Grade 9

TEAM #

#### **Mathematics Tournament 2016**

No calculators may be used on this part. All answers will be integers from 0 to 999, inclusive. One (1) point for each correct answer.

Name School	_ Score
Time Limit: 45 minutesLower Division	Answer Column
1. Given three real numbers, <i>x</i> , <i>y</i> , and <i>z</i> , define $(x, y, z)^* = \frac{xy+yz+zx}{x^2+y^2+z^2}$ . The value of $(3, 2, 4)^*$ is calculated and expressed as $\frac{a}{b}$ in simplest form. Compute $a + b$ .	1.
2. If the numerical value of the area of a circle is 12 times the numerical value of its circumference, compute the length of the radius of the circle.	2.
3. For a rectangular solid, the surface area of the top face is $28 ft^2$ , the surface area of the front face is $24 ft^2$ , and the surface area of a side face is $42 ft^2$ . Compute the volume of the rectangular solid.	
4. If x is a positive integer, for how many values of x is $0.25(x - 8) \ge 0.5x - 16$ ?	4.
5. Given parallelogram <i>ABCD</i> . Altitudes $\overline{AF}$ and $\overline{AE}$ are drawn to $\overline{DC}$ and $\overline{BC}$ , respectively. If $AB = 12$ , $AF = 6$ , $FC = 4$ , and $AE = h$ , compute 100 <i>h</i> .	5.
6. Three-quarters of the first of three integers is equal in value to both $\frac{6}{5}$ of the second integer and $\frac{9}{2}$ of the third integer. When the third integer is subtracted from the second integer, the result is 44. Compute the smallest of the three integers.	6.
7. For three consecutive integers, the positive difference of the cubes of the two larger consecutive integers is 66 more than the positive difference of the cubes of the two smaller consecutive integers. Compute the median integer.	7.
8. In degrees, compute the measure of an angle for which the square of the measure of its complement is 15° more than twice the measure of its supplement.	8.

9

#### **Mathematics Tournament 2016**

Grade 9

Time Limit: 45 minutes Lo	wer Division	Answer Column
9. Compute the units digit of the product: 3	$^{2015} \cdot 17^{2016} \cdot 29^{2017}$ .	9.
10. If $x$ is 10% larger than $y$ , $y$ is 20% larg $x$ is $p$ % smaller than $w$ , compute $p$ .	er than $z$ , $z$ is 25% smaller than $w$ , and	10.
11. Compute the sum of the prime factors of	899.	11.
12. A sequence can be called "boring" if all of hundredth term of a non-boring arithmet terms form a geometric sequence, and wi	ic sequence whose first, second, and fifth	12.
13. A baseball fan is somewhere between his stadium, he can walk directly to the stadi his bicycle to the stadium. He rides 7 tim require the same amount of time. If the r distance from the stadium is written as $\frac{a}{b}$	um or he can walk home and then ride es as fast as he walks, and both choices atio of his distance from his home to his	13.
14. A checkerboard has 13 rows and 17 columns. Each square is numbered consecutively beginning in the top left and continuing across the rows from left to right in each row. For example, row 1 is numbered 1, 2, 3,, 17; row 2 is numbered 18, 19, 20,, 34; row 3 is numbered 35 to 51, etc. If the squares are then numbered consecutively beginning in the top left and continuing from the top down in each of the columns, the numbering would be different. However, some squares would still have the same number. Compute the sum of the numbers that are in the same position no matter which of the two directions is used to number the squares.		14.
15. Three fair dice are tossed and the product faces is even. The probability that the sum expressed as $\frac{a}{b}$ in simplest form. Compu	m of those numbers is also even is	15.

