TWENTY SEVENTH ANNUAL NCIML

Nassau Mathematics Tournament At SUNY Old Westbury

FEBRUARY 5, 2010
SUNY OLD WESTBURY

NASSAU COUNTY
INTERSCHOLASTIC MATHEMATICS LEAGUE

NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE

President: ***Kevin Meyers*****, The Wheatley School**
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2nd VP: ***Art Kalish*****, Syosset H.S.**
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Leon LaSpina, Bethpage HS
Donna Martin, New Hyde Park HS
Tom Weiswange, Great Neck South HS
Linda Litvack, Great Neck North HS
Ronni David
Marilyn Wecksell, Manhasset HS
Christine Healy, Bethpage HS
David Sime, New Hyde Park HS
Dr. Jong Pil Lee, SUNY College at Old Westbury

Special Thanks to Dr. Calvin Butts, Dr. Jong Pil Lee, Mr. Michael Dolan, the business office personnel and the custodial staff of SUNY College at Old Westbury for their especially supportive services and assistance.

NASSAU MATHEMATICS TOURNAMENT 2009 FINAL STANDINGS

UPPER DIVISION		
	TEAM	SCORE
1	Great Neck South HS	140
2	Syosset HS	129
3	Bethpage HS	125
4	Chaminade HS	123
4	Great Neck South HS	117
6	Bellmore JFK HS	115
7	Jericho HS	114
7	Roslyn H.S.	114
9	Planview JFK HS	113
10	New Hyde Park HS	112
10	Oceanside HS	112
10	Hericks HS	112
10	Syosset HS	112

LOWER DIVISION		
	TEAM	SCORE
1	Great Neck South	133
2	Jericho	126
3	Roslyn	123
4	Great Neck South	120
4	Herricks	120
4	Wheatley	120
7	Bethpage	113
7	Great Neck North	113
9	Port Washington	102
10	Syosset	100

GRADE 09			
	STUDENT	SCHOOL	SCORE
1	Tiffany Herang	Great Neck South	21
2	Keaton Stubis	Great Neck South	20
3	Jan Gong	Garden City	18
4	Joe Na	Jericho	17/5
5	Hilary Mogul	Schreiber	17/4
6	Ryan Lee	Great Neck South	16/6
7	Daniel Wickes	Wheatley	16/6
8	Yifan Zhou	Jericho	16/4
8	Colin Deicke	Wheatley	16/4
10	Dan Alanko	Bethpage	16/3

GRADE 10			
	STUDENT	SCHOOL	SCORE
1	Brian Chen	Roslyn HS	20
2	Keaton Stubis	Great Neck South	19
3	Dan Alanko	Bethpage	18
4	Max Lin	Herricks HS	17
4	Colin Deicke	Wheatley	17
6	Brandon Shih	Great Neck South	16
6	Jon Goldman	Plainview JFK	16
6	Ryan Lee	Great Neck South	16
6	Ben Shin	Great Neck South	16
10	Darwin Shen	J.F.K. Bellmore	15
10	Tiffany Huang	Great Neck South	15

GRADE 11			
	STUDENT	SCHOOL	SCORE
1	Kenneth Cheng	Great Neck South	22
2	Jing Jian	Lynbrook	21
3	Joseph DiCapua	Chaminade	18
3	Michael Hodgson	New Hyde Park	18
5	Cheska Hong	Great Neck South	17
6	Kevin Chiu	Farmingdale	16
7	Daniel Suh	Great Neck South	15
7	Stephen Chow	Hericks HS	14
9	Aashiq Dheera	Great Neck South	14
10	Abhay Goyal	Hericks	14
10	David Melnick	Great Neck North	14
10	John Sullivan	Syosset	14
10	Lawrence Tsang	Great Neck South	14
10	Max Rubin	Roslyn	14
10	Nick Wu	Hericks	14

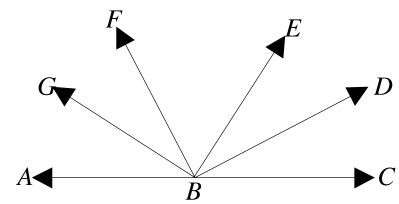
GRADE 12			
	STUDENT	SCHOOL	SCORE
1	Adam Gleitman	Oceanside	22
2	Kevin Sackel	Bethpage	21
3	Victor Chen	Great Neck South	20
4	Eric Cheong	Syosset	19
5	Katherine Zhu	Plainview	17
6	Sender Klein	Rambam	16
6	Stephanie Wang	Roslyn	16
6	Ben Aronow	Friends Academy	16
9	Alice Yau	Syosset	15
9	Devon Chu	Herricks	15
9	Marianne Thaila	New Hyde Park	15
9	Michael Strandberg	Chaminade	15

Grade 9

1. The positive difference between $3\sqrt{75}$ and $\sqrt{48}$, expressed in simplest form, can be written as $a\sqrt{b}$. Find the value of $a + b$.

2. Compute the number of centimeters in the height of a cylinder if the height and radius are equal and the total surface area of the cylinder is 1156π square centimeters.

3. If $a\Delta b = \dfrac{a^2 + 2b}{a + b^2}$, compute the value of $3\Delta(2\Delta 4)$.

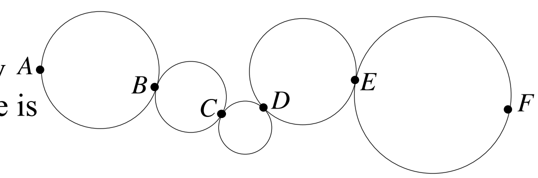


4. In the diagram, $\overleftrightarrow{ABC}$ with $\overrightarrow{BD} \perp \overrightarrow{BF}$, $\overrightarrow{BE} \perp \overrightarrow{BG}$ and $m\angle DBC = 20°$. If $\overrightarrow{BG}$ bisects $\angle FBA$, compute the number of degrees in $\angle DBE$.

5. If $(x + y)^2 = 53$ and $xy = 11$, compute the value of $x^2 + y^2$.

6. The longer dimension of a rectangle is 4 more than twice the shorter side. If the number of square units in the area is 56 more than 5 times the number of linear units in the perimeter, calculate the measure of the longer dimension.

7. As shown in the diagram each pair of circles are tangent at the labeled points *B*, *C*, *D*, and *E*. A path is traced from *A* to *F* entirely on the circumferences of the circles so that no portion of any circle is covered more than once. Compute the number of possible paths.



8. The coordinates of the vertices of ΔABC are *A*(1, 2), *B*(17, 24), and *C*(19, –17). Median $\overline{CD}$ is drawn to side $\overline{AB}$. Find the y-coordinate of the y-intercept of the line containing the median $\overline{CD}$.

9. Students from three different schools worked together on a summer project. Nine students from Archer School each worked for 5 days, seven students from Barker School each worked 4 days, and four students from Carver School each worked 8 days. The total amount paid for the students' work was $2010. If each student received the same daily salary, compute the total dollar amount earned by the students at Barker.

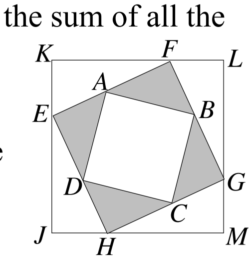
10. A 4-digit number is formed with the units digit odd, the tens digit even, the hundreds digit a multiple of three, and the thousands digit prime. Compute the number of possible numbers satisfying these conditions if repetition is allowed.

11. A lucky year is one in which at least one date has the following property: the product of the month times the day equals the last two digits of the year. For example 1956 was lucky since July 8 results in $7 \times 8 = 56$, the last two digits of the year. Determine the first year after 2010 that is not a lucky year and give the last two digits of the year.

