Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM #

Mathematics Tournament 2012

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. If $(2x+5)(3x-2)(4x-3)$ is written in the form ax^3+bx^2+cx+d, what is the value of d?	1.
2. If the difference $4\sqrt{98}-\frac{2}{3}\sqrt{72}$ is expressed in simplest radical form, $a\sqrt{b}$, find $a+b$.	2.
3. In ΔABC, altitude $\overline{AD}$ is drawn to base $\overline{BC}$. If $AD = 8$, $AC = 10$, and $AB = 17$, find BC.	3.
4. There are 180 girls and 20 boys in a room. How many girls must leave the room so that 84% of the people left in the room are girls?	4.
5. If $n=3^x+3^x+3^x$ and $n^2=9^k$, find the numerical value of $k-x$.	5.
6. The product of two positive integers is 96. If neither is a square or a cube of an integer and neither is a factor of the other, find the sum of the two integers.	6.
7. In a golf tournament, each match groups four people together so that one person wins and goes on to another match in the next round, while the other three lose and are eliminated. The tournament continues until only one person remains undefeated. If 256 players enter the tournament, how many matches must be played to determine the winner?	7.
8. The line $y = mx + 12$ with $m > 0$, forms a triangle with the coordinate axes. If the perimeter of the triangle is 36, and m is expressed in simplest form as $\frac{a}{b}$, find $a + b$.	8.

Turn Over

Mathematics Tournament 2012

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. If $2a + 3b + 4c = 30$ and $4a + 3b + 2c = 42$, find $a + b + c$.	9.
10. The average of a and b is c. The average of a and c is 1 more than b. The average of b and c is 2 more than twice a. If a, b, and c are real numbers, compute $\lvert a \rvert$.	10.
11. A lattice point in the coordinate plane is a point whose coordinates are integers. How many lattice points in the first quadrant does the graph of $20x + 11y = 2011$ pass through?	11.
12. Let (a, b) be an ordered pair of positive prime integers. If $a^b \cdot b^a = 6272$, find $a + b$.	12.
13. If $x > y$, $x^2 - xy = 75$, and $xy - y^2 = 50$, find $x + y$.	13.
14. Find the length of the radius of a circle inscribed in a right triangle whose sides are 9, 40, and 41.	14.
15. A sequence consists of five positive increasing consecutive even integers. If the first integer is divisible by 10, the second integer divisible by 8, the third integer divisible by 6, the fourth integer divisible by 4, and the fifth integer divisible by 2, find the smallest possible value for the first integer.	15.

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Nassau County Interscholastic Mathematics League

Grade 10

TEAM # ☐

Mathematics Tournament 2012

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. Compute the largest of six consecutive odd integers whose sum is –12.	1.
2. Find the area of the quadrilateral with vertices $A(-4, 0)$, $B(0, 1)$, $C(2, 0)$, and $D(0, -6)$.	2.
3. A clearance item at a store has been marked down several times. After 4 consecutive markdowns of 25% each, the sale price is \$81. Compute the number of dollars of the original selling price.	3.
4. Compute the degree measure of an acute angle whose complement is equal to its square root.	4.
5. A line has the equation $y = -3x + 9$. A second line whose equation is $ay - 8x = 48$ is perpendicular to the first line. Compute the y-intercept for the second line.	5.
6. Compute the number of inches, to the nearest whole number, in the length of a diagonal of a rectangular piece of paper that measures $8\frac{1}{2}$ inches by 11 inches.	6.
7. The sides of a right triangle can be expressed algebraically as $2x$, $3x - 1$, and $x + 11$, where $x > 6$. Compute the area of the triangle.	7.
8. In a class of 32 students, each student is given a standard die with faces 1 through 6, and each outcome is equally likely. If they all roll the dice and compute the sum of all the outcomes, what sum has the highest probability of occurring?	8.
9. In a regular hexagon, each side measures 10 cm. Let h represent the height of the hexagon when any of its sides is the base (the distance between a pair of parallel sides). Compute h^2.	

