

Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM #

Mathematics Tournament 2011

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for each correct answer.

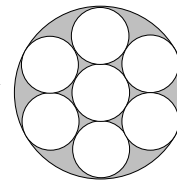
Name _____ School _____ Score _____

Time Limit: 45 minutes

Lower Division

Answer Column

1. If $\sqrt{8} + \sqrt{18} = \sqrt{c}$, compute c .	1.
2. If one root of $y = 2x^2 - 13x + k$ is $\frac{3}{2}$, compute the other root.	2.
3. Expressed in simplest radical form, the altitude of a triangle whose base is $3\sqrt{5}$ and whose area is 30 can be expressed as $a\sqrt{b}$. Compute $a + b$.	3.
4. A circle has a radius of r . The ratio of the area of the circle to its circumference is 5:2. Compute r .	4.
5. How many positive integer divisors does the number 2010 have?	5.
6. Three fair dice are tossed. If the probability that all three numbers showing are different is expressed as a fraction, $\frac{a}{b}$, compute $a + b$.	6.
7. Each of the small circles in the diagram has a radius of 1 unit. The innermost circle is tangent to the six circles that surround it, and each of the circles is tangent to the larger circle and to its small-circle neighbors. Compute x , if the area of the shaded region is πx .	7.
8. A car travels 1 mile in 1 minute. To travel 1 mile in 40 seconds, by what percent must the car increase its speed?	8.
9. Peter has twice as many brothers as sisters. His sister, Anna, has five times as many brothers as sisters. How many siblings does Peter have?	9.



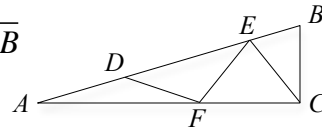
Turn Over

Time Limit: 45 minutes

Lower Division

Answer Column

10. Compute the positive integer x , which satisfies the equation $\frac{(200)^2 \times (700)^2}{(100x)^2} = (14)^2$.	10.
11. A car dealer sold 2 cars for \$9999 each. One sale made a profit of 10% while the other sale took a loss of 10%. What was the dollar amount of the overall profit or loss on the two deals?	11.
12. Find the sum of the first 15 odd integers that have a remainder of 2 when divided by 3.	12.
13. Mr. Rogers class is set up so that there are 5 groups of m students each. One day 6 students were absent. To create equal groups, he needed to increase the number of groups by 1 and decrease the number of students in each group by 2. How many students are in the class when no one is absent?	13.
14. Three cubes with edges 2 cm, 6 cm, and 8 cm are glued together at their faces. Find the minimum possible number of square centimeters in the surface area for the resulting figure.	14.
15. $\triangle ABC$ is a right triangle. Segments \overline{AD} , \overline{DF} , \overline{FE} , \overline{EC} , and \overline{CB} are all of equal length. Compute the measure of $\angle A$.	15.



Nassau County Interscholastic Mathematics League

10

Grade 10

 TEAM #

Mathematics Tournament 2011

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Name _____ School _____ Score _____

Time Limit: 45 minutes
Lower Division
Answer Column

1. Compute the largest of five consecutive integers whose sum is -10 .	1.
2. Three points $A(-6, 14)$, $B(18, -2)$, and $C(k, k)$ are collinear. Compute the value of k .	2.
3. Compute the degree measure of an angle whose supplement is 250% of its complement.	3.
4. Joan measured the length, width, and height of a right rectangular prism for the purpose of calculating its volume. Joan's measurements were 3, 3, and 22 respectively. It turns out that the actual volume of the prism is more than what Joan calculated. If she made a 1% relative error, what is the actual volume of the prism?	4.
5. Compute the area of an isosceles trapezoid if the lengths of the bases are 11 and 21 and the length of each of its diagonals is 20.	5.
6. Let the function f be defined by $f(x) = x + 3$. If $f(p) = 15$, what is the value of $f(3p)$?	6.
7. If $\frac{x - y}{x + y} = \frac{4}{5}$, compute the value of $\frac{x^2}{y^2}$.	7.
8. Find the value of k such that the point whose coordinates are $(0, k)$ will lie on the perpendicular bisector of the line segment whose endpoints are $(1, 1)$ and $(5, 3)$.	8.
9. A rectangle has a length of 3 cm and a width of $\sqrt{3}$ cm. Compute the product of the lengths of the two diagonals.	9.

