Nassau County Interscholastic Mathematics League

9

Grade 9

TEAM #

Mathematics Tournament 2006

No calculators may be used on this part.
Answer form specified in the problem must be used.
All answers must be exact or rounded to four or more significant digits.
One (1) point for each correct answer

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

1. If $2n + 3$ represents an odd integer, represent, as a binomial, the square of the next consecutive odd integer decreased by 25.	1.
2. In simplest form, $\sqrt{108}+\sqrt{72}$ can be written as $a\left(\sqrt{b}+\sqrt{c}\right)$. Compute $a^2+b^2+c^2$.	2.
3. Ken built a triangular pen for his pet rabbit. He had three boards already cut measuring 12 feet, 17 feet, and x feet. Compute the maximum number of possible whole number values for x.	3.
4. The supplement of the complement of any acute angle whose measure is x can be expressed in the form $90 + 5k$. Express k in terms of x.	4.
5. When $81x^4-16$ is factored completely, it can be expressed in the form $\left(ax^2+b\right)\left(cx+d\right)\left(cx-d\right)$. Compute $\left(a+b+c+d\right)^2$.	5.
6. One root of $12x^2-17x=40$ is $\frac{8}{3}$. Compute the other root?	6.
7. Compute the value of the expression $\left(1+\sqrt{2}\right)\left(1+3\sqrt{2}\right)\left(1-\sqrt{2}\right)\left(3\sqrt{2}-1\right)$.	7.
8. Compute the exact perimeter of an equilateral triangle whose area is $12\sqrt{3}$.	8.
9. Given that n is an integer and that $n<\frac{2^{-2}+3^{-2}}{4^{-2}}<n+1$, compute n.	9.

Turn Over

Mathematics Tournament 2006

Grade 9

Time Limit: 45 minutes **Lower Division** *Answer Column*

10. Given that $2x - 7y = 10$ and $7x + 10y = 30$, compute $11x - 4y$.	10.
11. Two unbiased 6-sided dice are rolled. Compute the probability that the sum of the numbers showing will be a prime number.	11.
12. The diagonal of a rectangle measures 25. If one dimension of the rectangle is 3 more than 3 times the other dimension, compute the perimeter of the rectangle.	12.
13. Compute the area of the triangle formed by the graphs of the equations $y = \frac{2}{3}x$, $y = -\frac{2}{3}x$, and $x = 9$.	13.
14. Solve for x: $\sqrt{\sqrt[3]{x-5}} = \sqrt{2}$.	14.
15. Curt and Art run a mile-long race. Art finishes 1 minute earlier than Curt because he runs 20% faster. Curt ran the mile in k minutes. Compute k.	15.

Nassau County Interscholastic Mathematics League

10

Grade 10

TEAM #

Mathematics Tournament 2006

No calculators may be used on this part.
Answer form specified in the problem must be used.
All answers must be exact or rounded to four or more significant digits.
One (1) point for each correct answer

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Lower Division** *Answer Column*

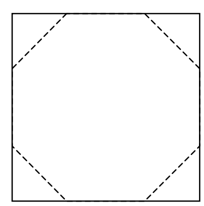
1. A line is given by the equation $y = -2x + 7$. A second line whose equation is given by $ay - 8x = 3$ is parallel to the first line. Compute the *y*-intercept of the second line.	1.
2. An interior angle of a regular pentagon (5 sides), an exterior angle of a regular decagon (10 sides), and an interior angle of a regular dodecagon (12 sides) have degree-measures in the extended ratio of *a:b:c*, where the greatest common factor of *a, b,* and *c* is 1. Determine the ratio *a:b:c*.	2.
3. In right ΔJKL, with the right angle at *J*, $m\measuredangle JKL = 60°$. P is a point on $\overline{JL}$ such that $\overline{KP}$ bisects $\measuredangle JKL$. If *JP* = 5, compute the sum of the squares of the lengths of the sides of ΔJKL.	3.
4. One root of $kx^2 + 15x + 9 = 0$ is 4 times the other root. Compute *k*.	4.
5. Compute the area of the region bounded by the graphs of the equations $y = 6$, $x = 10$, and $3x + 5y = 30$.	5.
6. Allison started from point A, traveled 6 miles north, 40 miles east, 18 miles north, and then returned in a straight line back to point A. The path of her journey created the boundary lines of two triangular plots of land. Compute the positive difference in their areas.	6.
7. $\overline{AB}$ has endpoints A(0, –4) and B(12, 2). Compute the *y*-intercept of the perpendicular bisector of $\overline{AB}$.	7.
8. Each letter in the equation $A \cdot 3^3 + B \cdot 3^2 + C \cdot 3 + D = 70$ represents an integer less than or equal to 2. Compute *A* + *B* + *C* + *D*.	8.

