Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM # ______

Mathematics Tournament 2013

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

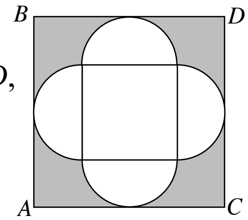
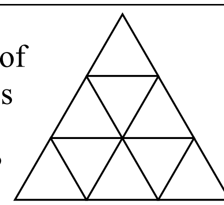
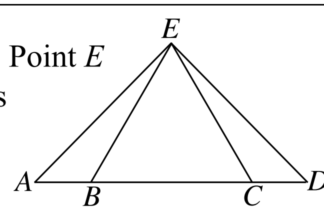
1. Find the maximum value of $y = -2x^2 + 4x + 5$.	1.
2. In a Midwest city, $\frac{1}{4}\%$ of the population have a rare type of flu. If there are 5200 people in the city, how many of them have this type of flu?	2.
3. Two cars are driving on the same highway in the same direction. The car traveling at 65 mph passes the car traveling at 50 mph. If they are traveling at constant rates, how many miles ahead of the 50 mph car is the 65 mph car $1\frac{1}{3}$ hours after he passes the 50 mph car?	3.
4. If the reciprocal of $0.0\overline{45}$ is expressed as a fraction, $\frac{a}{b}$, where *a* and *b* are relatively prime (fraction cannot be reduced), find the value of *a*.	4.
5. What is the smallest multiple of 12, 45, and 72?	5.
6. Find the product of the coordinates of the point of intersection of the two lines whose equations are $y + 2x = 24$ and $\frac{1}{11}x - \frac{1}{2}y = 0$.	6.
7. To determine the price of a new computer game, the retailer takes the distributor's price and adds 30%. The distributor sets his price by adding 10% to the manufacturer's price. If the game retails for $28.608, what was the manufacturer's price?	7.
8. Find the units-digit of the expansion of 47^{2013}.	8.

Turn Over

Mathematics Tournament 2013

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. Around the outside of a 6×6 square, 4 semi-circles are constructed with the sides of the square as their diameters. Another square, $ABCD$, has its sides parallel to the first square and each side is tangent to one of the semi-circles. The shaded area can be expressed as $n + m\pi$. Compute $n + m$. 	9.
10. N is the number of 4-digit numbers whose thousands-digit is even, hundreds-digit is odd, and all four digits are different. Find the sum of the digits of N.	10.
11. Sarah has five green cards numbered 1 through 5 and four yellow cards numbered 3 through 6. She stacks the cards so that the colors alternate and the number on each green card evenly divides into the number on each neighboring yellow card. What is the sum of the numbers on the middle three cards?	11.
12. The sum of two 5-digit numbers, NMT12 and NMT13, is 129425. Compute $N + M + T$.	12.
13. A large equilateral triangle is formed using toothpicks to create rows of small equilateral triangles. The figure below shows 3 rows of triangles with 5 small triangles on the base row. If this pattern continues, how many toothpicks are needed to construct a large triangle of 150 rows? 	13.
14. The sum of two numbers is 8 and their product is 3. Compute the sum of the squares of the two numbers.	14.
15. Points A, B, C, and D lie on a line with $AB = CD$ and $BC = 12$. Point E is not on the line and $BE = CE = 10$. The perimeter of ΔAED is twice the perimeter of ΔBEC. Find the length of $\overline{AB}$. 	15.

10

Nassau County Interscholastic Mathematics League

Grade 10

TEAM #

Mathematics Tournament 2013

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. The parabola whose equation is $y=\frac{2}{3}x^2+4x-1$ has a vertex with coordinates (h, k). Compute the product hk.	1.
2. What is the least common multiple of 8, 18, and 20?	2.
3. Including 1 and 2013, how many different positive integers are factors of 2013?	3.
4. Solve for x: $\sqrt{\sqrt{x}-\frac{11}{9}}=\frac{5}{3}$.	4.
5. What is the y-intercept of the line that passes through (4, –2) and is perpendicular to the line whose equation is $y=\frac{1}{3}x-7$?	5.
6. When $\sqrt[3]{640x^{20}}$ is converted to simplest radical form, it can be expressed as $ax^b\cdot\sqrt[3]{cx^d}$ where a, b, c, and d are integers. Compute the product of a, b, c, and d.	6.
7. A data set consists of all the consecutive integers from 18 through 299 inclusive. If each integer appears exactly once, compute the upper quartile for this set?	7.
8. Let D equal the degree measure of the only obtuse angle whose degree measure is a perfect cube. Compute $D^{1/3}+D^{2/3}+D^{3/3}+D^{4/3}$.	8.
9. Find the area of an isosceles trapezoid with a diagonal that measures 10, an altitude that measures 8, and one of its bases measures 5.	