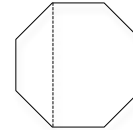
Turn Over

Time Limit: 45 minutes

Lower Division

Answer Column

<p>10. A triangle whose area is 240 sq cm undergoes a dilation of constant $\frac{3}{4}$. Find the number of square centimeters contained within the image.</p>	10.
<p>11. A train traveled the distance from Akron to Bedford at 60 mph, and then traveled the distance back to Akron at 90 mph. Compute the average speed of the train, in mph, for the entire round trip.</p>	11.
<p>12. A rectangular prism and a cube have equal surface areas. Two opposite faces of the rectangular prism are squares with unknown sides while the height of the prism is 22 units. If each edge of the cube is twice the length of each unknown edge of the prism, compute the number of square units in the surface area of the cube.</p>	12.
<p>13. A regular octagon is divided into a trapezoid and a hexagon as shown. If the side of the octagon is 10, the area of the trapezoid is what percent of the area of the original octagon.</p>	13.
<p>14. Point P with coordinates $(5, 0)$ is rotated 90° counterclockwise about the point whose coordinates are $(2, 4)$. If the coordinates of the result, P', are (a, b), compute the value of the product ab.</p>	14.
<p>15. Four unknown integers can be expressed algebraically as $\frac{x}{3}$, $(x+1)$, $2x$, and $\frac{27}{x}$. If the mean of these numbers is 5, compute the median of the four numbers.</p>	15.



Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM #

Mathematics Tournament 2011

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Name _____ School _____ Score _____

Time Limit: 45 minutes

Upper Division

Answer Column

1. A cube is inscribed in a sphere of radius $4\sqrt{3}$. Compute the volume of the cube.	1.
2. By how much does the product of the roots of $3x^2 + 14x - 5 = 0$ exceed the sum of the roots of the same equation?	2.
3. In a 15 mile race in which each runner runs at a constant rate for the entire race, Betty beats Carole by 5 miles and Betty beats Diane by 7 miles. By how many miles can Carole beat Diane in a 15 mile race?	3.
4. Two circles with radii 18 inches and 8 inches are tangent externally to each other. Compute the length of a line segment whose endpoints are the points of tangency of one of the common external tangent lines.	4.
5. A sequence is recursively defined as $a_1 = -50$ and $a_n = a_{n-1} + 2n$, for $n \geq 2$. Find the fifth term of the sequence.	5.
6. Compute $\frac{2!}{1!} - \frac{3!}{2!} + \frac{4!}{3!} - \dots + \frac{200!}{199!}$, where the signs alternate between addition and subtraction.	6.
7. The lengths of the diagonals of a rhombus are 10 inches and 20 inches. The area of a circle inscribed in the rhombus is $k\pi$ square inches. Compute k .	7.
8. Point P whose coordinates are $(55, -4)$ is rotated 90° in a counterclockwise direction about point Q whose coordinates are $(-5, 6)$. The coordinates of the resulting point P' are (a, b) . Compute $2(a + b)$.	8.
9. If $2^x = \frac{32y}{3}$ and $3^x = 81y$, then compute x^y .	9.

Time Limit: 45 minutes

Upper Division

Answer Column

10. If $\log(5x + 130) - \log(8x - 2) = 1$, then compute $5x + 130$.	10.
11. Compute the product of 100 and the sum of the roots of the equation: $ 2x - 1 + x - 2 = 4$.	11.
12. Compute the degree-measure of acute angle x , such that $(\csc x + \cot x)(1 - \cos x) = \cos 42^\circ$.	12.
13. In right $\triangle ABC$, $AC = 6$, $CB = 8$, and $AB = 10$. A circle is inscribed in $\triangle ABC$ such that it is tangent to \overline{AC} at point P and tangent to \overline{AB} at point Q . If the length of \overline{PQ} can be written in the form $\frac{k}{\sqrt{5}}$, compute the value of k .	13.
14. If $f(x) = \frac{2x + 1}{3x - 23}$, find $f^{-1}(3)$.	14.
15. Three fair dice of different colors are rolled. If, in simplest form, the probability is $\frac{p}{q}$ that the product of the three resulting numbers is 24, compute $p + q$.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2011

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Name _____ School _____ Score _____