12. Compute the 2010th digit found in the decimal expansion of 5 divided by 37.

13. In the polynomial expansion of $(x-1)(x-2)(x-3)\ldots(x-2009)(x-2010)$, compute the sum of all the coefficients including the constant term.

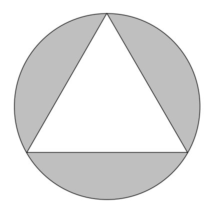
14. Square *ABCD* is inscribed in square *EFGH*, which is inscribed in square *JKLM*. If *JM* = 17, *JH* = 5, *CH* = 4, and *q* = the area of the shaded regions and *p* = the area of the unshaded regions compute *p* – *q*.



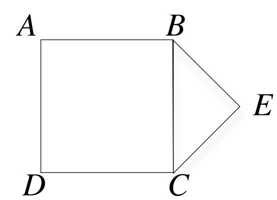
15. A boat travels at a speed of 18 mph in still water. It travels 35 miles upstream and then returns to its original position in a total of 4 hours. Compute the speed of the current in miles per hour.

Grade 10

1. A certain square has the same number of square units in its area as the number of linear units in its perimeter. Compute the number of units in the length of one side of the square.
2. Find the greatest possible integral value of *b* such that $x^2 + bx + 12 = 0$ will have imaginary roots.
3. Compute ${}_{2010}C_{99} - {}_{2010}C_{1911}$.
4. At a local office building, employee ID numbers are three digits long. The first digit must be odd, the next two digits can be any digit from 0 – 9, but none of the three digits may repeat. What is the total number of possible employee ID numbers?
5. A Geometry class of 30 students took a 5-question quiz, with each correct answer worth 2 points. Each answer was either right or wrong. The teacher noticed that two students received a grade of 2, four students a grade of 4, six students a grade of 6, eight students a grade of 8, and ten students a grade of 10. Compute the positive difference between the upper and lower quartiles.
6. The graph of a parabola has the equation $f(x) = x^2 - 5x + c$, and $f(c) = 32$. If $f(x) > 0$ for all values of *x*, compute the value of $f(-2)$.
7. Compute the value of *k* that would make the following system of equations have only one distinct solution: $y = 3x^2 - 4x + 10$ and $y = 2x + k$ (*k* is a constant).
8. A probability experiment involves rolling two dice at the same time. Each die contains six equally likely outcomes $\{1, 2, 3, 4, 5, 6\}$. The probability of rolling two different numbers is *n* times the probability of rolling two of the same numbers. Compute *n*.



9. The diagram shows an equilateral triangle inscribed in a circle of radius 8. The area of the shaded region is calculated and expressed in the form $a\pi - b\sqrt{c}$. If *a*, *b*, and *c* are all integers and *c* is prime, compute $a + b + c$.
10. How many distinct prime numbers (positive) are factors of 2010?
11. How many distinct non-prime positive integers are factors of 2010?
12. Compute the number of integer solutions for the inequality $|3x + 11| \le 17$.



13. In the diagram at the right, each side of square *ABCD* measures 12 cm. ΔBCE is an isosceles right triangle with base $\overline{BC}$. Compute the number of square centimeters in the area of pentagon *ABECD*.
14. Two opposite vertices of a rectangle have coordinates (–2, 0) and (8, 0). Another vertex of the rectangle lies on the *y*-axis at (0, *c*). If *c* is positive, compute *c*.
15. The first five terms of a sequence are $i\sqrt{2}, -2, -2i\sqrt{2}, 4, 4i\sqrt{2}$, where $i = \sqrt{-1}$, and each successive term is calculated by multiplying the previous term by $i\sqrt{2}$. Compute the positive difference between the sixth and twelfth terms.

Grade 11

1. Solve: $\log_9 x = \frac{3}{2}$.

2. An angle with a degree measure of 220° has a radian measure of $\frac{p}{q}\pi$ radians, where p and q are relatively prime (no prime factors common to p and q). Compute $p + q$.

3. Compute the number of degrees in the average of all the solutions to $\cos(2x) = \sin x$ when $0° \le x \le 360°$.

4. A rectangular table 2 feet by 4 feet is set into a corner of a rectangular room. A round table is then placed in the room so that it touches one corner of the rectangular table and also touches the two walls adjacent to the corner. Compute the number of feet in the radius of this table.

5. Compute the real value of x: $\log_5(x^2 - 64) - \log_5(x + 8) = \log_8 1$.

6. The probability that event A will occur at any time is $\frac{2}{5}$. If the probability that A occurs 2 out of 5 times can be represented by $\frac{m}{n}$ in lowest terms, compute $n - m$.

7. Two poles are 51 feet apart. The height of one pole is 28 feet and the height of the second pole is 40 feet. A wire is to be strung from the top of one pole to the ground and then up to the top of the second pole. What is the minimum number of feet required to accomplish this task?

8. The number $2\sqrt{2-\sqrt{3}}$ may be written in the form $\sqrt{a} - \sqrt{b}$. Compute $a - b$.

9. Solve for $2x$: $3^x + 3^{x-2} = 30\sqrt{3}$.

10. $\frac{3x+1}{x^2 - 3x - 28} = \frac{A}{x-7} + \frac{B}{x+4}$ is an identity (true for all defined values of x). Calculate the value of $A - B$.

11. In a circle, two chords each of length 24 inches, intersect perpendicularly to form segments of length 7 inches and 17 inches. Compute the number of inches in the radius of the circle.

12. Compute m when $\log_{64} c - \log_{32} c + \log_{16} c - \log_8 c + \log_4 c = \frac{m \log_2 c}{120}$.

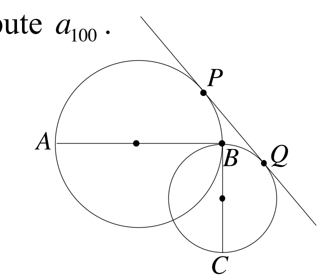
13. The sum of the infinite series $1 - \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} - \frac{1}{243} + \frac{1}{729} - \ldots$ in which each "+" term is followed by two "–" terms can be expressed as $\frac{p}{q}$ in lowest terms. Find $p + q$.

14. Find the number of degrees in the sum of all the solutions of $\sqrt{2}(\sin\theta - \cos\theta) = \sqrt{3}$, if $0° \le \theta \le 360°$.

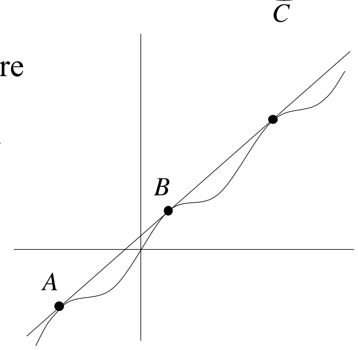
15. The diagonals of a quadrilateral are 12 and 16. One of the angles formed by the diagonals is 30°. Compute the area of the quadrilateral.

