Nassau County Interscholastic Mathematics League

9

Grade 9

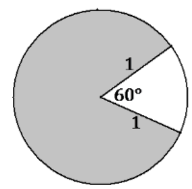
TEAM #

Mathematics Tournament 2015

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

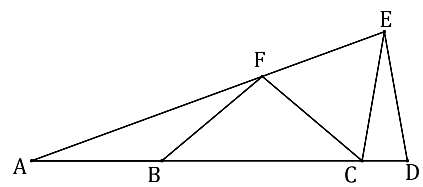
1. If $x = -5$, compute the value of $\vert\, 2 - \vert 2 + x \vert\, \vert$.	1.
2. In an old arcade game, the "monster" is a shaded sector of a circle whose radius is 1 cm as shown in the diagram. The missing piece of the circle (the mouth) has a central angle of 60°. If the perimeter of the monster is expressed in simplest form as $\frac{a}{b}\pi + c$, compute $a + b + c$. 	2.
3. If $a:b:c = 3:1:5$ and $\frac{2a+3b}{4b+3c} = \frac{e}{f}$, where $\frac{e}{f}$ is in simplest form, compute $e + f$.	3.
4. Aesha sells an item at $7 less than the list price and receives 10% of her selling price as commission. Emily sells the same item at $15 less than the list price and receives 20% of her selling price as commission. Find the list price if they both receive the same commission.	4.
5. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point E. If $m\angle AED = 10y - 2x$, $m\angle AEC = 4y - x$, and $m\angle CEB = 10x + 2y$, compute $x + y$.	5.
6. The length of a rectangle is 3 more than twice the width. If the area of the rectangle is 13 less than 4 times the perimeter, find the length of the rectangle.	6.
7. How many different diagonals can be drawn in a convex 14-gon?	7.
8. Compute the area of the triangular region bounded by the lines $x + y = 4$, $y = 2x + 1$, and $x = 5$.	8.

Turn Over

Mathematics Tournament 2015

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. When Kayla walks to school, she averages 90 steps per minute. Each of her steps is 75 cm long and it takes her 16 minutes to get to school. Her brother, Justin, going to the same school by the same route, averages 100 steps per minute. His steps are 60 cm long. How many minutes does it take Justin to get to school?	9.
10. Compute the number of digits in the product $5^{25} \times 8^{12}$.	10.
11. If $x + y = 6$ and $xy = 3$, compute $x^3 + y^3$.	11.
12. Triangle ADE is an isosceles triangle with vertex angle A. If $AB = BF = FC = CE = ED$, find the $m\angle A$. 	12.
13. Compute the positive difference between the numerator and the denominator of the product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$ when the product is expressed as a single fraction in simplest form.	13.
14. Let $x = .123456789101112131415\cdots997998999$. The digits of x are obtained by writing the integers from 1 through 999 in order. Find the 2015th digit to the right of the decimal point.	14.
15. A box contains 11 balls that are numbered $1, 2, 3, \cdots, 11$. If 6 balls are drawn simultaneously, the probability that the sum of the numbers on the drawn balls is odd is computed and expressed in simplest $\frac{a}{b}$ form. What is the value of a ?	15.

10

Nassau County Interscholastic Mathematics League

Grade 10

TEAM # ______

Mathematics Tournament 2015

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** __________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. One year, a town's population increased by 60%. The following year, the population increased by 40%. If, over this 2-year period, the population increased by $n\%$, compute n.	1.
2. How many distinct prime numbers are factors of 2015?	2.
3. The length of a diagonal of one square equals the length of a side of another square. The larger square has an area that is n times the area of the smaller square. Compute n.	3.
4. On a number line, the points B, C, D, E, F, G, and H divide the line segment $\overline{AJ}$ into 8 congruent segments. If the coordinate of A is -24, and the coordinate of D is -7.5, compute the coordinate of J.	4.
5. Point P, with coordinates (8, 8), is rotated 135° counterclockwise about the origin to point P'. If the coordinates of P' are (x, y), compute the product of x and y.	5.
6. If A is a constant, and if $\frac{A}{x+3} + \frac{5}{x-1} = \frac{12x+8}{x^2+2x-3}$, compute A.	6.
7. A given octagon contains interior angles that are either acute or obtuse. One student measures each interior angle in degrees and finds the average of the measures of these angles. Another student measures each exterior angle in degrees, one at each vertex, and finds the average of the measures of those angles. What is the number of degrees in the positive difference between their two averages?	7.
8. The product of two positive numbers is 324. The reciprocal of one of these numbers is 9 times the reciprocal of the other. Compute the sum of the two numbers.	8.

