Compute $10a + b$.

12.

13. Let z_1 and z_2 be complex numbers such that $|z_1| = \sqrt{363}$ and $\frac{z_2}{z_1} = 2(\cos 60^\circ + i \sin 60^\circ)$. Compute $|z_2 - z_1|$.

13.

14. In a regular octagon, every third vertex is connected with line segments to form an 8-pointed star (called an octagram). The ratio of the area of the large octagon to the area of the small octagon at the center of the octagram (shaded in the diagram) is, in simplest form, $1 : (a - b\sqrt{c})$. Compute $100a + 10b + c$.



14.

15. A line drawn tangent to the graph of $y = \frac{1}{x^2}$ at a point in the first quadrant intersects the x -axis and the y -axis at points Q and P , respectively. The minimum possible distance PQ , in simplest form, is $\frac{a\sqrt{b}}{c}$. Compute $100a + 10b + c$.

15.

Nassau County Interscholastic Mathematics League

M

Mathletics

TEAM #

Mathematics Tournament 2018

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Name _____ School _____ Score _____

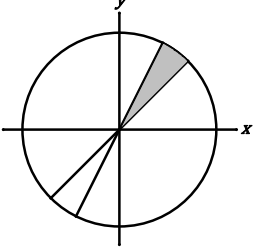
Time Limit: 30 minutes

Answer Column

1. Compute the value of k so that the equations $2x + 5y = 10$ and $6x + 15y = k$ both represent the same line.	1.
2. Compute the positive integer solution to $(x + 10)^2 - 1007(x + 10) = 2018$.	2.
3. The parabola $y = \frac{1}{3}x^2 + bx + c$ intersects the x -axis at $(-6, 0)$ and reaches a minimum at $(4, -3)$. Compute the x -coordinate of the other x -intercept.	3.
4. The number $\left(\frac{20}{18}\right)^2$ is written as a repeating decimal. Compute the sum of the first 100 digits after the decimal point.	4.
5. The function f is defined by: $f(x) = \begin{cases} x - 1 & \text{if } x \leq 3 \\ 2f(x - 1) & \text{if } 3 < x < 6 \\ f(x - 1) + f(x - 2) + 4 & \text{if } x \geq 6 \end{cases}$ Compute the value of $f(9)$.	5.

Time Limit: 30 minutes

Answer Column

<p>6. The image of point $A(3, 5)$ under dilation D is $A'(3, 5)$, and the image of point $B(4, 6)$ is $B'(6, 8)$. Under this same dilation, the image of the line $3x + 5y = 30$ is $ax + by = c$, where a, b, and c are positive integers with no common factors other than 1; i.e. $\text{GCF}(a, b, c) = 1$. Compute $a + b + c$.</p>	6.
<p>7. The numeral $2018!$ is written in base 16 or hexadecimal notation. Compute the number of consecutive zeroes at the right of the numeral.</p>	7.
<p>8. The base of a solid pyramid created by a 3-D printer is a regular hexagon with side length 10 cm. The distance from any vertex on the base to the apex of the pyramid is 20 cm. If the density of the 3-D printed pyramid is 0.25 g/cm^3, compute the mass of the hexagonal pyramid, rounded to the nearest whole gram, if necessary.</p>	8.
<p>9. In the diagram, the shaded region is a sector of the circle $x^2 + y^2 = 2018$ that is bounded by the lines $y = x$ and $y = 2x$. Compute, to the nearest integer, the area of this sector.</p>	 <p>9.</p>
<p>10. David, Wanlin, and Brandon attend a banquet where they are assigned seats at a circular table with 10 seats. They enter together and expect to be seated together. The seats have already been assigned randomly. The probability that the three students will in fact be seated together is $\frac{1}{n}$. Compute n.</p>	10.



Team Problem Solving

TEAM #

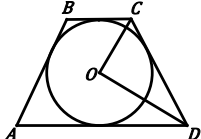
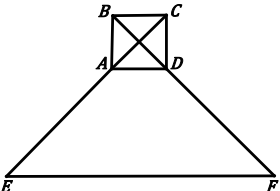
Mathematics Tournament 2018

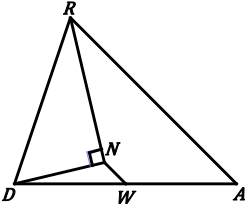
HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
 Calculators may be used on this part.
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 Three (3) points per correct answer.

