9

Nassau County Interscholastic Mathematics League

Grade 9

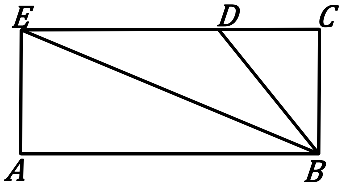
TEAM #

Mathematics Tournament 2017

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** __________

Time Limit: 45 minutes

Lower Division	*Answer Column*
1. Compute the area of ΔABC whose vertices are $A(-2,3)$, $B(-1,7)$ and $C(8,-1)$.	1.
2. Compute the sum of three consecutive positive odd integers if the square of the second integer exceeds three times the sum of the largest and smallest integers by 135.	2.
3. If p and r are the roots of the equation $3x^2 - x - 10 = 0$, compute the value of the product $(p-2)(r-2)$.	3.
4. In the diagram, $\overline{BE}$ is a diagonal of both rectangle $ABCE$ and trapezoid $ABDE$. If $AE = 10, BE = 26$, and $ED = 16$, compute the area of ΔBCD. 	4.
5. When $16\sqrt{243}$ is divided by $3\sqrt{72}$, the quotient, in simplest radical form, is $a\sqrt{b}$. Compute $a+b$.	5.
6. The surface area of a sphere is 144π in^2. If the volume of this sphere is $k\pi$ in^3, compute k.	6.
7. Using the letters of the word "SQUARE," compute the number of 6-letter arrangements that can be made if each letter is used exactly once, if each arrangement begins with a vowel, and if the vowels and consonants alternate.	7.
8. When the radius of a right circular cylinder is increased by 25%, the volume of the cylinder is unchanged, and the height of the cylinder is decreased by k%. Compute k.	8.

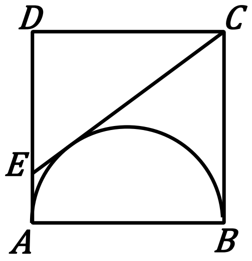
Turn Over

9

Mathematics Tournament 2017

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. The sum of two numbers, a and b, is 8, and their product is 3. Compute the value of $a^3 + b^3$.	9.
10. If A, B, and C represent three numbers such that $1001C - 2002A = 4004$ and $1001B + 3003A = 5005$, compute the average of A, B, and C.	10.
11. Given the equation $\frac{2016}{2017}x - 1 + \frac{1}{x} = 0$ with roots r_1 and r_2. If the sum of the reciprocals of r_1 and r_2, in simplest form, is p, compute p.	11.
12. Compute the units digit of 2017^{2017}.	12.
13. Bryanna and Sunil run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Bryanna has run 100 meters. The next time they meet, Sunil has run 150 meters past their first meeting point. If each one runs at a constant speed, compute, in meters, the length of the track.	13.
14. Samuel has 5 green cards numbered 1 through 5, and 4 yellow cards numbered 3 through 6. He stacks the cards so that the colors alternate and so that the number on each green card divides evenly into the number on each neighboring yellow card. Compute the sum of the numbers on the three middle cards.	14.
15. In the diagram, each side of square $ABCD$ has a length of 4. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. Compute the length of $\overline{CE}$. 	15.

10

Nassau County Interscholastic Mathematics League

Grade 10

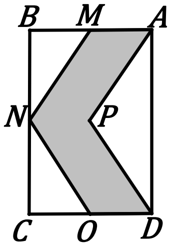
TEAM # ☐

Mathematics Tournament 2017

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** __________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. In the figure of rectangle $ABCD$, $AB=12$ and $BC=18$, where points M, N, and O are midpoints of the sides $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ respectively, and point P is the center of the rectangle. Compute the area of the shaded region $AMNODP$. 	1.
2. The least common multiple of the positive integers $3, m$, and n is 27. The greatest common divisor of the same three positive integers is 3. Compute the minimum possible value of $3mn$.	2.
3. Compute the length of the smaller dimension of a rectangle whose perimeter is 74 and whose area is 340.	3.
4. Compute a positive integer that has exactly 8 factors if two of them are 15 and 35.	4.
5. Each edge of a pyramid whose base is a square has a length of 4. If its volume, in simplest form, can be expressed as $\frac{a\sqrt{b}}{c}$, compute $a+b+c$.	5.
6. Three numbers form an arithmetic sequence with a common difference of 12. If the second and the third numbers are interchanged, the new numbers form a geometric sequence. Compute the sum of the absolute values of the numbers.	6.
7. A regular hexagon and an equilateral triangle are inscribed in a circle the length of whose radius is 2. If the area of the equilateral triangle is E, and the area of the regular hexagon is H, compute the product, EH.	7.
8. If $f(x+1)=ax^2+bx+c$, $f(0)=201$, and $f(2)=7$, compute $a+c$.	8.