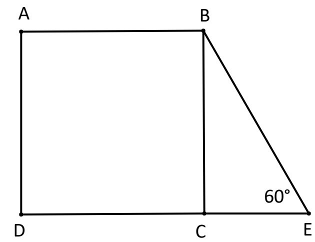
Turn Over

10

Mathematics Tournament 2015

Grade 10

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. In the diagram, the length of each side of square $ABCD$ is $6\sqrt{3}$. The measure of $\angle E = 60°$. If the area of trapezoid $ABED$, in simplest radical form, is $x + y\sqrt{z}$, compute $x + y + z$. 	9.
10. Ricky is 35 years younger than Robert. Robert's age 7 years from now will exceed 5 times Ricky's age 15 years ago by 5 years. Compute the sum of their present ages.	10.
11. Three vertices of a parallelogram are $A(-7,-1)$, $B(3,-3)$, and $C(12,3)$. The fourth vertex can be either one of two distinct points: (a,b) or (c,d). Compute the product $abcd$.	11.
12. Consider the system of equations: $y = 2x^2 + 8x + 11$ and $y = 3x + k$, where k is a constant. Find the smallest integer value of k for which this system of equations will have two distinct real solutions.	12.
13. In the coordinate plane, two opposite vertices of a square are $(-17,-2)$ and $(4,3)$. The midpoint of the side of the square that lies entirely in the third quadrant has coordinates (x,y). Compute the product of x and y.	13.
14. A circle of radius length 6 is cut into three sectors, each of which has a central angle measure of 120°. One of these sectors is used to create a right circular cone where the sector forms the lateral surface area of the cone. If the length of the height of the cone is expressed in simplest radical form as $a\sqrt{b}$, compute $a + b$.	14.
15. When a weighted six-sided number cube is rolled, there are six possible outcomes numbered 1 through 6. However, these outcomes are not equally likely. The probability of rolling a prime number is twice the probability of rolling a non-prime number. If the probability of rolling the same number on two rolls is expressed in the form $\frac{a}{b}$ where a and b have no common factors, compute $a + b$.	15.

Nassau County Interscholastic Mathematics League

11	**Grade 11**	**TEAM #**

Mathematics Tournament 2015

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. If $f(x)=\frac{1}{2}x-16$ and $g(x)=\frac{1}{4}x+20$, compute $g\big(f(24)\big)$.	1.
2. Compute the sum of the integral solutions of $\frac{x+9}{x-3}\le 5$ on the interval $1\le x\le 10$.	2.
3. A cheerleading squad has 10 girls and 5 boys. Compute the number of 4-member teams consisting of 2 girls and 2 boys.	3.
4. If d varies inversely as x^2 and if $d=100$ when $x=4$, compute the positive value of x when $d=64$.	4.
5. When the denominator of $\frac{2}{9-\sqrt{13}}$ is rationalized and the fraction is in simplest form, compute the new denominator.	5.
6. If the lengths of the sides of a triangle are integers and two of the sides have lengths 15 and 20, compute the maximum possible perimeter of the triangle.	6.
7. A given line segment contains the points $(2,-5)$ and $(-3,10)$. Compute the y-intercept of the line that is perpendicular to the given line and that contains the point $(-6,5)$.	7.
8. If $\log_b 5+\log_b 7+\log_b 11=1$, compute b.	8.

Turn Over

Mathematics Tournament 2015

Grade 11

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. The roots of $5^x + 5^{3-x} = 30$ are $x = a$ and $x = b$. Compute $a^3 + b^3$.	9.
10. Compute the area of the region bounded by the x-axis and the graph of $y = \|2x - 5\| - 4$.	10.
11. A sequence is defined by $a_1 = 342$ and $a_n = \begin{cases} \frac{1}{4}a_{n-1} \text{ if } \frac{1}{4}a_{n-1} \text{ is an integer} \\ 2a_{n-1} \text{ if } \frac{1}{4}a_{n-1} \text{ is not an integer} \end{cases}$ Compute a_{2015}.	11.
12. For square $OABC$, the coordinates of vertices O and B respectively are $(0,0)$ and $(0,4)$. In the same plane, equilateral triangle DEF has vertex D $(4,0)$ where $\overline{DF}$ is on the x-axis and where point D is between points O and F. The y-coordinate of vertex E is 4. When the length of the shortest horizontal line segment that can be drawn from a point on the square to a point on the triangle is expressed in simplest $\frac{a+b\sqrt{c}}{d}$ form, compute $a + b + c + d$.	12.
13. Find the area of a triangle if the lengths of its medians are 30, 30, and 48.	13.
14. If $\cos^4 x + \sin^4 x = \frac{3}{4}$ and $0° \leq x \leq 360°$, compute the average of all of the values of x that satisfy these conditions.	14.
15. The line $y = x + b$ is tangent to the circle $x^2 + y^2 = 4$. Compute b^2.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2015