Team Copy School _____ Score _____

Time Limit: 60 minutes

Answer Column

<p>1. Compute the sum of the mean, median, and mode for the data: 32, 36, 47, 35, 33, 36, 34, 35, and 36.</p>	<p>1.</p>
<p>2. Compute the least whole number N, such that $1120 \div N$ is a perfect square.</p>	<p>2.</p>
<p>3. Two different solutions of salt and water are combined to form one solution. The initial solutions contain 6 quarts of a 44% salt solution and 9.5 quarts of a 30% salt solution. The mixture contains 15.5 quarts of an $n\%$ salt solution. Compute n to the nearest whole number.</p>	<p>3.</p>
<p>4. If a right triangle with integer sides has a perimeter and area that have equal numerical values, we can call that triangle “amazing.” There are exactly two such “amazing” right triangles. Compute the sum of their perimeters.</p>	<p>4.</p>
<p>5. Circle O is inscribed in isosceles trapezoid $ABCD$ with $AB = CD$. If the area of $\triangle COD$ is 24, compute the area of trapezoid $ABCD$.</p>	 <p>5.</p>
<p>6. The graph of $y = x + 8 + 2x + 48$ passes through quadrants I, II, and III. Compute the number of lattice points it contains <u>in</u> quadrant II. [A lattice point is a point on a graph grid, both of whose coordinates are integers.]</p>	<p>6.</p>
<p>7. The diagonals of unit square $ABCD$ are extended to form trapezoid $ADFE$. If the height of the trapezoid is h and its area is 6 times the area of the square, compute $10h$.</p>	 <p>7.</p>
<p>8. The sum of the positive divisors of 240 is 744. If the sum of the reciprocals of these divisors of 240 is written in simplest $\frac{a}{b}$ form, compute $a + b$.</p>	<p>8.</p>
<p>9. If the center of the circle defined by $x^2 + y^2 - 8x + 12y + 3 = 0$ has coordinates (a, b), and the radius has length r, compute $a + b + r$.</p>	<p>9.</p>
<p>10. Compute the sum of all of the roots of $x^{\log x} = \frac{x^3}{100}$.</p>	<p>10.</p>

<p>11. In $\triangle DRA$, W is the midpoint of \overline{DA}, \overline{RN} bisects $\angle DRA$, $RD = 85$, and $RA = 121$. If $\overline{DN} \perp \overline{RN}$, compute NW.</p>	 <p>11.</p>
<p>12. Compute the number of integers between 1 and 2500 that have exactly 3 positive factors.</p>	<p>12.</p>
<p>13. In the multiplication shown, each different letter represents a different digit. Compute the sum of the digits in the word STRAP.</p>	$ \begin{array}{r} P \ A \ R \ T \ S \\ \times \quad 4 \\ \hline S \ T \ R \ A \ P \end{array} $ <p>13.</p>
<p>14. If $\log_3 \sqrt{x} \cdot \log_7 12 \cdot \log_{12} 81 = 4$, compute x.</p>	<p>14.</p>
<p>15. The bases of trapezoid $TRAP$ have lengths 14 and 35. The diagonals intersect at point X. A line is drawn through point X parallel to the bases of the trapezoid. The line intersects leg \overline{PT} in point O, and the line intersects leg \overline{AR} in point E. Compute OE.</p>	<p>15.</p>
<p>16. A jar filled with jelly beans has between 450 and 500 candies. The number of jelly beans is simultaneously 3 more than a multiple of 5, 3 more than a multiple of 6, and 3 more than a multiple of 8. Compute the number of jelly beans in the jar.</p>	<p>16.</p>
<p>17. If $\sec x - \tan x = 2$, compute the value of $24(\sec x + \tan x)$.</p>	<p>17.</p>
<p>18. The function f satisfies the following two properties: <i>i</i>) $f(0) = -1776$, and <i>ii</i>) $f(x + y) = x + f(y)$. Compute $f(2018)$.</p>	<p>18.</p>
<p>19. A fair six-sided die is rolled three times. The probability that at least one "2" is rolled, given that the sum of the first two rolls is equal to the number shown on the third roll, is written as $\frac{p}{q}$ in simplest form. Compute $p + q$.</p>	<p>19.</p>
<p>20. The coordinates of $\triangle ABC$ are $A(0,0)$, $B(33,0)$, and $C(0,20)$. Point A is reflected over \overline{BC} and its image is point A'. Point B is reflected over \overline{AC} and its image is point B'. Point C is reflected over \overline{AB} and its image is point C'. Compute the area of $\triangle A'B'C'$.</p>	<p>20.</p>



Team Problem Solving

TEAM #

Mathematics Tournament 2018

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Calculators may be used on this part.