9

10	Grade 10 TEA	M #	
Mathen	Mathematics Tournament 2016 No calculators may be used on this part. All answers will be integers from 0 to 999, inclusive. One (1) point for each correct answer.		
Name _	School	Score	
Time Lii	nit: 45 minutes Lower Division	Answer Column	
1. Com	pute the perimeter of a rhombus if the lengths of its diagonals are 20 and 48.	1.	
after	coordinates of point <i>P</i> are (5,2) and the coordinates of the image of point <i>P</i> applying the composite transformation $T_{-3,-1} \circ D_2$ are $(a,b)$ . Suppose <i>ab</i> .	2.	
3. Com	pute the sum of the distinct prime factors of 16,380.	3.	
	pute the value of x that makes the following an arithmetic sequence: $-7$ ), $(5x + 2)$ , $(9x - 13)$ .	4.	
subt	r the fraction $\frac{3}{5}$ , a number is added to the numerator and the same number is racted from the denominator, the result is equal to the reciprocal of the inal fraction. Compute the number.	5.	
	secants, $\overline{ABC}$ and $\overline{ADE}$ , are drawn to a circle from external point $A$ . B = 6 and $BC = AD = 3$ , compute $AE$ .	6.	
7. Com	pute the number of sides of a convex polygon that has 20 diagonals.	7.	
8. Com	pute $f(7)$ , if $f(2x - 5) = 8x^2 - 26x + 66$ .	8.	

Turn Over

Tournament 2016

Grade 10

Time Limit: 45 minutes Lower Division	Answer Column
9. The centroid of $\triangle ABC$ is 5 units from vertex <i>A</i> . If $AB = AC$ and $BC = 16$ , compute the area of $\triangle ABC$ .	9.
10. The endpoints of a diameter of a circle have coordinates (3,4) and (-9, -1). A diagonal of a square inscribed in this circle coincides with a diameter of the circle. If the area of the square in simplest form is $\frac{p}{q}$ , compute $p + q$ .	10.
11. The accompanying diagram contains $\triangle ABC$ , $\triangle BCD$ , $\overline{BED}$ , and $\overline{AEC}$ , with $CD = 2\sqrt{3}$ , $BC = 6$ , $m \not AAEB = 50^{\circ}$ , and $m \not BCD = 90^{\circ}$ . Compute the $m \not ACB$ .	11.
12. For the equation $3 - \frac{5x+3}{x-3} = \frac{x}{x+3}$ , let <i>P</i> be the product of the roots and let <i>S</i> be the sum of the roots. Compute <i>P</i> - <i>S</i> .	12.
13. In the accompanying diagram, circles <i>A</i> , <i>B</i> , and <i>C</i> are externally tangent in pairs. If the radii of the circles are 4, 6, and 8, respectively, then the area of $\triangle ABC$ in simplest form is $p\sqrt{q}$ . Compute $pq$ .	13.
14. Compute the number of arrangements of all of the letters of the word SCHOOLS for which there are two consecutive O's but no two consecutive S's.	14.
15. If two positive integers are relatively prime, then their greatest common factor is 1. Compute the minimum sum of three positive integers that are all three relatively prime, but for which any pair of these three positive integers is not relatively prime.	15.

10

Grade 11

TEAM #

#### **Mathematics Tournament 2016**

11

No calculators may be used on this part. All answers will be integers from 0 to 999, inclusive. One (1) point for each correct answer.

Na	me School	Score
Tii	ne Limit: 45 minutes Upper Division	Answer Column
1.	Compute the largest prime factor of 1287.	1.
2.	When the denominator of $\frac{2}{(\sqrt{19} - \sqrt{3})(\sqrt{41} - \sqrt{5})}$ is rationalized and the resulting fraction is written in simplest form, compute the new denominator.	2.
3.	If $f(x) = \frac{5x-1}{x-2}$ and $f(g(x)) = x$ , compute $g(14)$ .	3.
4.	One of the factors of $125x^3 - 64$ is of the form $ax^2 + bx + c$ , where <i>a</i> , <i>b</i> , and <i>c</i> are integers. Compute $a + b + c$ .	4.
5.	In a classroom, there are more than 10 students. When groups of 6 students are formed, there are exactly 5 students left without a group. When groups of 10 students are formed, there are exactly 9 students left without a group. Compute the smallest possible number of students in the classroom.	5.
6.	The expressions 1, $k$ , and $2k + a$ , in this order, and where $k$ and $a$ are real numbers, are three consecutive terms of both an arithmetic sequence and a geometric sequence. Compute $k + a$ .	6.
7.	Let $a_n = a_{n-1} \cdot i$ , where $i = \sqrt{-1}$ . If $a_1 = 2 - 3i$ , compute $\sum_{n=1}^{2016} a_n$ .	7.
8.	A cup, as shown, is a right circular cone with a radius of length 100 and a depth of length 300. It is completely filled with water. If Noah drinks half of the volume of water, then the depth of the water, in simplest form, will be $p\sqrt[3]{q}$ . Compute $p + q$ .	8.