Turn Over

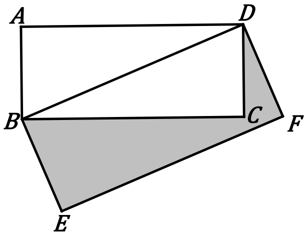
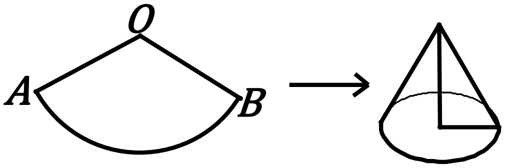
10

Mathematics Tournament 2017

Grade 10

Time Limit: 45 minutes

Lower Division

	Answer Column
9. In the figure, rectangle $ABCD$ and rectangle $BEFD$ are similar, where $\overline{BD}$ is a diagonal of rectangle $ABCD$. If $AB = 5$ and $BC = 12$, then the area of the shaded region $BEFDC$ is expressible in simplest form as $\frac{p}{q}$. Compute $p + q$. 	9.
10. As shown in the diagram, sector AOB is used to create a right circular cone. The lengths of the radius and height of the cone are 5 and 12, respectively. If the perimeter of the sector is $a + b\pi$, compute $a + b$. 	10.
11. The endpoints of $\overline{AB}$ have coordinates $A(3,0)$ and $B(-1,-3)$. The image of $\overline{AB}$ after a rotation about point Q is $\overline{A'B'}$ whose endpoints are $A'(-1,-8)$ and $B'(2,-12)$. If the coordinates of point Q are (x,y), compute $x^2 + y^2$.	11.
12. If the roots of $(c-2)^3 + (c-8)^3 = (2c-10)^3$ are x, y, and z, where $x < y < z$, compute $x^3 + y^2 + z$.	12.
13. If the infinite continued fraction equation, $x = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots}}}}}$ is expressed as $x = a + \sqrt{b}$, where $\sqrt{b}$ is in simplest radical form, compute $a + b$.	13.
14. When the sum $4^{17} + 6^{17}$ is divided by 25, compute the remainder.	14.
15. From 4 boys and 4 girls, choose 4 teams of 2 students per team. The probability, in simplest form, of choosing exactly 2 teams each with 1 boy and 1 girl is $\frac{p}{q}$. Compute $p + q$.	15.

Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM #

Mathematics Tournament 2017

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ________________________ **School** ________________________ **Score** ________

Time Limit: 45 minutes **Upper Division**

	Answer Column
1. If $2^x = 8^{y+2}$ and $27^{2x} = 9^y$, compute the value of $\lvert x + y \rvert$.	1.
2. If ΔABC is inscribed in circle O, the measure of arc BC is 120°, $AB = 12$, $AC = 8$, and the area of ΔABC is expressed in simplest form as $p\sqrt{q}$. Compute the product pq.	2.
3. When $(1 + i)^8$ is expanded and written in increasing powers of i, compute the middle term.	3.
4. Compute $\sum_{k=1}^{4} \cos^2\left(\frac{k\pi}{4}\right)$.	4.
5. For a given parabola $y = f(x)$, the coordinates of the y-intercept are $(0,6)$ and an equation of the axis of symmetry is $x = a$. Compute $3f(2a) - 5$.	5.
6. Compute the length of a radius of the circle that contains points whose coordinates are $(0,0)$, $(8,0)$ and $(0,6)$.	6.
7. Compute the number of integral solutions of the inequality $6x^2 - 63 < 13x$.	7.
8. Three terms of a geometric sequence of positive terms are $x - 5$, $x^2 - 25$, and $3x^3 - 15x^2 - 75x + 375$, in this order. Compute the positive value of x.	8.