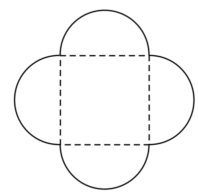
Turn Over

10

Mathematics Tournament 2013

Grade 10

Time Limit: 45 minutes | **Lower Division** | *Answer Column*

10. The following equation has two roots: $2x(5x-12)=(2x-3)(x+3)$. What percent of the smaller root is the larger root?	10.
11. The Sun is approximately 93 million miles from Earth and the Sun's light takes approximately 8 minutes to reach Earth. Based on these approximations, if we let Y represent the speed of light in miles per hour, compute $Y\times 10^{-6}$ rounded to the nearest integer.	11.
12. A right triangle has legs 30 and 40. Find the measure of the altitude drawn to the hypotenuse of the right triangle.	12.
13. An flower garden is designed by creating a square whose diagonal measures 12 meters and 4 semicircles placed on each of the four sides as in the diagram. If the area of the garden is expressed in the form $a + b\pi$, compute the positive difference between a and b. 	13.
14. Base your answer to the following problem by satisfying the 3 conditions below: * The difference between the roots is 18 * The axis of symmetry has the equation $x = 4$. * The y-intercept is 130. Find the y-coordinate of the vertex of this parabola.	14.
15. A standard die, with faces 1 through 6, is rolled 12 times. The sum of the reciprocals of each outcome is computed. The sum that is *most likely* is closest to what integer?	15.

Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM # ____

Mathematics Tournament 2013

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. Find the value of $5\log_5 125+4\log_4 \frac{1}{64}+6\log_a a^6$, where $a>1$.	1.
2. Find $\lvert 20+21i\rvert$. (Just as the absolute value of a real number x is the distance from the origin to the point on the number line whose coordinate is x, the absolute value of a complex number z is the distance from the origin to a point in the complex plane corresponding to z.)	2.
3. Two tangents to a circle from the same exterior point intercept a major arc of 240°. Find the number of degrees in the angle formed by the two tangents.	3.
4. In ΔABC, $AB = 24$, $BC = 40$, and $m\angle B = 30°$. Find the area of ΔABC.	4.
5. Find the sum of the integral solutions of $x^2-3x-28\ge 0$, if $1\le x\le 10$.	5.
6. Some people board an empty elevator on the first floor. Five exit on the second floor. Sixty percent of the remaining passengers exit on the third floor. The remaining two people exit on the fifth floor. How many people boarded the elevator on the first floor?	6.
7. How many of the integers between 1 and 600 inclusive are divisible by 3 or 5?	7.
8. $f(x)=x^2-3x+4$, $x\ge\frac{3}{2}$ and $g(x)=\frac{3+\sqrt{4x-7}}{2}$ are real-valued functions that are inverses of each other. Find the value of x for which the graphs of these functions intersect.	8.

Turn Over

Mathematics Tournament 2013

Grade 11

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. If $\sqrt{k}=\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}$, find k.	9.
10. A circle is inscribed in a triangle whose sides have length 5, 12, and 13. If the length of the line segment whose endpoints are the points of tangency on the sides of the triangle whose lengths are 5 and 13 is expressed in simplest $\frac{a\sqrt{b}}{c}$ form, find $a + b + c$.	10.
11. The aviary club has ten members including Heckle and Jeckle. A special committee of five members will be chosen. Heckle and Jeckle have stated that they will not serve together on the special committee. However, each one is willing to serve on the special committee without the other. In how many ways can the special committee be chosen?	11.
12. A circle is inscribed in a rhombus whose diagonals have length 6 and 8. If the area of the region inside the rhombus but outside the circle that is inscribed in the rhombus is expressed in the form $a-\frac{b}{c}\pi$, where $\frac{b}{c}$ is in simplest form, find $a + b + c$.	12.
13. If $\tan x+\tan y=24$ and $\cot x+\cot y=28$, find $\tan(x+y)$.	13.
14. The equations $(x-3)^4-(x-3)=0$ and $x^2-bx+c=0$ have two non-real roots in common. Find $b + c$.	14.
15. Start with a first square the length of whose sides are two. Form a second square whose vertices are the midpoints of the sides of the first square. Form a third square whose vertices are the midpoints of the sides of the second square. Continue this process forever. Find the sum of the areas of all the squares.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2013