11

Grade 11

Time Limit: 45 minutes Up	per Division	Answer Column
9. Let <i>a</i> and <i>b</i> be the only two integer solution $ x^2 - x - 6  < 1$ . Compute $a^2 + b^2$ .	tions of the inequality	9.
10. Compute the <i>x</i> -intercept of the function	$f(x) = \log_2(\log_3(\log_4(x-1))).$	10.
11. A circle is inscribed in a triangle whose si A second circle is circumscribed about this circumscribed circle to the area of the ins are relatively prime. Compute $a + b$ .	s triangle. The ratio of the area of the	11.
12. When $(2x + y + z)^5$ is expanded and like the terms, where k is an integer. Compu		12.
13. If $\cos^4(x) - \sin^4(x) = \frac{2}{3}$ for $0^\circ < x < 90^\circ$ simplest form. Compute <i>mn</i> .	P°, then $\sin(2x) = \frac{\sqrt{m}}{n}$ , where $\frac{\sqrt{m}}{n}$ is in	13.
14. Let $f(2n) = 2f(n) \cdot g(n)$ , where $(f(n))$ f(n) > 0, and $g(n) > 0$ for all $n$ . If $f(a)a$ and $b$ are relatively prime. Compute $a$	$f(4n) = \frac{3}{5}$ , then $f(4n) = \frac{a}{b}$ , where	14.
15. A function $f$ is non-constant, is defined f f(1 + x) = f(1 - x) and $f(6 + x) = f(6)compute the number of roots of f(x) = 6$	(-x). If $x = 0$ is a root of $f(x) = 0$ ,	15.



# Grade 12

TEAM #

#### **Mathematics Tournament 2016**

No calculators may be used on this part. All answers will be integers from 0 to 999, inclusive. One (1) point for each correct answer.

Na	ame School		Score
Tir	me Limit: 45 minutes Upper Division		Answer Column
1.	The city of Spenciv has a 25% sales tax on all products sold price, after any discounts have been applied. Kevin's Tires i discount their tires by $D$ % such that the final after-tax price original (undiscounted, untaxed) price. Compute $D$ .	n Spenciv wants to	1.
2.	Let $f(x) = \sin^4(x) + \cos^4(x) + 2\sin^2(x)\cos^2(x)$ . Compute	$e f'\left(\frac{2016\pi}{5}\right).$	2.
3.	Compute the number of solutions, $(x, y)$ , of $x^2 + y^2 = 50$ for $x$ and $y$ are integers.	or which both	3.
4.	Compute $\lim_{x \to 2} \frac{x^5 - 2x^4 + 3x^3 - 6x^2 + 8x - 16}{x^2 - 4}$ .		4.
5.	Given the equation $\log_2(x^3 - 1) - \log_2(x - 1) = \log_2(402)$	01). Compute <i>x</i> .	5.
6.	The angle, in degrees, represented by the expression $\sum_{k=0}^{2016}$ the angle $\theta$ , where $0^{\circ} \le \theta \le 360^{\circ}$ . Compute $\theta$ .	<i>k</i> ! is coterminal with	6.
7.	In the diagram, $\angle B$ and $\angle D$ are right angles, $\angle ACB \cong \angle ECD$ , $AB = 240$ , $ED = 135$ , and $BD = 200$ . Compute $AC + CE$ .		7.
8.	Let $S$ be the set of 60 points in the plane with polar coordinates $n$ is any integer. Compute the number of distinct regular p constructed using only vertices from within set $S$ . (Note: T polygons in different locations are considered distinct.)	olygons that can be	8.