Turn Over

11

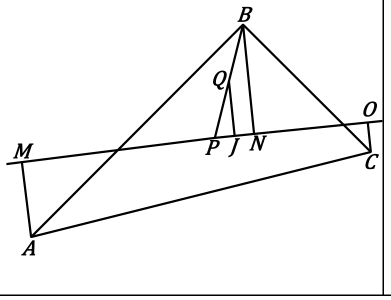
Mathematics Tournament 2017

Grade 11

Time Limit: 45 minutes

Upper Division

Answer Column

9. If $\log_{64}x+\log_4 x+\log_8 x=5$, compute x .	9.
10. Compute the number of possible domains for a function if each domain must be a subset of $\{1,2,3,4,5,6,7,8\}$.	10.
11. If angle A is an acute angle and $\sin A\cos A=\frac{60}{169}$, compute $13(\sin A+\cos A)$.	11.
12. Compute the units digit of $\frac{3^{33}+5^{12}}{3^{11}+5^4}$.	12.
13. The school copy room has two copy machines, both of which are used to complete one job. Working alone, machine A can complete the job in 3 hours, and machine B can complete the same job in 5 hours. Both machines start working at the same time and each machine works at a constant rate. While both machines are working on the same job, machine A stops for 45 minutes and machine B continues to work. Let $\frac{p}{q}$, in simplest form, represent the time, in hours, it takes for the job to be completed. Compute $p+q$.	13.
14. If $f\left(\frac{x-1}{2}\right)=\frac{14-2x}{x^2-2x+37}$ and $f(x)=-\frac{2}{29}$, compute the integral value of x.	14.
15. A line is drawn through point P, the centroid of ΔABC. On the drawn line, points M, N and O are the feet of perpendiculars from vertices $A, B,$ and C, respectively. Point Q is the midpoint of $\overline{BP}$ and point J is the foot of the perpendicular from point Q to $\overleftrightarrow{MO}$. If $QJ=12$, compute the sum $AM+BN+CO$. 	15.

12

Nassau County Interscholastic Mathematics League

Grade 12

TEAM #

Mathematics Tournament 2017

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School**____________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

<table>
<tr><td>1. Compute $2^{3^2} - (2^3)^2$.</td><td>1.</td></tr>
<tr><td>2. If $f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$, compute $f\,'(-4)$.</td><td>2.</td></tr>
<tr><td>3. For complex z, $f(z) = iz + 8$, where $i = \sqrt{-1}$. Compute $f((2 + 2i)^2)$.</td><td>3.</td></tr>
<tr><td>4. In regular hexagon $ABCDEF$, compute the measure of the acute angle, in degrees, formed by the intersection of the diagonals $\overline{AC}$ and $\overline{BF}$.</td><td>4.</td></tr>
<tr><td>5. If $\log(x + 24) + \log(x - 24) = 2$, compute x.</td><td>5.</td></tr>
<tr><td>6. Compute the number of six-digit numbers $(100000 \leq x < 1000000)$ whose digits are a permutation of the digits of 614209. Note: the number 614209 itself *does* count as a valid permutation.</td><td>6.</td></tr>
<tr><td>7. In ΔABC, $m\measuredangle A = 15°$, $m\measuredangle C = 90°$, and $AB = 16$. Compute the area of ΔABC.</td><td>7.</td></tr>
<tr><td>8. Given the values shown in the table. If $g(x) = f\Big(f\big(f(x)\big)\Big)$, compute $g'(1)$.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>$f(x)$</td><td>2</td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>$f'(x)$</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
</table></td><td>8.</td></tr>
</table>