10

Mathematics Tournament 2006

Grade 10

Time Limit: 45 minutes **Lower Division** *Answer Column*

9. If x and y are positive integers, $x + y < 20$, and $x > 7$, compute the greatest possible value of $x - y$?	9.
10. Compute the radius of a circle that is inscribed in a right triangle whose sides measure 5, 12, and 13.	10.
11. The probability that it will rain on any given day in April is $\frac{2}{3}$. Compute the probability that it will rain at least two days out of three consecutive April days.	11.
12. Compute the least positive integer that leaves a remainder of 1 when divided by each of the integers 2, 3, 4, 5, 6, and 7.	12.
13. The mean of the positive integers x, $2x$, $x - 2$, $3x - 8$, $x^2 + 2$, and $x^2 + 2x$ is 10. Compute the positive difference between the mode and the median.	13.
14. If x and y are integers, compute the number of ordered pairs (x, y) that satisfy the equation $24 + x^2 = y^2$.	14.
15. A square that measures 10 x 10 is transformed into a regular octagon by cutting off four congruent isosceles triangles, as seen in the diagram. An architect calculates the measures of the legs of each triangle and expresses this measurement in the form $a - b\sqrt{2}$. Compute the values of a and b and write them as the ordered pair (a, b). 	15.

Nassau County Interscholastic Mathematics League

11

Grade 11

TEAM #

Mathematics Tournament 2006

No calculators may be used on this part.
Answer form specified in the problem must be used.
All answers must be exact or rounded to four or more significant digits.
One (1) point for each correct answer

Name ________________ **School** ________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. The graph of the linear function $f(x)=\frac{1}{2}x+5$ is symmetric to the graph of $g(x)=ax+b$ about the line $y=x$. Determine the ordered pair (a, b).	1.
2. The solution set for the inequality $6x^2+7x>3$ can be expressed in the form $x<a$ or $x>b$. Compute the length of the interval from a to b.	2.
3. Given the periodic function defined by $c(x)=\frac{3}{5}\cos(8x)$. If a is the amplitude, b the frequency, and p the period, then a^2+b-p can be expressed in the form $\frac{g+h\pi}{k}$. Determine the ordered triple (g, h, k).	3.
4. Central $\measuredangle AOB$ of $\odot O$ intercepts $\overset{\frown}{AB}$ whose radian measure is $\frac{\pi}{3}$. If the area of $\odot O$ is 108π, compute the area of the region bounded by minor $\overset{\frown}{AB}$ and $\overline{AB}$.	4.
5. If $\log_7 y=3$, $\log_3\sqrt{x}=2$, $\log_z 8=3$, and $\log_4 64=a$, arrange the values $a, x, y,$ and z in order from smallest to largest.	5.
6. Given that one leg of a right triangle measures 1 less than three times the measure of the other leg, compute the length of the hypotenuse if its measure is 1 more than three times that of the shorter leg.	6.
7. Let $f(xy)=f(x)+f(y)$ for all integers x and y. Compute $f(64)$, given that $f(4)=5$.	7.
8. A non-zero sequence of real numbers is both arithmetic and geometric. Compute the quotient of any two consecutive terms.	8.