Grade 12

1. Let $f(x) = 421x^2 + Bx + C$, where B and C are constants. If $f(x) + f'(x) = 421x^2$ for all values of x, compute C.
2. Consider rectangle $ABCD$ with $AB = 22$ and $BC = 12$. Point E is located somewhere within rectangle $ABCD$. Find the sum of the areas of ΔABE and ΔDEC.
3. Compute $\lim_{x \to -30} \dfrac{x^2 + 97x + 2010}{x + 30}$.
4. Compute the sum of the solutions of $\left||x - 129| - 71\right| = 3$.
5. Let $f(x) = -\sin(2x)$. If $f^{(2010)}\left(\dfrac{\pi}{12}\right) = 128^k$, compute k. [Note: $f^{(n)}(x)$ denotes the n^{th} derivative of $f(x)$ with respect to x.]
6. Samantha is playing a game. She rolls a fair, 6-sided die and then flips a fair coin a number of times equal to the number that she rolled on the die. If the coin comes up heads every time she flips it, she wins the game. If the probability of her winning the game is $\dfrac{p}{q}$ in lowest terms, compute $p + q$.
7. Compute the number of terms in the simplest form of the expansion of the expression $(x-1)\cdot\left[(x+1)\cdot(x^2+1)\cdot(x^4+1)\ldots\left(x^{2^{2009}}+1\right)\cdot\left(x^{2^{2010}}+1\right)\right]$.
8. Let $f(x) = x^2e^x$. If the solutions of the equation $f'(x) = 899e^x$ are r and s, compute $|r - s|$.
9. Let S = the sum of 102 consecutive odd integers. Compute the largest positive integer which is a factor of *all possible values* of S.
10. Compute the smallest positive degree value of x that satisfies $(\sin x + \cos x)^2 = \dfrac{3}{2}$.
11. Compute the number of ordered triples (x, y, z) of positive integers that satisfy $x + y + 3z = 50$.
12. A sequence is defined by $a_1 = 1$, $a_4 = 10$, and $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 3$. Compute a_{100}.
13. Compute the value of x: $\log_7\left(x^{\log_7\left(x^{\log_7 x}\right)}\right) = 27$.
14. Two circles have diameters $\overline{AB}$ and $\overline{BC}$, as shown, and $\overline{PQ}$ is tangent to both circles. If $AB = 90$, $BC = 80$, and $\overline{AB} \perp \overline{BC}$, compute PQ.



15. The curve $y = 2.4x + \sin\left(\dfrac{7\pi}{11}x\right)$ is graphed as shown [the x- and y-axes are not drawn with the same scale]. A line is drawn tangent to the curve at an infinite number of points. Two successive points of tangency are labeled A and B. If $AB = \dfrac{p}{q}$ in lowest terms, compute $p + q$.



Mathletics

1. Given $8a+3b-45=7a+4b+55$, compute the value of $a-b$.

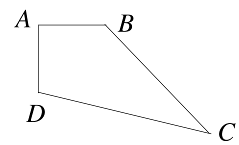
2. Find the area of the triangle formed by the lines $y=\frac{1}{3}x$, $y=3x$, and $x+y=60$.

3. Three cats are let loose in a mouse-infested high school: a Maine Coon, a Birman, and a Siberian. For every two mice the Birman catches, the Siberian catches three. The Maine Coon catches 50% more than the other two cats combined. If the three cats catch a total of 200 mice, how many more mice were caught by the Siberian than the Birman?

4. One root of the quadratic function $f(x)=3x^2-bx+c$, where b and c are integers, is $5+2i$. Compute $b+c$.

5. If $x^2+y^2-z^2+2xy=2010$ and $x+y-z=67$, compute the average of x, y, and z.

6. The distance from SUNY Old Westbury to SUNY Stony Brook is 32 miles. Yvette bikes from Old Westbury to Stony Brook, averaging 12 mph. On the return trip, she starts off at a pace that is $33\frac{1}{3}\%$ slower than her initial speed for the first 16 miles, and then her speed is reduced by another 25% for the final 16 miles. How many **minutes** did it take for Yvette to complete her entire round trip? Round your answer to the nearest minute.



7. In the figure at the right, $\angle A$ is a right angle, $AB = 5$, $AD = 12$, $BC = 84$ and $CD = 85$. Compute the number of square units in the area of quadrilateral $ABCD$. [Figure not drawn to scale.]

8. The lengths of two sides of an obtuse triangle are 10 and 13. How many different integer values are possible for the third side?

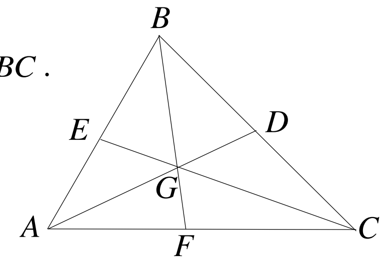
9. Three sets, A, B, and C, satisfy $A\cup B\cup C=\{1,2,3,4,5,6,7,8,9)$ and $A\cap B\cap C=\varnothing$. Let N be the number of ordered triples (A, B, C) of such sets that satisfy these conditions. Evaluate $\sqrt[3]{N}$ and, if necessary, round to the nearest integer.

10. Brian and Keaton both take a five-question quiz that they are completely unprepared for, so they guess randomly on all of the questions. Brian takes a True/False quiz while Keaton's quiz is multiple choice with four choices for each question. Let P be the probability, rounded to the nearest thousandth, that *at least* one of them scores above 60% on the quiz. Compute $100P$.

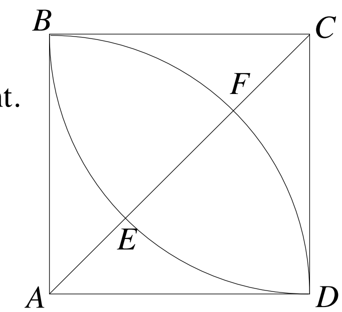
Team

1. Compute the area of the closed region bounded by $|x|+|y|=10$.
2. The 4-digit number $a47b$, where a and b are digits, is divisible by 6. Find the product of the values of a and b that create the largest such 4-digit number.
3. Find the sum of the roots of the equation $x^2-12=|x|$.
4. In triangle ΔABC with $m\angle B=90°$, both the altitude, $\overline{BD}$, and the median, $\overline{BF}$, are drawn. The bisector of $\angle DBF$ intersects the hypotenuse at point G. Compute the number of degrees in $\angle ABG$.
5. In the equation $x^2+kx+20=0$, the square of the difference of the two roots equals the product of the two roots. Calculate $|k|$.
6. Determine the integer between 801 and 899 that has an odd number of integral divisors. The number 1 and the number itself are considered to be integral divisors of the number.
7. An arithmetic sequence is a sequence in which there is a constant difference between successive terms as in 3, 5, 7 or 2, 12, 22. If the equations $ax + by = c$ and $dx + ey = f$ are formed by one continuous arithmetic sequence, $a, b, c, d, e,$ and f, with a solution (x, y), find $x + y$.
8. Solve for x: $\dfrac{(3x-2)^2-(2x-3)^2}{(x+1)^2}=0$.
9. Diameter $\overline{AB}$ of circle O is 40 inches long. Point C is a point on the circle such that $AC = 32$ inches. Tangent lines are drawn at points B and C and they intersect at F. Find the number of inches in the length of $\overline{OF}$.
10. In an isosceles triangle with a base of length 30 and legs of length 25, a point is chosen on the base. Perpendiculars from this point to the legs are drawn. Compute the sum of the lengths of these two line segments.
11. Find the sum of the ten least positive distinct integers S, such that 2^S-1 is a multiple of 7.
12. Let $100h+10t+u$ represent a three-digit number where h, t, and u are the digits. Rearrange the digits into all possible three-digit numbers, using each digit exactly once in each number. If the average of all such numbers is 333, compute the sum $h + t + u$.
13. Set $A = \{1, 2, 4\}$, set $B = \{1, 3, 9\}$, and set $C = \{1, 5\}$. If one number is selected from each set and the product of the three numbers is calculated, compute the sum of all possible products.
14. The sum of two positive fractions with denominators 5 and 9 is $\dfrac{154}{45}$. The product of their numerators is p. Calculate the maximum value of p.
15. Given two linear functions f and g such that for all x, $f(g(x))=g(f(x))=x$ and $f(0) = -2$ and $g(4) = 2$, compute the slope of the line defined by f.
16. The equation $x\sqrt{7}-2y\sqrt{7}-x+y=3\sqrt{7}+3$ has only one pair of integer solutions for x and y. Compute xy.