Turn Over

12

Mathematics Tournament 2017

Grade 12

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. If $x_1 = 2$ $x_1 + x_2 = 3$ $x_1 + x_2 + x_3 = 5$ ... $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 17$, such that $\sum_{k=1}^{n} x_k$ is the nth prime number, compute $10x_1 + 9x_2 + 8x_3 + 7x_4 + 6x_5 + 5x_6 + 4x_7$.	9.
10. Compute the distance between the points whose polar coordinates are $(3, 30°)$ and $(5, 150°)$.	10.
11. In a sequence of numbers, $a_{n+1} = \sum_{k=1}^{n} a_k$ for all $n \geq 1$. If $a_{2014} = 117$, compute a_{2017} .	11.
12. Ilene is playing a game. She rolls a fair six-sided die. If she rolls a 6, she wins. If she rolls anything else, she rolls again. If her two rolls now add up to exactly 6, she wins. If she fails to roll a 6 on one roll or a total of 6 on two rolls, she loses. The probability that Ilene wins, in lowest terms is $\frac{p}{q}$. Compute $p + q$.	12.
13. Let $f(x)$ be a function which has the property that, for all real a and b, $f(a + b) = f(a) + f(b)$. If $f(369) = 123$, compute $f(246)$.	13.
14. Consider the cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 6$, $y = 6$, and $z = 6$. The planes $x + y + z = 6$ and $x + y + z = 12$ cut the cube into three solids. Compute the volume of the largest of these three solids.	14.
15. A line, drawn tangent to the ellipse $5x^2 + 12y^2 = 83$ at the first-quadrant point (a, b), passes through the point $(4.15, 0)$. Compute $360ab$.	15.

M

Nassau County Interscholastic Mathematics League

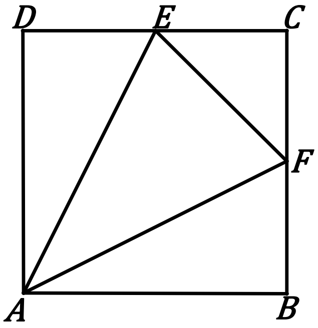
Mathletics

TEAM # ☐

Mathematics Tournament 2017

Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes	*Answer Column*
1. The function $g(x)$ has the property that $g(x) = 1 - g(x-1)$ for all positive integral values of x. If $g(1) = 314$, compute $g(2017)$.	1.
2. The positive root of the equation $(10x-3)^2 - (4x-9)^2 = 0$ is $\frac{a}{b}$, where a and b are positive integers that are relatively prime. Compute $a + b$.	2.
3. In square $ABCD$, $AB = 40$, and points E and F are the midpoints of sides $\overline{DC}$ and $\overline{BC}$, respectively. Compute the area of ΔAEF. 	3.
4. Adam has a circular swimming pool with an 18-foot diameter and a constant 4-foot depth. The bottom of the pool and the surface of the pool have the same area. Water begins to leak out of the pool from the bottom at a constant rate of 2 gallons/hour. If no water is added, compute the <ins>least</ins> number of complete days necessary for all of the water to drain from the pool. (note: 1 cubic foot = 7.614 gallons)	4.
5. Compute the number of positive integer divisors of 20^{17}.	5.

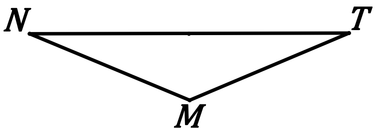
Turn Over

M

Mathematics Tournament 2017

Mathletics

Time Limit: 30 minutes *Answer Column*

6. Each term of the sequence $\{S_n\}$ is defined as the sum of n consecutive odd integers starting with 7. The first few terms are: $S_1 = 7$, $S_2 = 7 + 9 = 16$, and $S_3 = 7 + 9 + 11 = 27$. Compute the greatest integer n satisfying $S_n < 2017$.	6.
7. Compute the distance between the lines $24x + 7y = 31$ and $24x + 7y = 1331$.	7.
8. The circle $x^2 + y^2 = 2100$ contains the points A and B, each of which has an x-coordinate of 30. Lines l and m are tangent to the circle at points A and B, respectively, and intersect at point P. Compute the x-coordinate of point P.	8.
9. Twelve standard 6-sided dice are rolled. Let p represent the probability that exactly two dice land on each different numbered face. That is, exactly two dice land on the face numbered 1, two dice land on the face numbered 2, … , and two dice land on the face numbered 6. Compute $10{,}000p$ rounded to the nearest integer.	9.
10. Isosceles ΔNMT with $NT = 24$ and $NM = MT = 13$ is rotated about $\overleftrightarrow{NT}$ to form a 3-dimensional solid. The volume of this solid is $k\pi$, where k is an integer. Compute k. 	10.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

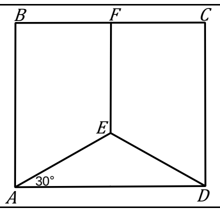
TEAM #

Mathematics Tournament 2017

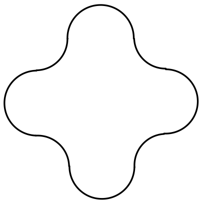
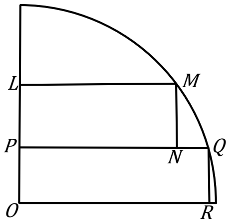
HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________________ **Score** __________