12

## **Mathematics Tournament 2016**

Grade 12

Time Limit: 45 minutes Upper D	ivision	Answer Column
9. Compute the closest integer to $\frac{1}{10}(\sqrt{3}+i)^{12}$ , v	where $i = \sqrt{-1}$ .	9.
10. For the equation $x^8 - Ax^7 + Bx^6 - Cx^5 + Dx^4$ the roots are all integers greater than 1. Comp		10.
11. Compute the number of distinct arrangements of that do <b>not</b> start with the letter A.	of the letters of the word ALFALFA	11.
12. Compute the sum of all values of $\theta$ , in the interinfinite sum, $1 + \sin\theta + \sin^2\theta + \sin^3\theta + \cdots = 0$		12.
13. The graphs of $y = x^2 + b$ and $y = 88x - x^2$ a Compute <i>b</i> .	re tangent to one another.	13.
14. Let $f$ be a function defined such that $f(x) \ge 0$ nonnegative $a$ and $b$ , $f(a + b) = f(a) + f(b)$	=	14.
15. If <i>A</i> is any positive real number, compute the g $f(x) = Ax^{2} - (A + 1)x + 144.$	reatest possible minimum value of	15.

## **Mathletics**

#### **Mathematics Tournament 2016**

Μ

Calculators may be used on this part. All answers will be integers from 0 to 999, inclusive. One (1) point for each correct answer.

Na	me School	Score
Tii	ne Limit: 30 minutes	Answer Column
1.	Given the equations $20x + 16y = 599$ and $16x + 20y = 121$ , compute the value of $x + y$ .	1.
2.	The coordinates of the endpoints of $\overline{AB}$ are $A(-2, -3)$ and $B(5, 11)$ . Point <i>C</i> is on $\overline{AB}$ so that the ratio $AC : CB$ is $3 : 4$ . If the coordinates of <i>C</i> are $(x, y)$ , compute $10x + y$ .	2.
3.	The length of the shortest segment from the point $(19, -1)$ to the graph of $(x-5)^2 + (y-1)^2 = 8$ , expressed in simplest form, is $p\sqrt{q}$ . Compute $p + q$ .	3.
4.	In the diagram, circle <i>O</i> is tangent to equilateral $\Delta NMT$ at point <i>D</i> , the foot of altitude $\overline{MD}$ . If the area of $\Delta NMT$ is $400\sqrt{3}$ , and the area of the circle is $n\pi$ , compute <i>n</i> .	4.
5.	Starting at the top of a down escalator, it takes Andrea 70 seconds to walk down the escalator to the bottom when it is not operating, and only 30 seconds to walk down the escalator (walking at the same pace) when it is operating. If, starting at the top of the same escalator, Yash just stands still on the operating escalator, it will take him $n$ seconds to reach the bottom. Compute 10 $n$ .	5.

TEAM #

# **Mathematics Tournament 2016**

Mathletics

Time Limit: 30 minutes

Time Limit. 50 minutes	miswer column
6. Compute the units digit of $2^{2016} + 0^{2016} + 1^{2016} + 6^{2016}$ .	6.
7. In $\Delta NMT$ , $\angle NTM$ is obtuse. The altitude of $\Delta NMT$ from vertex $N$ to the line containing $\overline{MT}$ intersects $\overline{MT}$ at point $A$ . Additionally, $NM = 17$ , $AT = 14$ , and $TM = 1$ . Compute the volume of the solid formed by rotating $\Delta NMT$ about altitude $\overline{NA}$ . Round your answer to the nearest integer.	7.
8. The figure shown is the union of a circle and two semicircles with diameters of 50 and 150. The ratio of the area of the shaded region to the area of the unshaded region, in simplest form, is $\frac{p}{q}$ . Compute $p + q$ .	8.
9. Ajay and Brandon challenge each other to a ping-pong match. Brandon is a much better player at first, but his arm tires quickly. Brandon wins the first game with probability 1, but the chance of Brandon's winning each subsequent game is reduced by 0.2 per game. They play 5 games. Let $p$ represent the probability that Brandon wins at least 4 of these games. Compute 1000 $p$ rounded to the nearest integer.	9.
10. A group of ten LED lights are placed in a row and each light can be switched between red and blue independently to create a pattern. Compute the number of possible patterns that have no two adjacent red lights.	10.