Mathematics Tournament 2006

Grade 11

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. Compute the sum of the roots of the equation $5^{2x+1} + 5 = 26 \cdot 5^x$.	9.
10. Compute the value of $(a+b)^2$ given that $ab = 10$ and $a^2b + ab^2 + a + b = 77$.	10.
11. Solve for the least positive *x*: $\dfrac{\sin(157°)\sin(53°) + \cos(23°)\sin(37°)}{\sin(102°)\cos(48°) + \sin(12°)\cos(138°)} = \tan(x°)$.	11.
12. Express the value of $\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$ as an integer.	12.
13. Given $f(x) = 2^x - x^2$ and *g(x)* is *f(x)* translated 2 units to the right and 3 units down, compute *g*(5).	13.
14. If $\log_{10} 2 = c$, express $\log_{10} 25$ in terms of *c*.	14.
15. Express the quotient $\dfrac{\sum_{k=1}^{2006} (i)^k}{\sum_{k=1}^{2006} (-i)^k}$ in *a* + *bi* form, where $i = \sqrt{-1}$.	15.

Nassau County Interscholastic Mathematics League

12

Grade 12

TEAM #

Mathematics Tournament 2006

No calculators may be used on this part.
Answer form specified in the problem must be used.
All answers must be exact or rounded to four or more significant digits.
One (1) point for each correct answer

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 45 minutes **Upper Division** *Answer Column*

1. Compute $\lim_{x\to\sqrt{2}} \dfrac{x^2-2}{x-\sqrt{2}}$.	1.
2. In a geometric sequence all of whose terms are positive, each term is one-half the average of the next two consecutive terms. The common ratio can be expressed in the form $\dfrac{a+\sqrt{b}}{c}$. Determine the ordered triple (*a, b, c*).	2.
3. Given $f(x)=\sin^2 x$ and $g(x)=\cos^2 x$, compute $f'(x)+g'(x)$ in simplest form.	3.
4. The circles defined by $x^2+y^2=9$ and $x^2+y^2-6x=0$ intersect at two points. Write an equation of the line containing the common chord of the two circles.	4.
5. Compute the distance between the points whose polar coordinates are (6, 17°) and (12, 77°).	5.
6. If $f(x)=\dfrac{2}{1-x}$ and $f(g(x))=\dfrac{2x+2}{x-1}$, then *g*(*x*) can be expressed in the form $\dfrac{a}{b+x}$. Compute *a* and *b* and give your result as the ordered pair (*a, b*).	6.
7. Functions $r(n)=\dfrac{n^2-25}{n^3-n}$ and $s(n)=\dfrac{\sqrt{4-n}}{\sqrt{n+1}}$ are defined on the set of integers. List the elements common to the domains of the two functions.	7.
8. If $y=\sin x$, compute $\dfrac{d^{2006}y}{dx^{2006}}$.	8.

Turn Over

Mathematics Tournament 2006

Grade 12

Time Limit: 45 minutes **Upper Division** *Answer Column*

9. Compute the slope of the line tangent to the curve $x^3y + y^2 = y - 10$ at the point on the curve where $y = -5$.	9.
10. The graph of $y = x^3 + ax^2 + bx - 148$ is tangent to the x-axis at (2, 0). Compute the value of the other x-intercept of the graph.	10.
11. Two positive real numbers x and y have the property that $x + ky = 12$. Given that the product xy has its maximum value when $y = 2$, compute k.	11.
12. Compute the infinite sum of $\frac{2}{3} - \frac{1}{2} + \frac{2}{9} - \frac{1}{4} + \frac{2}{27} - \frac{1}{8} + \frac{2}{81} - \frac{1}{16} + \ldots$.	12.
13. Determine an equation of the line tangent to the curve $y = x^4 - x^2$ at each of two points, one in quadrant III and the other in quadrant IV.	13.
14. Compute $h'(x)$, given that $h(x) = \ln\lvert\sin x\rvert + \ln\lvert x \cdot \cot x\rvert + \ln\lvert\sec x\rvert$. Leave your answer as a single fraction in reduced form.	14.
15. Consider the graph of the equation $f(x) = k - x^2$, where k is a positive integer. A line is constructed through the point (6, 0), tangent to the graph of f at a point in the first quadrant. Compute all possible values of k for which the point of tangency is a lattice point [a *lattice point* is a point whose coordinates are integers].	15.