Time Limit: 60 minutes

	Answer Column
1. Define $a \mathbin{@} b \mathbin{@} c$ as the mean of the the three numbers, a, b, and c. If c is a two-digit number, compute the greatest possible integer value of $34 \mathbin{@} 16 \mathbin{@} c$.	1.
2. If $\frac{p+q}{p-q} = \frac{279}{261}$, compute $\frac{p}{q}$.	2.
3. The coordinates of the midpoints of the sides of ΔABC are $(-2, 5)$, $(3, 4)$, and $(1, 1)$. If the coordinates of the vertices of ΔABC are $A(d, e)$, $B(f, g)$, and $C(h, k)$, compute $d + e + f + g + h + k$.	3.
4. Compute the value of b so that the parabola, defined by $f(x) = 5x^2 - bx + 45$, is tangent to the positive x-axis.	4.
5. If $\log_b(x) = 237$, compute the value of $\log_{\frac{1}{b}}\left(\frac{1}{x}\right)$.	5.
6. If $20^{x+y} \cdot 25^{x-2y} = 4^4 \cdot 10^{30}$, compute $x^2 - y^2$.	6.
7. In rectangle $ABCD$, point E is equidistant from A, D, and $\overline{BC}$. If $m\measuredangle EAD = 30°$, and the area of ΔEAD is 100 square units, compute the number of square units in the area of rectangle $ABCD$. 	7.
8. The coordinates of the endpoints of a line segment are $(21, 63)$ and $(909, 618)$. Compute the number of points on the line segment that have integer values for both the x- and y-coordinates. [Note: these points are called lattice points.]	8.
9. The endpoints of a diagonal of a rhombus have coordinates $(0, 10)$ and $(10, 0)$. If the coordinates of the endpoints of the other diagonal are (p, q) and (r, s), compute the sum $p + q + r + s$.	9.
10. The number 1881 is the sum of 19 consecutive integers. Compute the sum of the largest and smallest of these consecutive integers.	10.

Turn Over

11. Compute the greatest possible quotient when the five-digit number $1abcd$, whose missing digits are unique prime numbers, is divided by 495.	11.
12. If x and y are positive integers, compute the number of solutions, (x, y), for the equation $x^2 - y^4 = 3 \cdot 7 \cdot 29$.	12.
13. The cube of the sum of two integers exceeds the sum of the cubes of the two integers by 1404. If the sum of the two integers is 13, compute the product of the two integers.	13.
14. The quantity x varies directly as the sum of the two quantities y and z. The quantity y varies directly as the difference $z - x$. When $x = 20$, $y = -15$ and $z = 25$. Compute $(xy)^2$ when $z = 5$.	14.
15. Each arc in the diagram is either a quarter-circle with radius 1 or a semi-circle with radius 1. The area of the shape can be written in the form $a + b\pi$. Compute $a + b$. 	15.
16. If $\frac{1}{5} = \frac{1}{a} + \frac{1}{b}$, where a and b are different positive integers, compute ab.	16.
17. Compute the least integer greater than 100 that has exactly 6 positive integer divisors.	17.
18. Compute the value of n that satisfies the equation $2^n = 4^{12} + 8^8 + 16^6 + 64^4$.	18.
19. In the quarter-circle shown, rectangle $OPQR$ has $OP = 7$ and $OR = 24$. If, in rectangle $PLMN$, the ratio $\frac{LM}{LP} = \frac{5}{2}$, compute the area of rectangle $PLMN$. 	19.
20. There are three values of x for which the expressions $x^2 + 3x - 1$ and $2x^2 - 3x + 6$ will be pairs of consecutive positive odd integers. Compute the sum of the three pairs of consecutive odd integers.	20.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

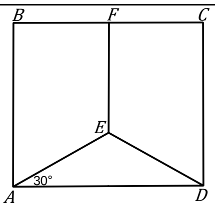
TEAM #

Mathematics Tournament 2017

DO <u>NOT</u> HAND IN THIS COPY. HAND IN THE ONE TEAM COPY.
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Individual Copy