17. In the diagram at the right, $\overline{BF}$ bisects $\angle ABC$ and $\overline{AD}$ is a median in ΔABC. The lengths of sides $\overline{AB}$ and $\overline{BC}$ are 6 and 8 respectively. When fully reduced, the ratio $\frac{AE}{EB} = \frac{p}{q}$. Compute $p + q$. (Hint: Use mass point geometry.)



18. In square $ABCD$, $AB = 40$. Quarter circles are drawn with centers at A and C and radius 40. Diagonal $\overline{AC}$ is also drawn as seen in the diagram at the right. The length of $\overline{EF}$ can be represented by the simplified expression $a - b\sqrt{c}$. Calculate the sum $a + b + c$.



19. Four men can build a brick wall 30 feet long, 8 feet high, and 3 feet deep in 18 days. Working at the same rate, how long will it take twelve men to construct a wall 600 feet long, 10 feet high, and 4 feet thick?

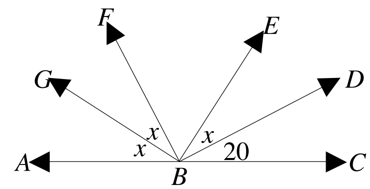
20. Every odd integer can be represented by the sum of two consecutive integers. For example 27 = 13 + 14. Many even integers may also be represented by the sum of three or more consecutive integers. For example 24 = 7 + 8 + 9. Find the largest three-digit integer that <u>cannot</u> be written as the sum of any number of consecutive integers.

Grade Level 9 - NMT 2010 Solutions

1. **14** $3\sqrt{75}-\sqrt{48}\Rightarrow 3\sqrt{25}\sqrt{3}-\sqrt{16}\sqrt{3}\Rightarrow 15\sqrt{3}-4\sqrt{3}=11\sqrt{3}$ and 11 + 3 = 14.

2. **17** The total surface area is the area found by adding the lateral area and the two base areas. Thus $SA=2\pi rh+2\pi r^2$. Since the height equals the radius the formula reduces to $SA=4\pi h^2$. Solve $1156\pi=4\pi h^2\Rightarrow h^2=289\Rightarrow h=17$.

3. **3** $2\Delta 4=\dfrac{4+8}{2+16}=\dfrac{12}{18}=\dfrac{2}{3}$ and $3\Delta\dfrac{2}{3}=\dfrac{9+4/3}{3+4/9}=\dfrac{81+12}{27+4}=\dfrac{93}{31}=3$.



4. **35** $\angle DBE\cong\angle FBG$ since both angles are complementary to $\angle FBE$. Since $\overrightarrow{BG}$ bisects $\angle FBA$, $\angle FBG\cong\angle GBA$. Solve $2x+90+20=180\Rightarrow x=35$.

5. **31** Multiply and substitute to solve: $(x+y)^2=x^2+2xy+y^2=x^2+2(11)+y^2=53\Rightarrow x^2+y^2=31$.

6. **36** Let x be the shorter side, then $2x+4$ is the longer side. Solve $x(2x+4)=5(6x+8)+56$. $2x^2+4x=30x+96\Rightarrow 2x^2-26x-96=0\Rightarrow(x-16)(x+3)=0$, so $x=16$ and $2x+4=36$.

7. **32** Since two paths lead out of each of the vertices A through E, $2^5=32$ paths are possible.

8. **40** The equation of the line containing points C an D can be found by calculating the coordinates of D and the slope of $\overleftrightarrow{CD}$ as follows: $D=\left(\dfrac{1+17}{2},\dfrac{2+24}{2}\right)=(9,13)$ and $m_{\overline{CD}}=\dfrac{13-(-17)}{9-19}=\dfrac{30}{-10}=-3$. Substitute into $y-y_1=m(x-x_1)$ to get $y-13=-3(x-9)$. Let $x=0$ to get the y intercept.

9. **536** Let w = daily wage per student, then $w(9\times5+7\times4+4\times8)=2010\Rightarrow w=\dfrac{2010}{105}$. Since the Barker students worked 28 days, multiply w by 28 to get 536.

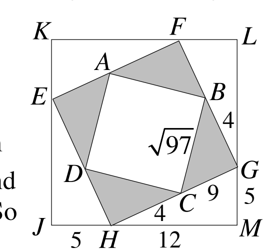
10. **300** Apply the Fundamental Counting Principle to get $4\times3\times5\times5=300$.

11. **37** From 2011 to 2031 are all lucky years determined by a day in January. The years 2032 to 2036 can be found on 2/16, 3/11, 2/17, 5/7, and 4/9. 2037 is not a lucky year.

12. **5** Do the division to get the repeating decimal $.135\overline{135}$. Since 2010 is a multiple of 3 the digit will be a 5 in the 2010th place.

13. **0** Notice that $(x-1)(x-2)=x^2-3x+2$ and $1-3+2=0$. $(x-1)(x-2)(x-3)=x^3-6x^2+11x-6$ and $1-6+11-6=0$. This pattern continues so the original sum is also 0. Proof requires math induction.

14. **145** <u>Method 1</u>: Given the figures are all squares, all the triangles that look congruent are congruent. Since JM = 17 and JH = 5, HM = 12. GM = 5 so ΔGMH is a 5-12-13 right triangle (Hypotenuse $\overline{HG}$ may also be found using the Pythagorean theorem). Since HC = 4 and HG = 13, CG = 9. Apply the Pythagorean theorem to ΔCGB to find $BC=\sqrt{97}$. Now the areas of all three squares may be found: $17^2=289$, $13^2=169$, $(\sqrt{97})^2=97$. Subtract the area of the middle square from the sum of the areas of the largest and smallest squares to get p, the area of the unshaded regions. To find q, the shaded area, subtract the area of the smallest square from the area of the middle square. So $p-q=217-72=145$.



<u>Method 2</u>: Find the area of the each shaded triangle (18) and each unshaded triangle (30). The area of the small square is 97. Then $p=4(30)+97=217$ and $q=4(18)=72$, so $p-q=217-72=145$.

15. 3 Let c = the speed of the current. Apply the formula $\text{time}=\dfrac{\text{distance}}{\text{rate}}$ to set up and solve the following:

$$\frac{35}{18-c}+\frac{35}{18+c}=4\Rightarrow 35(18+c)+35(18-c)=4(18-c)(18+c)\Rightarrow c^2=9.$$

Grade Level 10 - NMT 2010 Solutions

1. **4** Solve the equation $x^2=4x$ for the positive value of x.

2. **6** Set the discriminant, $b^2-4ac<0$, to get imaginary roots. $b^2-4(1)(12)<0 \Rightarrow b^2<48$. The greatest integer that satisfies this inequality is 6.

3. **0** Since 99 + 1911 = 2010, the two combinatorial numbers are equal.

4. **360** Apply the Fundamental Counting Principle to get $5 \cdot 9 \cdot 8 = 360$.

5. **4** In a data set that contains 30 items ranging from lowest to highest, the lower quartile is the 8th number and the upper quartile is the 22nd number (8th from the highest). Since the 8th lowest score is a 6 and the 22nd is a 10, the positive difference is 4.

6. **22** Since $f(c)=32, c^2-5c+c=32 \Rightarrow c^2-4c-32=0 \Rightarrow (c-8)(c+4)=0$, so $c = -4$ or 8. If $c = -4$, the y-intercept would be negative violating $f(x)>0$ for all x.

7. **7** Substitute and set equal to 0 to get $3x^2-4x+10=2x+k \Rightarrow 3x^2-6x+10-k=0$. A quadratic equation has only one distinct root when the discriminant is zero. So, $(-6)^2-4(3)(10-k)=0$. Solve for k: $36-120+12k=0 \Rightarrow k=7$.