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. Compute the 11^{th} term of the arithmetic sequence whose 1^{st} term is 39 and whose 31^{st} term is 2013.	1.
2. If $x + 2y + 3z = k$, $2x + 3y + 4z = 1337$ and $3x + 4y + 5z = 2013$, compute k.	2.
3. Let $f(x) = \left(x\sqrt{x} - 1\right)^2 + 2x^{3/2}$. Compute $f'(17)$.	3.
4. Compute $\sum_{n=672}^{2013} \log_{\sqrt{3}}\left(\frac{n}{n-1}\right)$.	4.
5. Compute $\lim_{x\to 5} \lim_{y\to 3} \frac{(x^2-25)(y^2-9)}{xy - 3x - 5y + 15}$.	5.
6. The area of regular hexagon *ABCDEF* is 999. Compute the area of rectangle *BCEF*.	6.
7. Compute the value of a such that the tangent line to the graph of $y = x^2 + 3x + 6$, at the point where $x = a$, does not intersect the line $37x - y = 2013$.	7.
8. Let $f(x) = \frac{1}{2}\sin(2012x)\sec(1006x)$. Compute $f'\left(\frac{\pi}{3018}\right)$.	8.
9. Compute the sum of the squares of the roots of $\left(x^2 - 7x\right)^2 + 22\left(x^2 - 7x\right) + 120 = 0$.	9.

Turn Over

Mathematics Tournament 2013

Grade 12

Time Limit: 45 minutes **Upper Division** *Answer Column*

10. Let $f(x)=\dfrac{17-2x}{342}$. Let $g(x)$ be a linear function such that $\left[(f\circ g)\circ(f\circ g)\right](x)=x$. Compute $\lvert g'(x)\rvert$.	10.
11. Let k be the absolute minimum value of $f(x)=\dfrac{\sqrt{x}}{x+1}$. Compute the value of 720k.	11.
12. Compute ${}_{16}C_{13}+{}_{16}C_{14}+{}_{17}C_{15}$.	12.
13. Define a sequence of points $P_0, P_1, P_2, \ldots$ by the rules: $P_0=(153,222)$, $P_1=(208,308)$, and for $n\geq 2$, $P_n=$ the image of P_{n-2} after a 90° rotation about P_{n-1}. Compute the sum of the x and y coordinates of P_{2013}.	13.
14. The tangent lines to the graph of $y=-x^4+5x^2-4$ at the points where $x=-3$ and $x=3$ intersect at (a, b). Compute b.	14.
15. In quadrilateral $ABCD$, $AB=AC=AD$, $m\angle BAD=120°$, $CD=20$, and $BC=12$. Compute BD.	15.

M

Nassau County Interscholastic Mathematics League

Mathletics

TEAM #

Mathematics Tournament 2013

Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes *Answer Column*

1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Turn Over

M

Mathematics Tournament 2013

Mathletics

Time Limit: 30 minutes *Answer Column*

6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

TEAM # ______

Mathematics Tournament 2013

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________ **Score** ________

Time Limit: 60 minutes *Answer Column*

1. Complete the number puzzle with only one digit in each box and then calculate the sum of all the digits. Place the sum in the answer box.

1	2	3	4
5			
6			

Across
1. Smallest 4-digit Fibonacci number.
5. A multiple of 11.
6. A 4-digit prime number.

Down
1. The square of a prime number.
2. A perfect cube.
3. The largest 3-digit prime number.
4. A palindromic number.

1.

2. How many positive integers less than 1000 contain the digit 7? 2.

3. A rectangle with sides in the ratio 1:10 is inscribed in a circle. Find, to the nearest whole number, the ratio of the area of the circle to the area of the rectangle. 3.