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

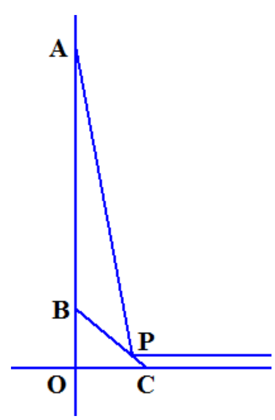
1. If $x + 2y + 3z = 87$, $3x + y + 2z = 92$, and $2x + 3y + z = 73$, compute $x + y + z$.	1.
2. Compute the positive value of x for which $\log_2(8x^2) = 17$.	2.
3. The number of distinct arrangement of the letters of the word CALCULUS is $n!$, where $n!$ is n factorial. Compute n.	3.
4. Compute the sum of the squares of the solutions to $x^{3/2} + 77x^{-1/2} = 18\sqrt{x}$.	4.
5. Let $f(x) = e^{4\ln x}$. Compute $f\,'(6)$.	5.
6. Right ΔABC has $m\angle C = 90°$ and $AC = 6$. Angle bisector $\overline{AD}$ is drawn to side $\overline{BC}$. If $AD = BD$, compute the square of the area of ΔABC.	6.
7. If $i = \sqrt{-1}$, compute $\sum_{k=2}^{1000} i^{k!}$	7.
8. Compute the sum of the squares of the roots of $(x^2 + 5x)^2 = 36$.	8.
9. A rectangular prism has integer dimensions and a volume of 2015. Compute the smallest possible total edge length of the prism.	9.

Turn Over

Mathematics Tournament 2015

Grade 12

Time Limit: 45 minutes **Upper Division** *Answer Column*

10. The semicircular graph of $y = \sqrt{25 - x^2}$ has diameter $\overline{AB}$ with endpoints $A(-5,0)$ and $B(5,0)$. Point P is moving along the curve such that its x-coordinate is increasing at a rate of 2015 units per second. Let $\theta = m\angle APB$. Compute $\frac{d\theta}{dt}$ (in radians per second) at the moment when P is at location $(3,4)$.	10.
11. Compute the distance between the ordered pairs at which the removable discontinuities of $y = \frac{12x^3 - 84x^2 + 72x}{5x^2 - 35x + 30}$ occur.	11.
12. Let $g(x)$ be a continuous function such that $g(1776 - x) = -g(x)$ and for which $g(a) = 0$ for exactly one value of a. Compute a.	12.
13. Let $f(x)$ be a differentiable function such that $f'(x) = xf(x)$ and $f(5) = 5$. Compute $g'(5)$, where $g(x) = f(f(x))$.	13.
14. Square $P_1P_2P_3P_4$ has a perimeter of 712. Vectors $\overrightarrow{P_1P_2}$, $\overrightarrow{P_1P_3}$, $\overrightarrow{P_1P_4}$, $\overrightarrow{P_2P_3}$, $\overrightarrow{P_2P_4}$, and $\overrightarrow{P_3P_4}$ are constructed. Let $\vec{v}$ be the sum of all 6 of these vectors. Compute the magnitude of $\vec{v}$.	14.
15. A mirror lies along the line $\frac{x}{6} + \frac{y}{5} = 1$ in the first quadrant. A beam of light comes in along the line $y = 1$, strikes the mirror at point P, reflects off the mirror, and strikes the y-axis at point A (see diagram). If $\cos(\angle BAP) = \frac{m}{n}$ in lowest terms, compute $m + n$. 	15.