8. **5** $P(\text{different})=\frac{6}{6}\cdot\frac{5}{6}=\frac{30}{36}$ and $P(\text{same})=\frac{6}{6}\cdot\frac{1}{6}=\frac{6}{36}$. Therefore the probability that they are different is 5 times greater than the probability that they are the same.

9. **115** The area of an equilateral triangle can be found using $A_{eq\Delta}=\frac{s^2\sqrt{3}}{4}$, where s is a side of the triangle. Using the relationships of the sides in a 30°-60°-90° triangle we can calculate that a side of the triangle is $8\sqrt{3}$. Therefore the area of the shaded region can be found as follows: $A_{shaded\ region}=A_{circle}-A_{\Delta}=\pi(8)^2-\frac{(8\sqrt{3})^2\sqrt{3}}{4}=64\pi-48\sqrt{3}$ and 64 + 48 + 3 = 115.

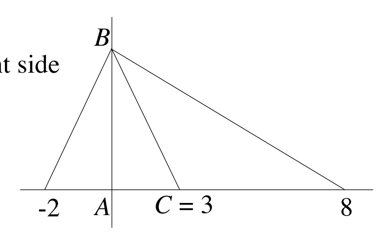
10. **4** The prime factorization of 2010 is $2 \cdot 3 \cdot 5 \cdot 67$.

11. **12** Each prime factor can appear or not appear in each factor. Therefore there are $2^4=16$ possible factors from which we must subtract the four prime factors to get the answer 12.

12. **12** $|3x+11| \le 17 \Rightarrow -17 \le 3x+11 \le 17 \Rightarrow -28 \le 3x \le 6 \Rightarrow -\frac{28}{3} \le x \le 2$. The only integers satisfying these conditions are $-9, -8, -7, \ldots, 0, 1,$ and 2.

13. **180** $A=A_{square}+A_{\Delta}=12^2+\frac{1}{2}(6\sqrt{2})^2=144+36=180$

14. **4** Method 1: The slope of one side of the rectangle is $\frac{c}{2}$ and the slope of the adjacent side is $\frac{c}{-8}$. Since these sides are perpendicular their slopes are negative reciprocals so $\frac{c}{2}\cdot\frac{c}{-8}=-1 \Rightarrow c^2=16 \Rightarrow c=\pm 4$.



Method 2: Draw a diagram and note that ΔABC is a 3-4-5 right triangle, where C is the midpoint of the diagonal.

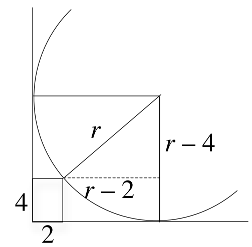
15. **72** Every even position is a power of –2. Term 6 is –8 and term 12 is 64. The positive difference is 72.

Grade Level 11 - NMT 2010 Solutions

1. **27** $\log_9 x=\frac{3}{2}\Rightarrow x=9^{3/2}=\left(\sqrt{9}\right)^3=27$

2. **20** To convert degrees to radians multiply by $\frac{\pi}{180}$. So $220°=220\cdot\frac{\pi}{180}=\frac{11}{9}\pi$ radians. 11 + 9 = 20.

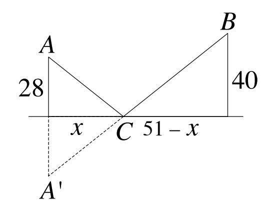
3. **150** Replace $\cos(2x)$ with $1-2\sin^2 x$ and solve the resulting quadratic equation, $2\sin^2 x+\sin x-1=0$. $(2\sin x-1)(\sin x+1)=0\Rightarrow\sin x=\frac{1}{2}$ or -1. So x = 30°, 150°, and 270°. $\frac{30+150+270}{3}=150$.



4. **10** Examine the diagram at the right. Apply the Pythagorean theorem to calculate r. $(r-2)^2+(r-4)^2=r^2\Rightarrow r^2-12r+20=0$. Factor and solve: $(r-10)(r-2)=0\Rightarrow r=10$ or $r=2$. Reject r = 2.

5. **4** $\log_5\left(x^2-64\right)-\log_5(x+8)=\log_8 1\Rightarrow\log_5\frac{x^2-64}{x+8}=0\Rightarrow\log_5(x-8)=0\Rightarrow x-8=5^0=1$. Therefore x = 9.

6. **409** $P(A\text{ 2 out of 5 times})={}_5C_2\cdot(P(A))^2\cdot(P(\sim A))^3=10\cdot\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^3=\frac{216}{625}$ and 625 – 216 = 409.

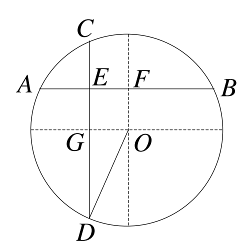


7. **85** Refer to the diagram at the right. Reflect point A over the ground line to find A'. The shortest distance from B to the ground to A is the length BA'. To find the location of point C, note that the triangles are all similar and solve the proportion $\frac{28}{x}=\frac{40}{51-x}$. $2040-40x=28x\Rightarrow x=30$. Now apply the Pythagorean theorem or recognize that the two triangles are multiples of 3-4-5 producing 30-40-50 and 21-28-35. Finally A'C + CB = 35 + 50 = 85.

8. **9** Set the two expressions equal to each other and then square both sides. $2\sqrt{2-\sqrt{3}}=\sqrt{a}-\sqrt{b}\Rightarrow4\left(2-\sqrt{3}\right)=a-2\sqrt{ab}+b\Rightarrow8-4\sqrt{3}=a+b-2\sqrt{ab}$. Set a + b = 8 and $4\sqrt{3}=2\sqrt{ab}\Rightarrow ab=12$. Solve by inspection to get a = 6 and b = 2. So a – b = 4. Note: This question is a direct result from solving for sin(15°) using both the half angle formula and the formula for sin(45° – 30°).

9. **7** $3^x+3^{x-2}=30\sqrt{3}\Rightarrow3^x+\frac{3^x}{3^2}=10\cdot3\cdot3^{1/2}\Rightarrow3^x\left(\frac{10}{9}\right)=10\cdot3^{3/2}\Rightarrow3^x=9\cdot3^{3/2}=3^{7/2}$. $x=\frac{7}{2}\Rightarrow2x=7$.

10. **1** Multiply both sides of the equation by the LCD to get $A(x+4)+B(x-7)=3x+1$. Since this is an identity it is true for all values of x. Let x = –4 to get B = 1 and let x = 7 to get A = 2. 2 – 1 = 1.



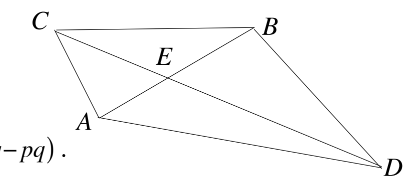
11. **13** Draw 2 diameters (dotted lines) each parallel to one of the chords as seen in the diagram. Since a diameter drawn perpendicular to a chord bisects the chord AF = FB = 12 and CG = GD = 12. We also know that AE = 7, therefore EF = 5. In ΔOGD, OG = 5, GD = 12, so the radius OD = 13.

12. **46** Use the base change formula to convert all of the logs to base 2. This will result in the following: $\frac{\log_2 c}{\log_2 64}-\frac{\log_2 c}{\log_2 32}+\frac{\log_2 c}{\log_2 16}-\frac{\log_2 c}{\log_2 8}+\frac{\log_2 c}{\log_2 4}=\frac{m}{120}\cdot\frac{\log_2 c}{\log_2 2}$. Now divide by $\log_2 c$ and convert all the denominators to integers. $\frac{1}{6}-\frac{1}{5}+\frac{1}{4}-\frac{1}{3}+\frac{1}{2}=\frac{m}{120}\Rightarrow20-24+30-40+60=m\Rightarrow m=46$.