Time Limit: 60 minutes *Answer Column*

1. Define $a \text{ @ } b \text{ @ } c$ as the mean of the the three numbers, a, b, and c. If c is a two-digit number, compute the greatest possible integer value of $34 \text{ @ } 16 \text{ @ } c$.	1.
2. If $\frac{p+q}{p-q} = \frac{279}{261}$, compute $\frac{p}{q}$.	2.
3. The coordinates of the midpoints of the sides of ΔABC are $(-2, 5), (3, 4)$, and $(1, 1)$. If the coordinates of the vertices of ΔABC are $A(d, e)$, $B(f, g)$, and $C(h, k)$, compute $d + e + f + g + h + k$.	3.
4. Compute the value of b so that the parabola, defined by $f(x) = 5x^2 - bx + 45$, is tangent to the positive x-axis.	4.
5. If $\log_b(x) = 237$, compute the value of $\log_{\frac{1}{b}}\left(\frac{1}{x}\right)$.	5.
6. If $20^{x+y} \cdot 25^{x-2y} = 4^4 \cdot 10^{30}$, compute $x^2 - y^2$.	6.
7. In rectangle $ABCD$, point E is equidistant from A, D, and $\overline{BC}$. If $m\measuredangle EAD = 30°$, and the area of ΔEAD is 100 square units, compute the number of square units in the area of rectangle $ABCD$. 	7.
8. The coordinates of the endpoints of a line segment are $(21, 63)$and $(909, 618)$. Compute the number of points on the line segment that have integer values for both the x- and y-coordinates. [Note: these points are called lattice points.]	8.
9. The endpoints of a diagonal of a rhombus have coordinates $(0, 10)$ and $(10, 0)$. If the coordinates of the endpoints of the other diagonal are (p, q) and (r, s), compute the sum $p + q + r + s$.	9.
10. The number 1881 is the sum of 19 consecutive integers. Compute the sum of the largest and smallest of these consecutive integers.	10.

Turn Over

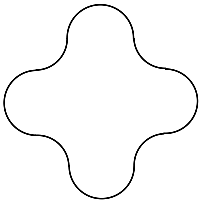
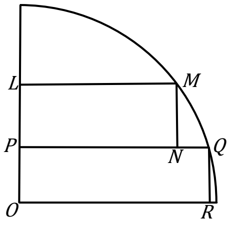
T

Mathematics Tournament 2017

Team Problems

Time Limit: 60 minutes

Answer Column

11. Compute the greatest possible quotient when the five-digit number $1abcd$, whose missing digits are unique prime numbers, is divided by 495.	11.
12. If x and y are positive integers, compute the number of solutions, (x, y), for the equation $x^2 - y^4 = 3 \cdot 7 \cdot 29$.	12.
13. The cube of the sum of two integers exceeds the sum of the cubes of the two integers by 1404. If the sum of the two integers is 13, compute the product of the two integers.	13.
14. The quantity x varies directly as the sum of the two quantities y and z. The quantity y varies directly as the difference $z - x$. When $x = 20$, $y = -15$ and $z = 25$. Compute $(xy)^2$ when $z = 5$.	14.
15. Each arc in the diagram is either a quarter-circle with radius 1 or a semi-circle with radius 1. The area of the shape can be written in the form $a + b\pi$. Compute $a + b$. 	15.
16. If $\frac{1}{5} = \frac{1}{a} + \frac{1}{b}$, where a and b are different positive integers, compute ab.	16.
17. Compute the least integer greater than 100 that has exactly 6 positive integer divisors.	17.
18. Compute the value of n that satisfies the equation $2^n = 4^{12} + 8^8 + 16^6 + 64^4$.	18.
19. In the quarter-circle shown, rectangle $OPQR$ has $OP = 7$ and $OR = 24$. If, in rectangle $PLMN$, the ratio $\frac{LM}{LP} = \frac{5}{2}$, compute the area of rectangle $PLMN$. 	19.
20. There are three values of x for which the expressions $x^2 + 3x - 1$ and $2x^2 - 3x + 6$ will be pairs of consecutive positive odd integers. Compute the sum of the <u>three pairs</u> of consecutive odd integers.	20.

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2017

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit:	*Answer Column*
1.	1.

Name ____________________ **School** ____________________ **Score** ________

Time Limit:	*Answer Column*
2.	2.

Name ____________________ **School** ____________________ **Score** ________

Time Limit:	*Answer Column*
3.	3.