Μ

#### **Team Problem Solving**

#### TEAM #

## **Mathematics Tournament 2016**

#### HAND IN ONLY **ONE** ANSWER SHEET PER TEAM

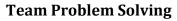
Calculators may be used on this part.

All answers will be integers from 0 to 999, inclusive.

Three (3) points per correct answer.

Те	eam Copy School	Score
Tir	me Limit: 60 minutes	Answer Column
1.	The fraction $\frac{a}{b}$ is midway between $\frac{3}{7}$ and $\frac{4}{5}$ . If the only common positive factor of <i>a</i> and <i>b</i> is 1, compute $a + b$ .	1.
2.	The perimeter of a square equals the perimeter of a triangle. The lengths of two sides of the triangle are 8 and 17, and the length of a side of the square is 10. The area of the square is $k$ square units greater than the area of the triangle. Compute $k$ .	2.
3.	One sixteenth of $2^{56}$ can be written as $2^m$ . Compute <i>m</i> .	3.
4.	The points $A(1, 4)$ , $B(3, 10)$ , and $C(5, 12)$ do not lie on a line. An equation for the line of best fit is $3y - 6x = 8$ . Let $W$ equal the average of the slopes of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ . Compute the difference between $W$ and the slope of the line of best fit.	4.
5.	A point that is always on the line of best fit has an <i>x</i> -coordinate that is the average of the <i>x</i> -coordinates of the given data points, and a <i>y</i> -coordinate that is the average of the data points' <i>y</i> -coordinates. Let $(p, q)$ be that point on the line of best fit for the points $(15, -6)$ , $(12, 0)$ , $(5, 5)$ , and $(-1, 10)$ . Compute $p + q$ .	5.
6.	The unique prime factorization of 2016 is $2^5\cdot 3^2\cdot 7$ . Compute the number of positive divisors of 2016.	6.
7.	Use the definition that a trapezoid is a quadrilateral with <i>at least</i> one pair of sides parallel to compute the area of the smallest trapezoid whose perimeter is numerically equal to its area.	7.
8.	A square is inscribed in a semicircle of radius 10 as seen in the diagram. To the nearest whole number, compute how much greater the area of the semicircle is than the area of the square.	8.
9.	A solid sphere is sliced into two parts. The cross-section common to the two parts is a circle of radius 156 cm. If the height of one of the parts is only 104 cm, compute the number of centimeters in the radius of the sphere.	9.

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TEAM #

## **Mathematics Tournament 2016**

DO <u>NOT</u> HAND THIS COPY IN. HAND IN THE ONE TEAM COPY. Calculators may be used on this part. All answers will be integers from 0 to 999 inclusive. Three (3) points per correct answer.

## **Individual Copy**

Tir	ne Limit: 60 minutes	Answer Column
1.	The fraction $\frac{a}{b}$ is midway between $\frac{3}{7}$ and $\frac{4}{5}$ . If the only common positive factor of $a$ and $b$ is 1, compute $a + b$ .	1.
2.	The perimeter of a square equals the perimeter of a triangle. The lengths of two sides of the triangle are 8 and 17, and the length of a side of the square is 10. The area of the square is $k$ square units greater than the area of the triangle. Compute $k$ .	2.
3.	One sixteenth of $2^{56}$ can be written as $2^m$ . Compute <i>m</i> .	3.
4.	The points $A(1, 4)$ , $B(3, 10)$ , and $C(5, 12)$ do not lie on a line. An equation for the line of best fit is $3y - 6x = 8$ . Let $W$ equal the average of the slopes of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ . Compute the difference between $W$ and the slope of the line of best fit.	4.
5.	A point that is always on the line of best fit has an <i>x</i> -coordinate that is the average of the <i>x</i> -coordinates of the given data points, and a <i>y</i> -coordinate that is the average of the data points' <i>y</i> -coordinates. Let $(p,q)$ be that point on the line of best fit for the points $(15, -6)$ , $(12, 0)$ , $(5, 5)$ , and $(-1, 10)$ . Compute $p + q$ .	5.
6.	The unique prime factorization of 2016 is $2^5\cdot 3^2\cdot 7$ . Compute the number of positive divisors of 2016.	6.
7.	Use the definition that a trapezoid is a quadrilateral with <i>at least</i> one pair of sides parallel to compute the area of the smallest trapezoid whose perimeter is numerically equal to its area.	7.
8.	A square is inscribed in a semicircle of radius 10 as seen in the diagram. To the nearest whole number, compute how much greater the area of the semicircle is than the area of the square.	8.
9.	A solid sphere is sliced into two parts. The cross-section common to the two parts is a circle of radius 156 cm. If the height of one of the parts is only 104 cm, compute the number of centimeters in the radius of the sphere.	9.