13. **41** Create three infinite geometric series and apply the formula $S_{\infty}=\frac{a}{1-r}$, where a is the first term and r is the constant ratio. $1+\frac{1}{27}+\frac{1}{729}+\ldots=\frac{1}{1-\frac{1}{27}}=\frac{27}{26}$, $\frac{1}{3}+\frac{1}{81}+\frac{1}{27\cdot 81}+\ldots=\frac{\frac{1}{3}}{1-\frac{1}{27}}=\frac{9}{26}$, and $\frac{1}{9}+\frac{1}{243}+\frac{1}{27\cdot 243}+\ldots=\frac{\frac{1}{9}}{1-\frac{1}{27}}=\frac{3}{26}$. Now $\frac{27-9-326}{26}=\frac{15}{26}$ and 15 + 26 = 41.

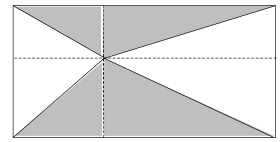
14. **270** Square both sides of the equation to solve for θ. $2\left(\sin^2 -2\sin\theta\cos\theta+\cos^2\theta\right)=3$. Since $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin(2\theta)$, substitute to get $\sin(2\theta)=-\frac{1}{2}$. 2θ = 210°, 330°, 570°, or 690° so θ = 105°, 165°, 285°, or 345°. However, when θ = 285° or 345° the $\sin\theta$ is a negative number and the $\cos\theta$ is positive so $\sin\theta-\cos\theta$ is negative which cannot be. Thus the only values for θ are 105° and 165° and 105 + 165 = 270.

15. **48** Let $AE = p$ and $CE = q$, then $EB = 12 - p$ and $ED = 16 - q$. Use the formula $A_{triangle}=\frac{1}{2}ab\sin C$ to find the area of each of the four triangles in terms of p and q. $A_{\Delta AEC}=\frac{1}{2}pq\sin 30°=\frac{1}{4}pq$. $A_{\Delta BEC}=\frac{1}{2}(12-p)q\sin 150°=\frac{1}{4}(12q-pq)$. $A_{\Delta EBD}=\frac{1}{2}(12-p)(16-q)\sin 30°=\frac{1}{4}(192-12q-16p+pq)$. $A_{\Delta AED}=\frac{1}{2}p(16-q)\sin 150°=\frac{1}{4}(16p-pq)$. The sum of all four areas is the area of the quadrilateral so $A_{quad}=\frac{1}{4}\left(pq+(12q-pq)+(192-12q-16p+pq)+(16p-pq)\right)=48$. It is interesting to note that as long as the angle between the diagonals is 30°, the area of the quadrilateral will always equal one-fourth the product of the diagonals. The proof is left up to the reader.



1. **842** $f'(x)=842x+B$, so $f(x)+f'(x)=421x^2+Bx+C+842x+B=421x^2$. Since this is an identity, $Bx+C+842x+B=0$ which means $B = -842$ and $C = 842$.

2. **132** Method 1: Since E can be any point in the rectangle, let it be the intersection of the diagonals. In that case it is easy to see that the area in question is half the area of the rectangle.



Method 2: In general, draw lines parallel to the sides of the rectangle through point E. The four resulting rectangles are each bisected by the lines joining E to the vertices of the given rectangle.

3. **37** Factor and cancel. $\lim_{x\to-30} \dfrac{x^2+97x+2010}{x+30} = \lim_{x\to-30} \dfrac{(x+30)(x+67)}{x+30} = \lim_{x\to-30} (x+67) = 37$.

4. **516** Recall $|w|=3$ means that $w = \pm3$. Therefore $\big||x-129|-71\big|=3 \Rightarrow |x-129|-71=\pm3 \Rightarrow |x-129|=74 \vee |x-129|=68$. Solving these equations we get x = 55, 203, 61, and 197. The sum of the roots is 516. [Note: a generalization of this problem, i.e $\big||x-a|-b\big|=c$, results in the solution $x = 4a$ for all values of a, b, and c.]

5. **287** Examine the first few derivatives to find the following cyclic pattern:

z	Ordered triples	Count
16	(1, 1, 16)	1
15	(1, 4, 15), (2, 3, 15), (3, 2, 15), (4, 1, 15)	4
14	(1, 7, 14), (2, 6, 14), …, (7, 1, 14)	7

Each successive row adds 3 more triples so when $z = 1$, there are 46 terms. The sum of the arithmetic sequence $1+4+7+\ldots+46=\dfrac{16}{2}(1+46)=8\cdot47=376$.

6. **149** The probability of throwing n heads is $\left(\dfrac{1}{2}\right)^n$. The probability for each number on a die is $\dfrac{1}{6}$. So $P(\text{win})=\dfrac{1}{6}\left(\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{32}+\dfrac{1}{64}\right)=\dfrac{1}{6}\left(\dfrac{63}{64}\right)=\dfrac{21}{128}$. Furthermore 21 + 128 = 149.

7. **2** The product $(x-1)(x+1)=x^2-1$, $\left(x^2-1\right)\left(x^2+1\right)=x^4-1$ and $\left(x^4-1\right)\left(x^4+1\right)=x^8-1$, etc. so the final product will only consist of 2 terms.

8. **60** $f'(x)=x^2e^2+2xe^x=\left(x^2+2x\right)e^x=899e^x$. Since $e^x\neq0$, $x^2+2x-899=0$. If you recognize that $899=30^2-1^2=(30-1)(30+1)$, then $x^2+2x-899=(x+31)(x-29)$ and the two roots are 29 and –31. Thus $|29-(-31)|=60$.

9. **204** Method 1: Summing the numbers –101 – 99 – … + 99 + 101 doesn't help in the calculation, but summing up – 99 – 97 – … + 97 + 99 + 101 + 103 = 101 + 103 = 204 yields the least positive sum for S. Adding $2k$ to each of the terms will not change the fact that 204 will be a factor of S since there are 102 terms and the sum will be $204 + 204k = 204(k + 1)$.

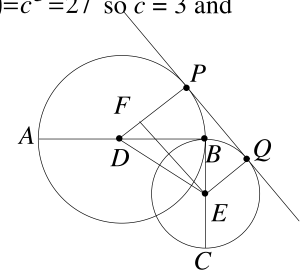
Method 2: The sum of an arithmetic sequence can be found using $S = \dfrac{n}{2}\left(a_1+a_n\right)$, where a_1 is the first term and $a_n=a_1+(n-1)d$ is the n^{th} term. Let $a_1=2k+1$, $a_{102}=(2k+1)+101\cdot2=2k+203$, so $S = \left((2k+1)+(2k+203)\right)\dfrac{102}{2} = 51(4k+204) = 204(k+51)$. Hence 204 is the LCD of all such terms.

10. **15** Expand and simplify $(\sin x+\cos x)^2=\sin^2 x+\cos^2 x+2\sin x\cos x=1+\sin(2x)=\frac{3}{2}$. Therefore $\sin(2x)=\frac{1}{2}2x=30°\Rightarrow x=15°$.

11. **376** Given $z = 16$, there is only one pair of integers for x and y (1, 1, 16). When $z = 15$ there are four pairs of numbers for x and y (1, 4, 15), (2, 3, 15), (3, 2, 15), and (4, 1, 15). For each increase in z there are three additional pairs of integers for x and y. Therefore the total number of triplets can be found by summing $1 + 4 + 7 + \ldots 46 = \frac{16}{2}(1+46)=376$.