Turn Over

Mathematics Tournament 2012

Grade 10

Time Limit: 45 minutes **Lower Division** *Answer Column*

10. A jar contains only nickels, dimes, and quarters. The quantities of each type of coin are in the same ratio as their respective denominations. If the total value in the jar is \$18, find the total number of coins.	10.
11. Compute the least positive integer that is divisible by 1, 2, 3, 4, 5, 6, and 7.	11.
12. For positive integers a and b, it is true that $a^2 = 55 + b^2$. There are two possible solutions for b. Compute the positive difference between these two values.	12.
13. An equilateral triangle has an area of $12\sqrt{3}$. The perimeter of the same triangle can be expressed as $m\sqrt{3}$. Compute m.	13.
14. The mean of the four positive integers x, $(x + 2)$, $(x^2 + 4)$, and $2x^2$ is 5.5. Compute the positive difference between the mode and the median.	14.
15. A pyramid is formed with a rectangular base and 4 isosceles triangles. The length and width of the rectangle are 18 and 10, and the height of the pyramid is 12. Compute the total surface area of the pyramid.	15.

Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM #

Mathematics Tournament 2012

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ____________________ **School** ____________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. In a circle, chords $\overline{AB}$ and $\overline{CD}$ intersect in the interior of the circle at point E. If $AE = 10$, $EB = 2$, $CD = 9$, and $DE < CE$, find DE.	1.
2. Find the only positive integer solution to $3^{2x^2-9x} = \frac{1}{81}$.	2.
3. If $f(x) = \sqrt{x-2}$ and $f(g(x)) = x$, compute g(3).	3.
4. If the angles in the following equation are acute: $\sec(3x-20)^\circ = \csc(2x+25)^\circ$, compute x.	4.
5. The solution set of $4x-12>0$ and $x^2-12x<-20$ may be expressed as $a<x<b$, where a and b are integers. Find the product $a \cdot b$.	5.
6. Compute $\frac{1}{1+\frac{1}{\sqrt{2}}} \cdot \frac{1}{1-\frac{\sqrt{2}}{2}}$.	6.
7. Evaluate $\frac{1}{50}\sum_{n=1}^{100}(3n-1)$.	7.
8. What is the only positive integer solution to $\frac{\sqrt{8+x}}{4} = \frac{6}{2+\sqrt{8+x}}$.	8.

Turn Over

Mathematics Tournament 2012

Grade 11

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. When the following equation is solved for x on the interval $[0, 2\pi)$: $\cot(2x)+\tan(2x)=2$, the sum of all the roots may be expressed as $\frac{p}{q}\pi$. If $\frac{p}{q}$ is fully reduced, find the product $p\cdot q$.	9.
10. A bag contains 6 red and 4 blue marbles that are identical in every way except for color. Four are drawn out of the bag one at a time without replacement. If, in simplest form, the probability is $\frac{p}{q}$ that they are alternately of different color, find $p + q$.	10.
11. For all real numbers x and y, the function f has the property that $f(x+y)=f(x)+f(y)$. If $f(1)=4$, find $f(100)$.	11.
12. Each of angles x and y has a measure between 0° and 360°, $x \neq y$, $\sin x \neq 0$, $\sin y \neq 0$, and $\sin(2y) \neq 0$. If $\frac{\sin(2x)}{\sin(2y)} = \frac{\sin x}{\sin y}$, then compute $x + y$ in degrees.	12.
13. Solve $\log_2(x+3)+\log_{\sqrt{2}}(x+3)+\log_{1/2}(x+3)=14$.	13.
14. The area of an equilateral triangle with exactly one vertex on each of the lines $y = 3$, $y = 0$, and $y = -1$ can be expressed as $\frac{k\sqrt{3}}{3}$. Find k.	14.
15. Quadrilateral $ABCD$ is a parallelogram with an area of 240. Point P lies on $\overline{BC}$ such that $BP = \frac{1}{4}BC$. Point R lies on $\overline{PD}$ such that $DR = \frac{3}{4}PD$. Find the area of ΔPAR.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2012

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. If $f(x)=3x+5$ and $g(x)=\dfrac{x-5}{3}$, compute $\dfrac{f(g(2012))}{4}$.	1.
2. For what value of c is $x - 5$ a factor of x^3-2x^2-cx?	2.
3. If $f(n+1)=\dfrac{3f(n)-1}{3}$ for all integers $n \geq 1$ and $f(1) = 10$, find $f(10)$.	3.
4. If $f(x)=\cos x$, compute $f^{(2012)}(2012\pi)$, where $f^{(n)}$ denotes the n^{th} derivative of f with respect to x.	4.
5. Compute the area of ΔABO if the polar coordinates of A are (10, 23°), the polar coordinates of B are (10, 113°), and the polar coordinates of C are (0, 0°).	5.
6. In a geometric sequence, the sum of the first and third terms is 65 and the sum of the second and fourth terms is 97.5. Compute the sum of the first three terms.	6.
7. Compute the y-intercept of the line tangent to the circle defined by $x^2+4x+y^2+6y-60=0$ at the point (6, 0).	7.
8. If $f(x)=(2x^2-3)^3$, compute $f'(2)$.	8.
9. How many factors does the number whose prime factorization is $2^2 \cdot 3^3 \cdot 5^5 \cdot 7^7$ have?	9.