M

Nassau County Interscholastic Mathematics League

Mathletics

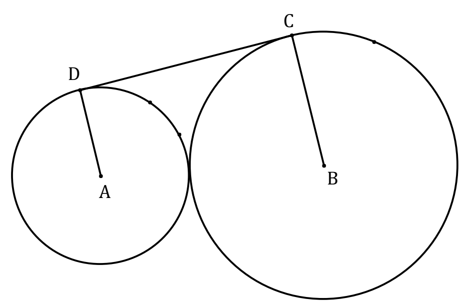
TEAM #

Mathematics Tournament 2015

Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes *Answer Column*

1. Compute the only integer solution of the equation $(3x-31)^2-17(3x-31)=60.$	1.
2. If the length of a diameter of a circle increases by 60%, then the area of the circle increases by n%. Compute n.	2.
3. Three positive integers, $x, y,$ and z, with $x<y<z$ have a sum less than 1000. What is the largest possible value of y ?	3.
4. In the diagram, circles A and B are tangent to each other and to line segment $\overline{DC}$ at points D and C, respectively. If circle A has a radius of length 4, and circle B has a radius of length 6, compute $(DC)^2$.  	4.
5. Let N represent the number of possible passwords that can be formed using 6 uppercase letters followed by 2 digits. Let M represent the number of possible passwords that can be formed using 6 letters, where both uppercase and lowercase letters can be used. If repetition of both letters and digits is allowed, compute $\frac{(N-M)}{26^5}$.	5.

Turn Over

M

Mathematics Tournament 2015

Mathletics

Time Limit: 30 minutes *Answer Column*

6. The 11^{th} term of an arithmetic sequence is 19.7 and the 17^{th} term is 20.15. Compute the 2015^{th} term.	6.
7. If $f(x) = 3x + 2$, we can find a number, t, such that $f\big(f(f(t))\big) = 929$. If we express $t = \frac{a}{b}$ as a fraction in lowest terms, compute $a + b$.	7.
8. Every possible permutation of the letters in the word **SILENT** is arranged alphabetically to form a sequence of "words": {**EILNST, EILNTS, EILSNT, … , TSNLIE**}. If the n^{th} term of the sequence is **LISTEN**, compute n.	8.
9. The square base of a pyramid has an area of 100. Each of the four edges joining the vertices of the base to the top vertex has a length of 10. Compute the volume of the pyramid rounded to the nearest integer.	9.
10. Let $N = \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \cdots + \frac{1}{\sqrt{2015}+\sqrt{2014}}$. Compute $10N$, rounded to the nearest integer.	10.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2015

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________ **Score**________

Time Limit: 60 minutes *Answer Column*

1. A sequence of integers is formed by adding two consecutive terms to get the next term. If the eighth term is 8 and the tenth term is 10, what is the fourth term?	1.
2. Find the sum of the roots of the equation $x - 6\sqrt{x} + 8 = 0$.	2.
3. Compute the maximum area of a right triangle if the sum of the areas of the squares drawn on the three sides is 400.	3.
4. The positive difference between the product of the digits of a two-digit number and the number itself is 14. Find the sum of all two-digit numbers that satisfy this property.	4.
5. The letters: $p, q, r,$ and s represent positive integers. If $p = q^2$, $r = s^4$, and $p - r = 73$, compute $q - s$.	5.
6. There are an infinite number of solutions to the equation $\left(\sqrt{9+\sqrt{81-x^2}}\right) \cdot \left(\sqrt{9-\sqrt{81-x^2}}\right) = x$. Compute the sum of the smallest and largest of these solutions.	6.
7. Two intersecting lines drawn on a sheet of paper divide the paper into 4 regions. What is the maximum number of regions that can be formed when five lines are drawn on a sheet of paper?	7.
8. Find the product of all the solutions to the equation $\lvert 6 - \lvert x \rvert \rvert = 3$.	8.
9. Find the area of the smallest rectangle that can completely contain four non-overlapping equilateral triangles, each of area 17.	9.
10. The lengths of the three sides of a triangle are integers. If one side of the triangle has length 8 and the product of the lengths of the other two sides is 255, find the area of the triangle.	10.
11. Compute the product of all of the real solutions to the equation $(\log_3 x^2)^2 = 9$.	11.

Turn Over

12. The x-intercept and the y-intercept are equal in the linear equation $6y + 3x - 7 = 4k(2y - x + 5)$. Compute the sum of the x- and y- intercepts.	12.
13. The lengths of the congruent sides of an isosceles triangle are each 18. The perpendicular bisector of one leg passes through the midpoint of the base of the triangle. Compute the area of the triangle.	13.
14. Determine the maximum number of non-overlapping circles of radius 1 inch that can fit into a rectangle whose dimensions are 5.5 inches by 10.5 inches.	14.
15. An equation of a line has the following properties: (a) The sum of the y-intercept and the slope is $47/6$ and (b) the sum of the y-intercept and the reciprocal of the slope is $32/9$. If the slope is positive, compute the product of the slope and the y-intercept.	15.
16. The 6-digit number $4c8ddc$ is a multiple of 33 and c and d are digits. If $c < d$, compute $c + d$.	16.
17. Let T_n be the nth triangular number (the nth triangular number is the sum of $1 + 2 + 3 + \cdots + n$). Compute the difference ${T_8}^2 - {T_7}^2$.	17.
18. Given $180° \leq y \leq 270°$, $270° \leq x \leq 360°$, and $\log_3 \tan y = \log_2 \cos x = -\frac{1}{2}$. Compute the sum of the number of degrees of x and y.	18.
19. The product of three consecutive positive integers is 24 times the average of the three integers. Find the product of the three integers.	19.
20. Complete the number puzzle and then compute the sum of all of the digits in the grid.	20.