#### **Mathematics Tournament 2016**

**Team Problems** 

Time Limit: 60 minutes	Answer Column
10. The coordinates of parallelogram <i>ABCD</i> are $A(-12, 45)$ , $B(24, -18)$ , $C(40, 10)$ , and $D(p, q)$ . Compute $pq$ .	10.
<ol> <li>Compute the largest three-digit integer such that if the integer is decreased by 6.25%, the result will also be an integer.</li> </ol>	11.
12. There are $10 \times 10 \times 10 = 1000$ ways to arrange three digits in a row using the numbers 0 through 9. When each of $1$ $1$ $2$ those arrangements is placed in a circle, many of the $3$ $2$ $2$ $3$ $1$ $3$ clockwise orders are the same. For example, in the diagram, A B C clockwise arrangements A and B are considered different, but clockwise arrangements A and C are the same. Compute the number of unique clockwise arrangements of the three digits.	12.
13. Consider the equation: $sin(x^{\circ}) = sin(x^{r})$ where $x^{\circ}$ means $x$ degrees and $x^{r}$ means $x$ radians. The equation is true when $x = 0$ since $sin(0^{\circ}) = sin(0^{r}) = 0$ . It is also true for many other values of $x$ . Compute the smallest positive value of $x$ for which the equation is true, multiply the result by 100, and then round your answer to the nearest integer.	13.
14. The difference between the arithmetic mean and the geometric mean of two positive numbers is 16. If one of the numbers is 2, compute the other number.	14.
15. Without repetition, place the integers 1, 2, 3, 4, 5, 6, 7, and 8 in the eight circles so that no two adjacent circles (circles with a segment joining them) contain consecutive integers. Compute the sum of the numbers in the circles labeled <i>a</i> , <i>b</i> , <i>c</i> , and <i>d</i> .	15.
16. Triangle <i>ABC</i> is inscribed in circle <i>O</i> whose radius is $20(\sqrt{6} - \sqrt{2})$ . If $m \angle A = 45^{\circ}$ and $m \angle B = 30^{\circ}$ , compute <i>AB</i> .	16.
17. Using only the digits 1, 2, 3, 4, 5, and 6, compute the number of 4-digit numbers that can be formed with a greater number of odd digits than even digits.	17.
18. Compute the units digit in the expansion of $8^{8^8}$ . (Note that by convention, the exponentiation is performed from top to bottom. That is, $2^{3^4} = 2^{81}$ ).	18.
19. Let <i>a</i> , <i>b</i> , <i>c</i> , and <i>d</i> be 4 consecutive positive integers such that $a^3 + b^3 + c^3 = d^3$ . Compute $a + b + c + d$ .	19.
20. The table contains a multiplicative magic square. That means the products of the three numbers in every row, column, and the two major diagonals are the same. Compute the sum of the four missing numbers in the square.2933	20.

# Т

#### **Tie Breakers**

Mathematics Tournament A	2 <b>016</b> No calculators may be used on this par All answers will be integers from 0 to 999 in One (1) point for correct answer.	
Name	School	Score
Time Limit:		Answer Column
1.		1.
Time Limit.	School	Anguar Calumn
2.		2.
Name	School	Score
Name	School	