Time Limit: 45 minutes

Upper Division

Answer Column

1. Find the one-hundredth term of an arithmetic sequence whose first term is -8 and whose common difference is 7 .	1.
2. If $f(x) = (x^3 - 1)^2$, compute $f'(2)$.	2.
3. If $f(x) = 5x - 4$, $g(f(x)) = 50x + 75$, and the function $g(x)$ is defined by the equation $g(x) = ax + b$, where a and b are real numbers, compute $a + b$.	3.
4. Let the two roots of the equation $\sqrt{\frac{x+10}{x-2}} + \sqrt{\frac{x-2}{x+10}} = \frac{5}{2}$ be r and s . Compute $ r + s $.	4.
5. Find $\lim_{x \rightarrow -3} \frac{x^2 + 340x + 1011}{x + 3}$.	5.
6. Find the number of solutions to $3x + 5y = 2011$ such that x and y are positive integers.	6.
7. Compute $\frac{\sum_{k=1}^{10} \log\left(\frac{1}{2^k}\right)}{\log\left(\frac{1}{2}\right)}$.	7.
8. The line tangent to the graph of the function $y = f(x)$ at the point with coordinates $(10, 23)$ passes through the point with coordinates $(15, 68)$. Compute $f'(10)$.	8.
9. In $\triangle ABC$, $AB = 40$, $AC = 70$ and the length of the median \overline{AM} is 35 . Compute BC .	

Time Limit: 45 minutes

Upper Division

Answer Column

<p>10. If r and s are the roots of $x^2 - x + 8 = 0$, and the numerical value of $\frac{1}{r^2} + \frac{1}{s^2}$ is computed and simplified to $\frac{p}{q}$, compute $p + q$.</p>	10.
<p>11. If $\sin(x) + \cos(x) = -\frac{1}{\sqrt{41}}$ and $\pi < x < \frac{3\pi}{2}$, compute the value of $45 \cdot \tan(2x)$.</p>	11.
<p>12. Consider the function $f(x) = 24x - 9x^2$ on the interval $[1, 2]$. Compute the absolute value of the difference between the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ on the given interval.</p>	12.
<p>13. Four students are to take a test consisting of 100 questions of equal difficulty. The probability is that the first student will answer 50 questions correctly, that the second student will answer 40 questions correctly, that the third student will answer 25 questions correctly, and the fourth student will answer 20 questions correctly. Find the integer p, such that $\frac{p}{100}$ is the probability that at least one of the four students will answer the first question correctly.</p>	13.
<p>14. A sequence is defined recursively by $a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{if } a_n \text{ is an even number} \\ 3 \cdot a_n & \text{if } a_n \text{ is an odd number} \end{cases}$ and $a_1 = 15$. Compute a_{2011}.</p>	14.
<p>15. Find the area of the triangle in quadrant I bounded by the coordinate axes and the line tangent to the curve whose equation is $y = \frac{10}{x}$ at $x = 2011$.</p>	15.

Nassau County Interscholastic Mathematics League

M

Mathletics

TEAM #

Mathematics Tournament 2011

Calculators may be used on this part.
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One (1) point for each correct answer.

Name _____ **School** _____ **Score** _____

Time Limit: 30 minutes
Answer Column

<p>1. A bathroom venting fan should be powerful enough to remove the entire volume of air in the room 8 times per hour. The unit of measure of the power of a fan is cfm (cubic feet per minute). To the nearest integer, how many cfm would be required for a fan in a bathroom whose dimensions were 5 feet by 8 feet by 8 feet?</p>	1.
<p>2. The solution to the equation below can be written as p/q where p and q are relatively prime integers. (That means they have no common factors other than 1). Find $p + q$.</p> $4^x + 2^{2x} + 2^{2x+1} = \frac{16}{64^{4x}}$	2.
<p>3. A running track has a long straight path followed by a semicircular arc that brings the runner back to his starting point. If the total length of the track is 2011 feet, compute the number of feet in the straight part of the track to the nearest integer.</p>	3.
<p>4. At the Nassau Math Tournament last year, the average score of the top 10 teams was 117.3 and the average score of the top 6 teams was 125.5. What was the average score of the four teams that placed in seventh, eighth, ninth, and tenth positions?</p>	4.
<p>5. David's Donuts held a design your own donut contest. Each participant could start with either a vanilla or chocolate donut and then add any one of three fillings, or none at all. They then had to frost the donut with any one of 12 frostings as well as to cover the frosting with one of two types of glaze, or not use glaze at all. Compute the number of different types of donuts that could be designed.</p>	5.