4. Find the volume of a rectangular prism whose face diagonals are 10, 17, and $\sqrt{261}$. 4.

5. A triangle has vertices at (0, 0), (1, 1), and (9, 1). A vertical line divides the triangle into 2 regions of equal area. If $x = a$ is the equation of the vertical line, find a. 5.

6. Solve for n in the equation $\frac{1+3+5+\ldots+(2n-1)}{2+4+6+\ldots+2n} = \frac{497}{498}$. 6.

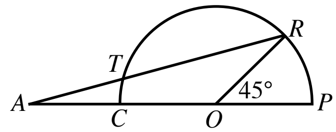
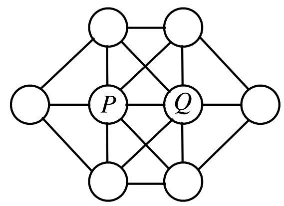
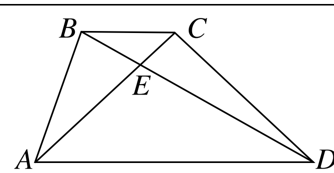
7. The graphs of the equations $y = \frac{1}{2}x - 18$ and $y = ax + b$ are symmetric about the line whose equation is $y = x$. Find the sum $a + b$. 7.

8. The ratios x/y and 3/7 are equal. The ratios $(x + 2y)/(3x + 7y)$ and a/b are equal. If the greatest common factor of a and b is 1, calculate $a + b$. 8.

9. What is the largest positive factor for all integers formed by subtracting a four-digit number from the number formed by reversing the digits? 9.

10. The line with equation $x + y = k$ is tangent to the circle $x^2 + y^2 = 100$. Compute the area of the triangle formed by the line and the two coordinate axes. 10.

Turn Over

11. A store prices an item in dollars and cents so that when 4% sales tax is added no rounding is necessary because the result is exactly n dollars where n is a positive integer. What is the smallest possible value of n?	11.
12. In rectangle $ABCD$ $AB = 288$ and $BC = 120$. Points M and N are placed on $\overline{AB}$ and $\overline{CD}$ respectively, such that $AMCN$ is a rhombus. Find MN.	12.
13. Given semicircle O with $m\angle ROP = 45°$, and $AT = OP$, determine the number of degrees in $m\angle CAT$. 	13.
14. Place the integers 1 through 8 in the circles so that no two consecutive integers are in circles connected by line segments. Find the sum of the two integers in circles P and Q. 	14.
15. A box contains 4 nickels, 5 dimes, and 6 quarters. Six coins are drawn without replacement, with each coin having an equal probability of being chosen. The probability that the value of the coins drawn is at least \$1.25 is a/b, where a and b are relatively prime (fraction is reduced completely). Find $a + b$.	15.
16. Point B lies on a circle whose center is $O(0, 0)$ with radius $\overline{OA}$. The coordinates of A are (100, 0). If the measure of $\angle AOB = 125°$, and the coordinates of the midpoint of $\overline{AB}$ are (p, q) find, to the nearest integer $p + q$.	16.
17. $\frac{8}{\log_2 x}+\frac{1}{2\log_{49} x}-\frac{2}{\log_8 x}=\frac{1}{\log_b x}$. Compute the value of b.	17.
18. In trapezoid $ABCD$ with diagonals intersecting at E, the area of ΔBEC is 16 and the area of ΔAED is 49. Find the area of the trapezoid. 	18.
19. In the sequence 5, 9, 4, … each term after the first two is equal to the term preceding it minus the term preceding that (4 = 9 – 5). Find the sum of the first one hundred terms.	19.
20. If $f(x)=ax^7-bx^5+cx+57$ and $f(17)=-28$, compute $f(-17)$.	20.

T

Nassau County Interscholastic Mathematics League

Team Problem Solving

TEAM #

Mathematics Tournament 2013

DO NOT HAND THIS COPY IN. HAND IN THE ONE TEAM COPY.
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Individual Copy

Time Limit: 60 minutes *Answer Column*

Turn Over

T

Mathematics Tournament 2013

Team Problems

Time Limit: 60 minutes

Answer Column

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2013

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

1.	1.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

2.	2.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

3.	3.