Turn Over

12	**Mathematics Tournament 2012**	**Grade 12**

Time Limit: 45 minutes **Upper Division** *Answer Column*

10. If $0° \le x < 360°$, find the sum of the degree-measures of the solutions of $\sin x + \cos x = 1 + \sin(2x)$.	10.
11. Find the maximum value of $f(x) = (6+x)(6-x)(x+2)(x-2)$.	11.
12. If $\sqrt[3]{2+\sqrt{x}} + \sqrt[3]{2-\sqrt{x}} = \sqrt[3]{16}$, solve for *x*.	12.
13. If $r = 28\sqrt{2}(\cos 118° + i\sin 118°)$ and $s = 4(\cos(-73°) + i\sin(-73))$ and $rs = x + yi$, compute $x + y$.	13.
14. If the length of the radius of regular octagon *STUVWXYZ* is 8, the area of the quadrilateral region bounded by $\overleftrightarrow{ST}, \overleftrightarrow{UV}, \overleftrightarrow{WX}$, and $\overleftrightarrow{YZ}$ is $k(2+\sqrt{2})$. Compute *k*.	14.
15. Square *PQST* is drawn on hypotenuse $\overline{PQ}$ of right ΔPQR and $\overline{RS}$ is drawn. If *PR* = 1 and *RQ* = 3, find *RS*.	15.

Nassau County Interscholastic Mathematics League

M

Mathletics

TEAM #

Mathematics Tournament 2012

Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes *Answer Column*

1. Find the least positive integer n, such that $2012n$ is a perfect square.	1.
2. If 1 US Dollar is equivalent to 0.67 Euros, 1 Euro is equivalent to 64.4 Indian Rupees, and 10 Rupees is equivalent to 1.44 Chinese Yuan, how many dollars are equivalent to 1500 Chinese Yuan? Round your answer to the nearest dollar.	2.
3. Peter took 5 tests in his English class and earned the grades 70, 75, 85, 96, and 97. He has two more test	3.
4. Points P, Q, and R, are placed at the midpoints of the sides of equilateral ΔABC. The perimeter of ΔABC is 45, and the area of ΔPQR can be expressed in simplest form as $\frac{a\sqrt{3}}{b}$. Compute $a + b$.	4.
5. How many integers, x, satisfy $\left\|x^3 + 2x^2 + 10x - 1\right\| \leq 2012$?	5.

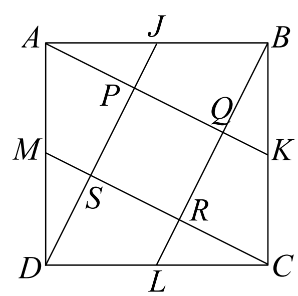
Turn Over

M

Mathematics Tournament 2012

Mathletics

Time Limit: 30 minutes *Answer Column*

6. The first few terms of the arithmetic sequence $\{a_n\}$ are shown below. Compute the least integer n such that $a_n > 2012$. 7, 18, 29, 40, …	6.
7. A circle is inscribed in right ΔABC with right angle C. The hypotenuse of the triangle is tangent to the circle at P, $AP = 30$, and $AB = 40$. Find the area of the triangle.	7.
8. A quadratic function contains the ordered pairs (0, 12), (1, –7), and (20, 12). The graph of this function intersects the line whose equation is $y = 12x - 100$ in quadrant IV at the point (a, b). Compute $a - b$.	8.
9. If three standard six-sided dice are rolled, the probability that the sum of the top faces is greater than 5 can be expressed as a fraction in lowest terms as $\frac{p}{q}$. Compute $p + q$.	9.
10. The area of square $ABCD$ is 625. The points J, K, L, and M are the midpoints of their respective sides. The points P, Q, R, and S are the intersections of the segments seen in the diagram. Compute the area of $PQRS$. 	10.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2012

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
Three (3) points per correct answer.