Time Limit: 30 minutes

Answer Column

6. Given $\triangle ABC$ with $BC = 10$, $AC = 6$, and D is on \overline{AC} such that $CD : DA = 1 : 2$. Median \overline{AE} intersects \overline{BD} at F . The ratio $DF : FB$ can be expressed in lowest terms as $a:b$. Compute $10a + b$.	6.
7. Point P is inside square $ABCD$ such that $AP = 4$, $BP = 10$, and $m\angle BAP = 45^\circ$. Determine, to the nearest integer, the area of the square.	7.
8. How many ordered pairs of positive integers satisfy $ x + 3y = 150$?	8.
9. The base of a square pyramid has an area of 100 and a volume of 400. The four faces are isosceles triangles. Compute the total surface area of the pyramid.	9.
10. Compute the distance between the lines $20x + 21y = 100$ and $20x + 21y = 941$.	10.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2011

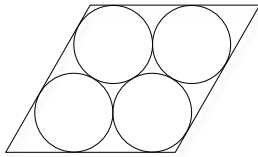
HAND IN ONLY ONE ANSWER SHEET PER TEAM
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 Three (3) points per correct answer.

Team Copy **School** _____ **Score** _____

Time Limit: 60 minutes

Answer Column

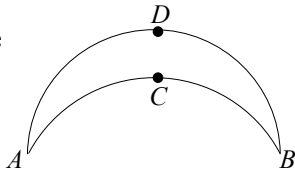
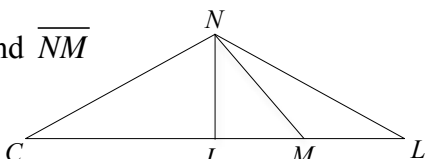
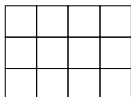
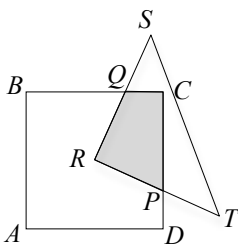
<p>1. Complete the number puzzle below and then calculate the value of the following: (sum of across numbers) – 5(sum of down numbers).</p> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="width: 45%;"> <p>Across</p> <p>1. A multiple of 101.</p> <p>5. A perfect cube with a digit that appears twice.</p> <p>6. A multiple of 83.</p> </div> <div style="width: 45%;"> <p>Down</p> <p>1. A palindromic perfect square.</p> <p>2. A Fibonacci number.</p> <p>3. A perfect square Fibonacci number.</p> <p>4. A palindromic perfect cube.</p> </div> </div> <div style="margin-top: 10px; text-align: center;"> <table border="1" style="border-collapse: collapse; font-size: 12px;"> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">3</td><td style="padding: 2px 5px;">4</td></tr> <tr><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> <tr><td style="padding: 2px 5px;">6</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> </table> </div>	1	2	3	4	5				6				1.
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<p>5. The four congruent circles in the diagram are tangent to each other and tangent to the sides of the rhombus. If the area of each circle is π sq units, and the number of square units in the rhombus can be represented by the irreducible expression $\frac{a + b\sqrt{3}}{c}$, calculate the sum $a + b + c$.</p>	5.												
<p>6. Five points have coordinates A(0, 0), B(10,0), C(20, –10), D(30, 10), and E(30, 20). These and only these points can be used to form exactly one pair of congruent triangles. Compute the area of one of those triangles.</p>	6.												