M

Nassau County Interscholastic Mathematics League

Mathletics

TEAM #

Mathematics Tournament 2006

Calculators may be used on this part.
One (1) point for each correct answer

Name ______________________ **School** ______________________ **Score** ________

Time Limit: 30 minutes *Answer Column*

1. When three integers are added together two at a time, the resulting sums are 31, 90, and 137. Compute the *greatest* of the three integers. (A) 39 (B) 89 (C) 98 (D) 129 (E) 137	1.
2. The lengths of the legs of a right triangle are 5 and $\sqrt{15}$. The length of the altitude drawn to the hypotenuse of the triangle can be written in the form $\frac{a\sqrt{b}}{c}$ where a, b, and c are integers and $b < 20$. Compute $a + b + c$. (A) 9 (B) 13 (C) 15 (D) 17 (E) 18	2.
3. When open, each of three valves *A*, *B*, and *C*, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 2 hours; with only valves *A* and *C* open, it takes 3 hours; with only valves *B* and *C* open, it takes 4 hours. Compute the number of hours it takes to fill the tank with only valves *A* and *B* open. (A) .2 (B) 2 (C) 2.1 (D) 2.4 (E) 10	3.
4. Compute the *minimum* distance between the graphs of the equations $x^2 + y^2 = 4$ and $(x+15)^2 + (y-8)^2 = 9$. (A) 8 (B) 10 (C) 12 (D) 15 (E) 17	4.
5. Given positive integers *a, b, c,* and *d*, which satisfy $\frac{a}{b} < \frac{c}{d} < 1$, which of the following expressions has the *greatest* value? (A) 1 (B) $\frac{b}{a}$ (C) $\frac{d}{c}$ (D) $\frac{bd}{ac}$ (E) $\frac{b+d}{a+c}$	5.

Turn Over

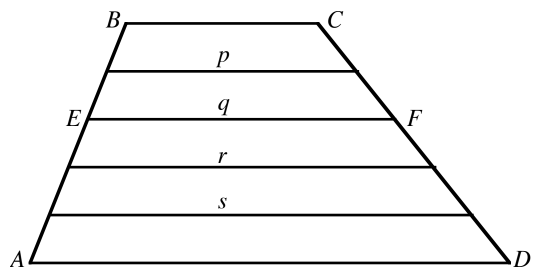
Time Limit: 30 minutes — *Answer Column*

6. A machine has 8 identical independent components. The probability that a component fails is $\frac{1}{2}$. In order for the machine to operate at least 6 of the 8 components must work. Compute the probability that the machine fails to operate to the *nearest thousandth*.

(A) 0.855 (B) 0.145 (C) 0.825 (D) 0.965 (E) 0.988

Answer: 6.

7. Trapezoid *ABCD* has $\overline{AD} \parallel \overline{BC}$. Lines *p*, *q*, *r*, and *s* are parallel to $\overline{AD}$ and cut off equal segments on $\overline{AB}$. If *AD* = 34 and *BC* = 14, compute EF.



(A) 16 (B) 18 (C) 20 (D) 22 (E) 24

Answer: 7.

8. If $G = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}$ and $R = \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}$, compute $G \cdot R$.

(A) 2.25 (B) 2.7 (C) 3.2 (D) $\frac{1+\sqrt{5}}{2}$ (E) $\frac{3+\sqrt{5}}{2}$

Answer: 8.

9. The positive integers 30, 36, 50 and *N* have the property that each integer divides the product of the three remaining integers. Compute the *minimum* value of *N*.

(A) 6 (B) 15 (C) 20 (D) 30 (E) 60

Answer: 9.

10. If $\frac{1}{a+c} = \frac{1}{a} + \frac{1}{c}$, compute $\left(\frac{a}{c}\right)^3$.

(A) –1 (B) 1 (C) 8 (D) $\frac{-1-i\sqrt{3}}{2}$ (E) $\frac{-1+i\sqrt{3}}{2}$

Answer: 10.