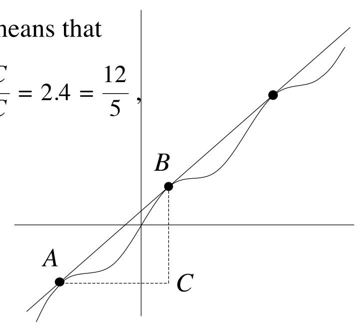
12. **298** Let the first four terms be 1, x, y, 10. By the given recurrence relation $y = 2x - 1$, and $10 = 2y - x$. Solving this system of equations yields $x = 4$ and $y = 7$. The sequence is therefore arithmetic and $a_{100}=1+(100-1)3=298$.

13. **343** Let $c=\log_7 x$. Then $\log_7\left(x^{\log_7\left(x^c\right)}\right)=\log_7\left(x^{c\cdot\log_7(x)}\right)=\log_7\left(x^{c\cdot c}\right)=c^2\log_7(x)=c^3=27$ so $c = 3$ and $x=7^3=343$.



14. **60** Call the centers of the circles D and E. Draw radii $\overline{DP}$ and $\overline{EQ}$ and draw $\overline{ED}$ and $\overline{EF}\perp\overline{DP}$ as seen in the diagram. Since $PFEQ$ is a rectangle, $PF = EQ = 40$ and $DF = 45 - 40 = 5$. Since ΔBDE is a right triangle we can find DE by applying the Pythagorean theorem, $DE^2=DB^2+EB^2=45^2+40^2$. In right triangle DEF we can find FE using $FE^2=DE^2-DF^2=45^2+40^2-5^2=2025+1600-25=3600$.
So $FE = PQ = 60$.

15. **321** The line is tangent to the curve at the same place on each "bump" of the curve and occurs periodically based upon the period of the function. The period is $2\pi\div\frac{7\pi}{11}=\frac{22}{7}$ based upon the x-axis. This means that $AC=\frac{22}{7}$ for two consecutive tangent points. Since the slope of the line is 2.4, $\frac{BC}{AC}=2.4=\frac{12}{5}$, which means that ΔABC is a 5-12-13 triangle, and so $AB=\frac{13}{5}AC=\frac{13}{5}\cdot\frac{22}{7}=\frac{286}{35}$.
Finally, $286 + 35 = 321$.



1. **100** Add $-7a-4b+45$ to both sides of the equation to result in $a - b = 100$.

2. **900** Find the intersection points of each pair of lines. They are (15, 45), (45, 15), and (0, 0).
Method 1: The resulting triangle is isosceles so the median to the base is also the altitude. Since the midpoint of the base is (30, 30), the altitude is $\sqrt{900+900}=30\sqrt{2}$. The length of the base is $\sqrt{(15-45)^2+(45-15)^2} = \sqrt{900+900} = 30\sqrt{2}$.
Then $A_\Delta=\frac{1}{2}\left(30\sqrt{2}\right)\left(30\sqrt{2}\right)=900$.

Method 2: Subtract the areas of the two smaller triangles from the area of the triangle determined by $x+y=60$ and the two coordinate axes. Each of the smaller triangles has an area of 450 while the area of the largest triangle is $A=\frac{1}{2}\cdot 60\cdot 60=1800$. The final area is $1800 - 2(450) = 900$.

3. **16** Let the number of mice consumed by the Birman and Siberian and Maine Coon be $2x$, $3x$, and $7.5x$. Solve the equation $2x+3x+7.5x=200\Rightarrow x=16$.

4. **117** The second root must be $5 - 2i$ therefore the sum of the roots is 10 and the product is 29. Apply the formulas $S=\frac{-b}{a}=\frac{b}{3}=10\Rightarrow b=30$ and $P=\frac{c}{a}=\frac{c}{3}=29\Rightarrow c=87$. Hence $b + c = 117$.

5. **10** Rewrite the first equation as $(x+y)^2-z^2=2010$. Factor the difference of two squares into $\left((x+y)+z\right)\left((x+y)-z\right)=2010\Rightarrow 67(x+y+z)=2010\Rightarrow x+y+z=30$, so the average of x, y, and z is $\frac{30}{3}=10$.

6. **440** She travels 12 mph for 32 miles, then $\frac{2}{3}(12)=8$ mph for 16 miles and finally 6 mph for the last 16 miles. Her total time is $\frac{32}{12}+\frac{16}{8}+\frac{16}{6}=7\frac{1}{3}$ hours = 440 minutes.

7. **576** Draw $\overline{BD}$. Since ΔABD is a right triangle, we find $BD = 13$. Notice that 13-84-85 satisfies the Pythagorean theorem so ΔDBC is also a right triangle. The area of each triangle can be found as follows: $A_\Delta=\frac{1}{2}(5)(12)=30$ and $A_\Delta=\frac{1}{2}(84)(13)=546$. Finally, $30 + 546 = 576$.

8. **11** Call the missing side c. Since the triangle is obtuse, $c^2>10^2+13^2\Rightarrow c>\sqrt{269}\approx 16.4$. By the Triangle Inequality Theorem, $c < 10 + 13$. So $16.4 < c < 23$. Six numbers satisfy this condition. A second possibility is that $c^2+10^2<13^2\Rightarrow c<\sqrt{69}\approx 8.3$. But we also know that $c + 10 > 13$. Thus $3 < c < 8.3$ and five numbers satisfy this condition. Together there are $6 + 5 = 11$ solutions.

9. **216** Count the number of possible sets that can contain each integer. The number 1 may be in any one set or in any two sets. Therefore there are 6 possible configurations that contain 1. Similarly each of the other numbers can be in any of 6 configurations. The total number of possibilities is 6^9 and the cube root of that is $6^3=216$.

10. **200** $P(\text{Brian fails})=1-\left[P(4\text{ right})+P(5\text{ right})\right]=1-\left[{}_5C_4\left(\frac{1}{2}\right)^5+{}_5C_5\left(\frac{1}{2}\right)^5\right]=1-\frac{6}{32}=\frac{26}{32}$ and
$P(\text{Keaton fails})=1-\left[P(4\text{ right})+P(5\text{ right})\right]=1-\left[{}_5C_4\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^4+{}_5C_5\left(\frac{1}{4}\right)^5\right]=1-\frac{16}{1024}=\frac{1008}{1024}$ The probability that at least one passes is $P = 1 - (.8125)(.9844) = .200$. So $1000P = 200$.

Team Problem Solving - NMT 2010 Solutions

1. **200** The graph defined by the given equation is a square with vertices at (±10, 0) and (0, ±10). The area may be found using $A=s^2=\left(10\sqrt{2}\right)^2=200$ or $A=\frac{1}{2}d^2=\frac{1}{2}(20)^2=200$.

2. **36** Since the number is divisible by 6 it is divisible by both 2 and 3. Every integer divisible by 2 has a unit's digit of 0, 2, 4, 6, or 8. Every integer divisible by three has the sum of its digits divisible by three as well. Since we want the largest possible integer, let a = 9 and then b must be 4, 7, or 1. The only integer satisfying both conditions for b is 4. The number is 9474.

3. **0** $x^2-12=|x|$ is equivalent to $x^2-x-12=0$ and $x^2+x-12=0$. The sum of the roots for the two equations is 1 and –1 respectively. Therefore the sum of all four roots is 0.

4. **45** Since no angle other than the right angle is given, you can assume a 45°-45°-90° triangle. In that case the median, altitude, and angle bisector are all the same line segment so $m\angle ABG = 45°$. In general, the angle bisector of the angle formed by the median and the altitude of a right triangle, when drawn from the right angle, will bisect the right angle as well. (Proof requires one to know that the median drawn to the hypotenuse is one-half the hypotenuse so $\angle C \cong \angle FBC$, and in addition, $\Delta ABD \sim \Delta ACB$ by AA. Therefore, $\angle FCB \cong \angle ABD$.)