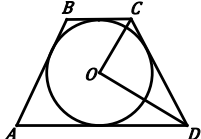
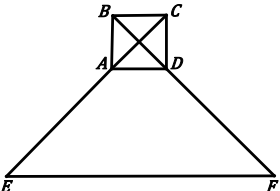
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Individual Copy

Time Limit: 60 minutes

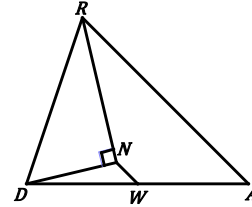
Answer Column

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<p>2. Compute the least whole number N, such that $1120 \div N$ is a perfect square.</p>	<p>2.</p>
<p>3. Two different solutions of salt and water are combined to form one solution. The initial solutions contain 6 quarts of a 44% salt solution and 9.5 quarts of a 30% salt solution. The mixture contains 15.5 quarts of an $n\%$ salt solution. Compute n to the nearest whole number.</p>	<p>3.</p>
<p>4. If a right triangle with integer sides has a perimeter and area that have equal numerical values, we can call that triangle “amazing.” There are exactly two such “amazing” right triangles. Compute the sum of their perimeters.</p>	<p>4.</p>
<p>5. Circle O is inscribed in isosceles trapezoid $ABCD$ with $AB = CD$. If the area of $\triangle COD$ is 24, compute the area of trapezoid $ABCD$.</p>	 <p>5.</p>
<p>6. The graph of $y = x + 8 + 2x + 48$ passes through quadrants I, II, and III. Compute the number of lattice points it contains <u>in</u> quadrant II. [A lattice point is a point on a graph grid, both of whose coordinates are integers.]</p>	<p>6.</p>
<p>7. The diagonals of unit square $ABCD$ are extended to form trapezoid $ADFE$. If the height of the trapezoid is h and its area is 6 times the area of the square, compute $10h$.</p>	 <p>7.</p>
<p>8. The sum of the positive divisors of 240 is 744. If the sum of the reciprocals of these divisors of 240 is written in simplest $\frac{a}{b}$ form, compute $a + b$.</p>	<p>8.</p>
<p>9. If the center of the circle defined by $x^2 + y^2 - 8x + 12y + 3 = 0$ has coordinates (a, b), and the radius has length r, compute $a + b + r$.</p>	<p>9.</p>
<p>10. Compute the sum of all of the roots of $x^{\log x} = \frac{x^3}{100}$.</p>	<p>10.</p>

Time Limit: 60 minutes

Answer Column

11. In $\triangle DRA$, W is the midpoint of \overline{DA} , \overline{RN} bisects $\angle DRA$, $RD = 85$, and $RA = 121$. If $\overline{DN} \perp \overline{RN}$, compute NW .



11.

12. Compute the number of integers between 1 and 2500 that have exactly 3 positive factors.

12.

13. In the multiplication shown, each different letter represents a different digit. Compute the sum of the digits in the word STRAP.

$$\begin{array}{r}
 P \ A \ R \ T \ S \\
 \times \quad 4 \\
 \hline
 S \ T \ R \ A \ P
 \end{array}$$

13.

14. If $\log_3 \sqrt{x} \cdot \log_7 12 \cdot \log_{12} 81 = 4$, compute x .

14.

15. The bases of trapezoid $TRAP$ have lengths 14 and 35. The diagonals intersect at point X . A line is drawn through point X parallel to the bases of the trapezoid. The line intersects leg \overline{PT} in point O , and the line intersects leg \overline{AR} in point E . Compute OE .

15.

16. A jar filled with jelly beans has between 450 and 500 candies. The number of jelly beans is simultaneously 3 more than a multiple of 5, 3 more than a multiple of 6, and 3 more than a multiple of 8. Compute the number of jelly beans in the jar.

16.

17. If $\sec x - \tan x = 2$, compute the value of $24(\sec x + \tan x)$.

17.

18. The function f satisfies the following two properties: *i*) $f(0) = -1776$, and *ii*) $f(x + y) = x + f(y)$. Compute $f(2018)$.

18.

19. A fair six-sided die is rolled three times. The probability that at least one "2" is rolled, given that the sum of the first two rolls is equal to the number shown on the third roll, is written as $\frac{p}{q}$ in simplest form. Compute $p + q$.

19.

20. The coordinates of $\triangle ABC$ are $A(0,0)$, $B(33,0)$, and $C(0,20)$. Point A is reflected over \overline{BC} and its image is point A' . Point B is reflected over \overline{AC} and its image is point B' . Point C is reflected over \overline{AB} and its image is point C' . Compute the area of $\triangle A'B'C'$.

20.

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2018

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

1.	1.
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Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

2.	2.
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Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

3.	3.
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