Turn Over

Time Limit: 60 minutes

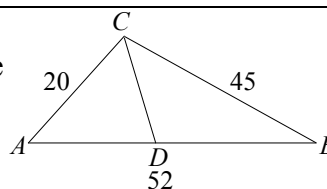
Answer Column

<p>7. There are four solutions to the system of equations $5x^2 + 7y^2 = 2453$ and $9x^2 - 3y^2 = 2169$. Given that x and y are both positive, compute $x + y$.</p>	7.
<p>8. Three bags of marbles are filled with different color marbles. Bag two has 3 times as many marbles as bag one and bag three has twice as many marbles as bag two. Half the number of marbles in bag one, two thirds of the number in bag two, and three fourths of the number in bag three are red marbles. What percent of all the marbles is red?</p>	8.
<p>9. The diagram at the right consists of two arcs and their respective midpoints. $m\widehat{ACB} = 90^\circ$ and $m\widehat{ADB} = 180^\circ$. Given that the radius of the circle whose arc is \widehat{ADB} is $34 + 17\sqrt{2}$, compute the distance from C to D.</p>	 <p>9.</p>
<p>10. Compute the sum of all the integers greater than 1 whose only prime factors are 3 or 7 and which do not have a perfect cube factor.</p>	10.
<p>11. The numbers 1, 2, 3, 4, 5, and 6 can be arranged to form 720 six-digit numbers. Compute the <u>reciprocal</u> of the probability that a number chosen from these 720 numbers has no two adjacent digits whose sum is a multiple of 2 or 3. For example do not count the number 346215 since $4 + 6 = 10$, a multiple of 2.</p>	11.
<p>12. In isosceles $\triangle NCL$ with $NC = NL$, \overline{NI} bisects \overline{CL} and \overline{NM} bisects \overline{IL}. Furthermore $NI = ML$. Compute $m\angle CNM - m\angle LNM$ in degrees.</p>	 <p>12.</p>
<p>13. The diagram at the right is comprised of 12 little squares. How many rectangles, including squares are in the diagram?</p>	 <p>13.</p>
<p>14. Square $ABCD$ has side length 28 and right isosceles $\triangle SRT$ has $RS = RT = 32$. Point R is at the center of the square and the triangle intersects the sides of the square at P and Q. If $QC = 8$, compute the area of the overlapped region (shaded in the diagram).</p>	 <p>14.</p>

Time Limit: 60 minutes

Answer Column

<p>15. A set of four dice is used to play a game. Each player chooses one die and rolls it. The highest number wins. The dice have the following sets of numbers on their faces: $A = \{6,6,2,2,2,2\}$, $B = \{5,5,5,1,1,1\}$, $C = \{4,4,4,4,0,0\}$, and $D = \{3,3,3,p,q,r\}$. The probability that A beats B = $2/3$ and the probability that B beats C = $2/3$. Determine integer values for p, q, and r such that the probability that C beats D = the probability that D beats A = $2/3$. Give as your answer the product pqr.</p>	15.
<p>16. A pizza number is the maximum number of pieces of pizza that you get when you cut a pizza with a single straight-line cut. Let $P(n)$ be the pizza number for n straight cuts. So $P(1) = 2$, $P(2) = 4$, and $P(3) = 7$. Find $P(10)$.</p>	16.
<p>17. The complex number $z = a + bi$. If the magnitude of $z = z = \sqrt{a^2 + b^2} = 13$ and $z + 3 + 3i = 17$, compute the value of $z + 16 + 16i$.</p>	17.
<p>18. Given $\triangle ACB$ with $AC = 20$, $CB = 45$, $AB = 52$ and \overline{CD} is the bisector of $\angle ACB$. If the perimeter of $\triangle BCD$ is 99, compute the perimeter of $\triangle ADC$.</p>	18.
<p>19. Determine the length of the graph of $3y + 4x - 4 + x - x + x - 3 + x - 3 = 0$.</p>	19.
<p>20. Given the system of equations below, compute $10x_1 + 15x_2 + 21x_3 + 28x_4 + 36x_5$.</p> $x_1 + 3x_2 + 6x_3 + 10x_4 + 15x_5 = 246$ $3x_1 + 6x_2 + 10x_3 + 15x_4 + 21x_5 = 160$ $6x_1 + 10x_2 + 15x_3 + 21x_4 + 28x_5 = 185$	20.



Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2010

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM

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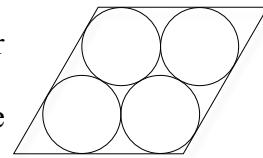
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Individual Copy - Do not hand this in.

Time Limit: 60 minutes

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<p>6. Five points have coordinates A(0, 0), B(10,0), C(20, -10), D(30, 10), and E(30, 20). These and only these points can be used to form exactly one pair of congruent triangles. Compute the area of one of those triangles.</p>	6.												

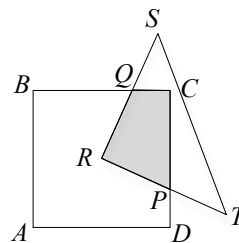
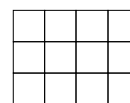
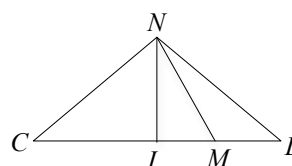
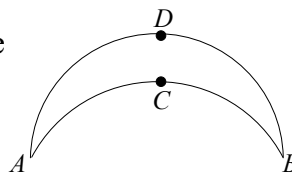