5. **10** Apply the quadratic formula to calculate the two roots and then subtract and square the result. Set that result equal to 20, the product of the two roots. Solve for k to get k = 10.

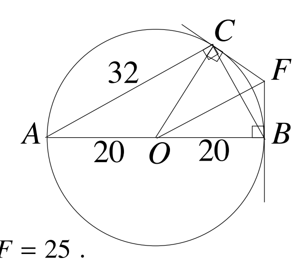
$$\left(\left(\frac{-k+\sqrt{k^2-80}}{2}\right)-\left(\frac{-k-\sqrt{k^2-80}}{2}\right)\right)^2=\left(\sqrt{k^2-80}\right)^2=k^2-80=20 .$$

6. **841** Generally integers have an even number of factors since dividing by one factor yields a second factor different from the divisor. The only integers with an odd number of factors are perfect squares since one factor, the square root of the integer, has itself as the other factor. The only perfect square in the given domain is 841, whose square root is 29.

7. **1** Select any specific pair of equations that are formed by 6 terms in an arithmetic sequence and solve them simultaneously. For example: $3x + 5y = 7$ and $9x + 11y = 13$. Multiply the first equation by –3 and add to the second equation. The result is $-4y = -8$, so $y = 2$. Substitute this into either of the original equations to get $x = -1$. Add the results together to get 1. Try to prove that the values for x and y are independent of the coefficients selected.

8. **1** Multiply both sides of the equation by $(x+1)^2$, noting that $x \neq -1$. You can square the remaining terms and combine like terms or factor according to the difference of two perfect squares.
$(3x-2)^2-(2x-3)^2=((3x-2)+(2x-3))\cdot((3x-2)-(2x-3))(5x-5)(x+1)$. Set each factor equal to zero to get $x = \pm 1$. Reject the negative value due to domain restrictions.

9. **25** Since the diameter is 40, each radius is 20. Since radii drawn to the point of tangency of tangent lines are perpendicular to the tangent, $\Delta COF \cong \Delta BOF$ by hypotenuse-leg. Therefore $\angle COF \cong \angle BOF$. The $m\angle COB=m\widehat{BC}$ and $m\angle A=\frac{1}{2}m\widehat{BC}$. Thus $\angle A \cong \angle FOB$. Since an angle inscribed in a semicircle is a right angle, $\angle ACB$ is a right angle and $\Delta ACB \sim \Delta OBF$. $\frac{AC}{AB} = \frac{OB}{OF} \Rightarrow \frac{32}{40} = \frac{20}{OF} \Rightarrow OF = 25$.



10. **24** Since no specific point was chosen on the base, let the point be a vertex of one of the base angles. In a triangle the product of any altitude by the corresponding base is a constant. Find the altitude to the base of the isosceles triangle and then solve for the other altitude. The altitude to the base of an isosceles triangle bisects the base forming two right triangles with legs of 15 and hypotenuse 25. The altitude is the missing leg and equals 20. Finally, $30 \times 20 = 25x$ so $x = 24$.

11. **57** Since $2^S-1=7k$, for integer k, one possible value for S is 3 ($2^3-1=7$). The next value for S is 6 and then 9 etc. So S is a multiple of 3. The sum $1+2+3+\ldots+10=10\left(\frac{10+1}{2}\right)=55$ which is not a multiple of 3. The next multiple of 3 is 57, so change 10 to 12 and the sum will be 57.

12. **9** The sum of all possible arrangements of the given three digit integers is $100(2h+2t+2u)+10(2h+2t+2u)+(2h+2t+2u)=111(2h+2t+2u)$. Since the average is 333, we get $\frac{111(2h+2t+2u)}{6}=333 \Rightarrow h+t+u=9$.

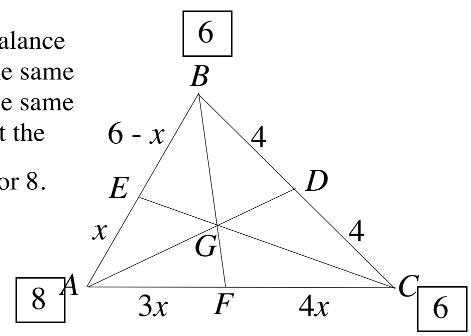
13. **546** Consider applying the distributive property to $(1 + 2 + 4)(1 + 3 + 9)(1 + 5)$. This will result in every possible combination of products where one number is selected from each package. Obviously the easiest way to multiply these terms together is to add first, resulting in $7 \cdot 13 \cdot 6 = 546$.

14. **121** Let $\frac{x}{5}+\frac{y}{9}=\frac{154}{45} \Rightarrow 9x+5y=154$. Create a table of integer values that satisfy this equation. Use the idea that the slope is $\frac{\Delta y}{\Delta x}=\frac{-9}{5}$ so that once you find one integer pair the other integer pairs are easily found. If $x = 1$, $y = 29$. Now add 5 to x and subtract 9 from y to get the next pair of integer solutions. The only possible pairs are (1,29), (6,20), (11,11), and (16, 2). The greatest product, 121, results from (11, 11).

15. **3** Functions f and g are inverses so if $g(4) = 2$, $f(2) = 4$. Use the two points $f(2) = 4$ and $f(0) = -2$ to find the slope of the line. Slope = $\frac{4-(-2)}{2-0} = 3$.

16. **54** Factor $x\sqrt{7}-2y\sqrt{7}-x+y=3\sqrt{7}+3$ into $\sqrt{7}(x-2y)+(-x+y)=3\sqrt{7}+3$. From this form the two equations $x - 2y = 3$ and $-x + y = 3$. Solve these to find $y = -6$ and $x = -9$, so $xy = 54$.

17. **7** Mass point geometry allows you to use weights at the vertices to counterbalance the distances to the "lever" points so that weight times distance remains the same on the two sides of the "lever". For example, the weights at B and C are the same since they are each 4 units from D. The angle bisector theorem tells us that the ratio $\frac{AF}{FC}=\frac{AB}{BC}=\frac{3}{4}$. Therefore the weight at A must be $\frac{4}{3}$ the weight at C or 8. Using weights again, $6BE = 8AE$, so $6(6 - x) = 8x$ or $x=\frac{18}{7}$ and $6-x=\frac{24}{7} \Rightarrow \frac{AE}{EB}=\frac{18}{24}=\frac{3}{4}$. The sum of 3 + 4 = 7.



18. **122** $EF=AC-(AE+FC)=40\sqrt{2}-\left(\left(40\sqrt{2}-40\right)+\left(40\sqrt{2}+40\right)\right)=80-40\sqrt{2}$. So $a = 80$, $b = 40$ and $c = 2$. $a+b+c=122$.

19. **200** By increasing the number of men by 3 fold, the number of days is decreased to 6 days. Increasing the length by 20 fold, the height by 25%, and the width by $33\frac{1}{3}\%$ requires one to multiply $6 \cdot 20 \cdot \frac{5}{4} \cdot \frac{4}{3}=200$ days.

20. **512** The only integers that cannot be written as the sum of consecutive integers are the powers of 2. The largest power of 2 in the given domain is 512. This can be proven in several ways. The sum of any two consecutive integers is always odd. The sum of three consecutive integers is always a multiple of 3. In general the sum of n consecutive integers where n is odd is always a multiple of that integer. If n is even, the sum is always $n(2a + 2n - 1)$, where a is the first number in the sequence. The second factor is always odd, hence it is not a power of 2.