Team Copy **School** ______________________ **Score** ________

Time Limit: 60 minutes *Answer Column*

1. Complete the number puzzle with only one digit in each box and then calculate the value of $A - 10B$ where
A = the sum of the three 4-digit across numbers and
B = the sum of the four 3-digit down numbers.

1	2	3	4
5			
6			

Across
1. An arithmetic sequence.
5. A palindromic number.
6. A geometric sequence.

Down
1. The sum of the three digits is even.
2. The start of 100π.
3. A power of two.
4. A multiple of 7.

1.

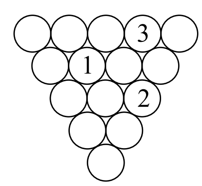
2. Six friends meet. If each one shakes hands with each of the others, how many handshakes are there all together?

2.

3. In an isosceles triangle each of the two congruent sides has a length of $17\sqrt{2}$. Find the length of the third side that will maximize the area of the triangle.

3.

4. Fill in the circles with the numbers 4 through 15 so that every number in the bottom four rows is the positive difference between the two numbers directly above it. Find the sum of the five (5) numbers on the leftmost edge of the triangular shape.



4.

5. Find the sum of all integer solutions of $|x-3|+|2x-6|>12$, given that $-10 \le x \le 10$.

5.

6. Compute $a + b + c$, given that $2a+5b+7c=23$ and $4a+b-c=1$.

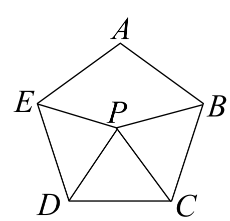
6.

7. The lengths of two sides of a triangle are 4π and $5\sqrt{2}$. How many triangles can be constructed if the third side of the triangle must be an integer?

7.

Turn Over

Time Limit: 60 minutes *Answer Column*

8. If A and B are digits and the number 123,456,7AB is divisible by 99, compute the product $A \cdot B$.	8.
9. How many diagonals are there in a prism with a pentagonal base? Make sure to count the base and face diagonals as well as the interior diagonals.	9.
10. Find the smallest integer greater than 2012 that is divisible by 2, 3, 4, 5, and 6.	10.
11. An equation of a parabola is $y = ax^2 + bx + c$. If the parabola intersects the x-axis at –4 and –8 and intersects the y-axis at –16, compute $\lvert a \cdot b \cdot c \rvert$.	11.
12. Three vertices of a rhombus lie on a circle and the fourth vertex is at the center of the circle. If the area of the rhombus is $32\sqrt{3}$ and the area of the circle is $k\pi$, compute k.	12.
13. The product $12_a \cdot 131_a = 2012_a$ is correct in base a. Find a.	13.
14. A pair of adjacent sides of a square are represented by $\sqrt{32 - x^2}$ and $\sqrt{4 - 3x}$. Find the perimeter of the square.	14.
15. There are two values for m that make the roots of the equation $mx^2 - 3x - 2 = 0$ differ by 1. Compute the sum of the two values of m.	15.
16. The first two terms of a geometric series are 5.03 and 35.21. If the sum of n terms of the series is 2012, compute n.	16.
17. Figure $ABCDE$ is a regular polygon and ΔPDC is equilateral. Compute the number of degrees in the measure of $\angle EPB$. (Note: picture is not drawn to scale.) 	17.

Go to Next Page

18. A deck of cards consists of 1 one, 2 twos, 3 threes, and so on up to and including 10 tens. There are no other cards in the deck and two cards are simultaneously chosen. If the probability that both cards have the same number is $\frac{p}{q}$, where p and q are relatively prime, find the value of $p + q$.	18.
19. Two masts on a schooner are 273 feet apart. The masts are 150 feet and 200 feet tall. A guy wire is to be connected from the top of each mast to a single point on the deck. What is the number of feet from the foot of the shorter mast to the point on the deck that will minimize the total length of wire?	19.
20. The sum of the solutions on the interval $[0, \pi]$ of $\frac{\cot^3 x \cdot \csc^5 x}{\sin x \cdot \tan x \cdot \sec^2 x} = 16\cot^8 x$ can be written as $n\pi$. Compute n.	20.

Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2012

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

1.	1.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

2.	2.

Name ______________________ **School** ______________________ **Score** ________

Time Limit: *Answer Column*

3.	3.