20. Complete the number puzzle and then compute the sum of all of the digits in the grid.

<u>Across</u>

1. A Fibonacci number.
5. A number divisible by 401.
6. Four consecutive digits but not in consecutive order.

<u>Down</u>

1. A Pythagorean triple.
2. A palindromic perfect square.
3. Each digit is a different cube.
4. A multiple of 11.

1	2	3	4
5			
6			

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

TEAM #

Mathematics Tournament 2015

DO NOT HAND THIS COPY IN. HAND IN THE ONE TEAM COPY.
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Individual Copy

Time Limit: 60 minutes *Answer Column*

1. A sequence of integers is formed by adding two consecutive terms to get the next term. If the eighth term is 8 and the tenth term is 10, what is the fourth term?	1.
2. Find the sum of the roots of the equation $x - 6\sqrt{x} + 8 = 0$.	2.
3. Compute the maximum area of a right triangle if the sum of the areas of the squares drawn on the three sides is 400.	3.
4. The positive difference between the product of the digits of a two-digit number and the number itself is 14. Find the sum of all two-digit numbers that satisfy this property.	4.
5. The letters, $p, q, r,$ and s represent positive integers. If $p = q^2$, $r = s^4$, and $p - r = 73$, compute $q - s$.	5.
6. There are an infinite number of solutions to the equation $\left(\sqrt{9+\sqrt{81-x^2}}\right) \cdot \left(\sqrt{9-\sqrt{81-x^2}}\right) = x$. Compute the sum of the smallest and largest of these solutions.	6.
7. Two intersecting lines drawn on a sheet of paper divide the paper into 4 regions. What is the maximum number of regions that can be formed when five lines are drawn on a sheet of paper?	7.
8. Find the product of all the solutions to the equation $\lvert 6 - \lvert x \rvert \rvert = 3$.	8.
9. Find the area of the smallest rectangle that can completely contain four non-overlapping equilateral triangles, each of area 17.	9.
10. The lengths of the three sides of a triangle are integers. If one side of the triangle has length 8 and the product of the lengths of the other two sides is 255, find the area of the triangle.	10.
11. Compute the product of all of the real solutions to the equation $(\log_3 x^2)^2 = 9$.	11.

Turn Over

T

Team Problems

12. The x-intercept and the y-intercept are equal in the linear equation $6y + 3x - 7 = 4k(2y - x + 5)$. Compute the sum of the x- and y- intercepts.	12.
13. The equal sides of an isosceles triangle each measure 18. The perpendicular bisector of one leg passes through the midpoint of the base of the triangle. Compute the area of the triangle.	13.
14. Determine the maximum number of non-overlapping circles of radius 1 inch that can fit into a rectangle whose dimensions are 5.5 inches by 10.5 inches.	14.
15. An equation of a line has the following properties: (a) The sum of the y-intercept and the slope is $47/6$ and (b) the sum of the y-intercept and the reciprocal of the slope is $32/9$. If the slope is positive, compute the product of the slope and the y-intercept.	15.
16. The 6-digit number $4c8ddc$ is a multiple of 33 and c and d are digits. If $c < d$, compute $c + d$.	16.
17. Let T_n be the nth triangular number (the nth triangular number is the sum of $1 + 2 + 3 + \cdots + n$). Compute the difference ${T_8}^2 - {T_7}^2$.	17.
18. Given $180° \le y \le 270°$, $270° \le x \le 360°$, and $\log_3 \tan y = \log_2 \cos x = -\frac{1}{2}$. Compute the sum of the number of degrees of x and y.	18.
19. The product of three consecutive positive integers is 24 times the average of the three integers. Find the product of the three integers.	19.

20. Complete the number puzzle and then compute the sum of all of the digits in the grid.

20.

Across

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Down

1. A Pythagorean triple.
2. A palindromic perfect square.
3. Each digit is a different cube.
4. A multiple of 11.

1	2	3	4
5			
6			

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2015

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

1.	1.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

2.	2.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

3.	3.