Turn Over

Time Limit: 60 minutes

Answer Column

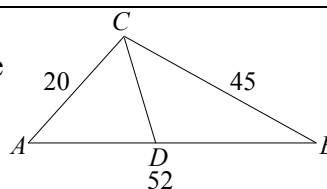
7. There are four solutions to the system of equations $5x^2 + 7y^2 = 2453$ and $9x^2 - 3y^2 = 2169$. Given that x and y are both positive, compute $x + y$.	7.
8. Three bags of marbles are filled with different color marbles. Bag two has 3 times as many marbles as bag one and bag three has twice as many marbles as bag two. Half the number of marbles in bag one, two thirds of the number in bag two, and three fourths of the number in bag three are red marbles. What percent of all the marbles is red?	8.
9. The diagram at the right consists of two arcs and their respective midpoints. $m\widehat{ACB} = 90^\circ$ and $m\widehat{ADB} = 180^\circ$. Given that the radius of the circle whose arc is \widehat{ADB} is $34 + 17\sqrt{2}$, compute the distance from C to D .	9.
10. Compute the sum of all the integers greater than 1 whose only prime factors are 3 or 7 and which do not have a perfect cube factor.	10.
11. The numbers 1, 2, 3, 4, 5, and 6 can be arranged to form 720 six-digit numbers. Compute the <u>reciprocal</u> of the probability that a number chosen from these 720 numbers has no two adjacent digits whose sum is a multiple of 2 or 3. For example do not count the number 346215 since $4 + 6 = 10$, a multiple of 2.	11.
12. In isosceles $\triangle NCL$ with $NC = NL$, \overline{NI} bisects \overline{CL} and \overline{NM} bisects \overline{IL} . Furthermore $\triangle NIM$ is isosceles. Compute $m\angle CNM - m\angle LNM$ in degrees.	12.
13. The diagram at the right is comprised of 12 little squares. How many rectangles, including squares are in the diagram?	13.
14. Square $ABCD$ has side length 28 and right isosceles $\triangle SRT$ has $RS = RT = 32$. Point R is at the center of the square and the triangle intersects the sides of the square at P and Q . If $QC = 8$, compute the area of the overlapped region (shaded in the diagram).	14.



Time Limit: 60 minutes

Answer Column

<p>15. A set of four dice is used to play a game. Each player chooses one die and rolls it. The highest number wins. The dice have the following sets of numbers on their faces: $A = \{6,6,2,2,2,2\}$, $B = \{5,5,5,1,1,1\}$, $C = \{4,4,4,4,0,0\}$, and $D = \{3,3,3,p,q,r\}$. The probability that A beats B = $2/3$ and the probability that B beats C = $2/3$. Determine integer values for p, q, and r such that the probability that C beats D = the probability that D beats A = $2/3$. Give as your answer the product pqr.</p>	15.
<p>16. A pizza number is the maximum number of pieces of pizza that you get when you cut a pizza with a single straight-line cut. Let $P(n)$ be the pizza number for n straight cuts. So $P(1) = 2$, $P(2) = 4$, and $P(3) = 7$. Find $P(10)$.</p>	16.
<p>17. The complex number $z = a + bi$. If the magnitude of $z = z = \sqrt{a^2 + b^2} = 13$ and $z + 3 + 3i = 17$, compute the value of $z + 16 + 16i$.</p>	17.
<p>18. Given $\triangle ACB$ with $AC = 20$, $CB = 45$, $AB = 52$ and \overline{CD} is the bisector of $\angle ACB$. If the perimeter of $\triangle BCD$ is 99, compute the perimeter of $\triangle ADC$.</p>	18.
<p>19. Determine the length of the graph of $3y + 4x - 4 + x - x + x - 3 + x - 3 = 0$.</p>	19.
<p>20. Given the system of equations below, compute $10x_1 + 15x_2 + 21x_3 + 28x_4 + 36x_5$.</p> $x_1 + 3x_2 + 6x_3 + 10x_4 + 15x_5 = 246$ $3x_1 + 6x_2 + 10x_3 + 15x_4 + 21x_5 = 160$ $6x_1 + 10x_2 + 15x_3 + 21x_4 + 28x_5 = 185$	20.



Nassau County Interscholastic Mathematics League

Tie Breakers

Mathematics Tournament 2011

No calculators may be used on this part.
All answers will be integers from 0 to 999 inclusive.
One (1) point for correct answer.

Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

1.	1.
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Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

2.	2.
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Name _____ **School** _____ **Score** _____

Time Limit:

Answer Column

3.	3.
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