Nassau County Interscholastic Mathematics League

T

Team Problem Solving

TEAM #

Mathematics Tournament 2006

HAND IN ONLY **ONE** ANSWER SHEET PER TEAM
Calculators may be used on this part.
Answer form specified in the problem must be used.. All answers must be exact
or rounded correctly to four or more significant digits.
Three (3) points per correct answer

Team Copy **School** ______________________________ **Score** ________

Time Limit: 45 minutes *Answer Column*

1. If $7^{3x} = 216$, compute 7^{2x+1}.	1.
2. Compute all x for which $\left(x^2+3x+7\right)^2 - 15 = 2x^2 + 6x + 14$.	2.
3. If five distinct integers are added four at a time, the five possible sums are 60, 68, 72, 80 , and 84. If S is the least of the five integers and L is the greatest, determine the ordered pair (S,L).	3.
4. At noon, the bottom of a 100-inch ladder placed against a vertical wall, is 28 inches from the base of the wall and sliding away from the base of the wall at the constant rate of 8 inches per hour. Compute the total number of inches the top of the ladder has moved down the wall between noon and 4 pm.	4.
5. For the past six years, the Exwhyzee company has experienced a sales increase of 8% per year and a profit of 12% per year on those sales. In 2005, its sales amounted to \$17,006,112. Compute the company's profit for the year **2003**.	5.
6. Compute the greatest negative integer, n, for which $n^2 - 16n + 105$ is divisible by 41.	6.
7. Determine the units digit in the expansion of 2007^{2006}.	7.
8. The number of degrees, x, y, and z, in the angles of a triangle satisfy the equations $x + 7y - 13z = 0$ and $3x - y - 6z = 0$. Determine the ordered triple (x,y,z).	8.
9. A lattice point is one whose coordinates are integers. Through how many lattice points in quadrant I does the graph of $20x + 6y = 2006$ pass?	9.

Turn Over

T

Mathematics Tournament 2006

Team Problems

Time Limit: 45 minutes *Answer Column*

10. In a race of 1820 yards, Alan beats Bob by 390 yards and Alan beats Charles by 610 yards. If each of the three contestants run at a uniform rate, by how many yards does Bob beat Charles?	10.
11. From least to greatest, *A, B,* and *C* are three consecutive positive integers. If $\sqrt{A+B^2+C^3}=A+B+C$, determine C.	11.
12. Two dimensions of a rectangular prism are 12 and 16. A diagonal of the prism measures 29. Compute the third dimension of the prism.	12.
13. Two numbers have a sum of 8 and a product of 20. Compute the sum of the reciprocals of their squares.	13.
14. Determine all x such that $x^{(x+1)^2}=x^{16}$.	14.
15. For $1<x<y$, let $S=\{x,1,x+y,y\}$. Compute the positive difference between the mean and the median of *S*.	15.
16. In a race of *d* meters in which each of the runners races at a uniform speed, A can beat B by 40 meters, B can beat C by 20 meters, and A can beat C by 56 meters. Compute *d*.	16.
17. With each purchase of a yard of merchandise, a certain merchant gives an additional *x* inches free to the customer. If, as a result of putting this business practice in force, the profit on the merchandise was reduced from 50% of the merchandise's cost to 20% of its cost, compute *x*.	17.
18. Find all ordered pairs of real numbers (x,y) such that $x^y=2$ and $(2x)^{y^2}=64$.	18.
19. Art is 17 years older than Sheila. If his age was placed to the right of hers, the result would be a 4-digit perfect square. The same statement could be made 13 years from now. What is Sheila's present age?	19.
20. Determine the result of the infinite sum: $\frac{2006}{9}-\frac{2006}{9^2}+\frac{2006}{9^3}-\frac{2006}{9^4}+\ldots+(-1)^{n+1}\frac{2006}{9^2}+\ldots$, where n is a positive integer.	20.