Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2023 – 2024

No calculators are allowed.

Time: 10 minutes **Name: ________________________________**

25. Compute the value of the sum $x + y$ if $3^x = 27^{y+1}$ and $4^y = 2^{x-9}$.

26. Fred, working alone, can build a dinosaur nest in 5 hours. Wilma, working alone, can build the same dinosaur nest in 4 hours. If Fred works alone for 2 hours and then Wilma and Fred work together, compute the number of **minutes** it will take for them to complete the job working together.

25. ☐

26. ☐

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27. Given that n and k are positive integers, and that $125! = 5^n(k)$. Compute the maximum possible value of n. (Note: $125!$ is 125 factorial).

28. A circle is tangent to a line whose equation is $y = -1.25x + 20.5$ at the point whose coordinates are $(10,8)$. The center of the circle is on the line whose equation is $y = x - 1$. Compute the square of the radius of the circle.

27.

28.

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29. The area of a square in which the length of its diagonal is one inch longer than the length of its side can be expressed as $a + b\sqrt{c}$ square inches in simplest radical form. Compute the product: abc.

30. Two people will meet for dinner. Each of them arrives at a random time between 7:00 PM and 8:00 PM. If the probability that they arrive within 15 minutes of each other, in simplest form, is $\frac{p}{q}$, compute the sum: $p + q$.

29. [ ]

30. [ ]

Solutions for Contest #5

25. $3^x = 27^{y+1} \rightarrow 3^x = (3^3)^{y+1} \rightarrow x = 3y + 3$ and $4^y = 2^{x-9} \rightarrow 2^{2y} = 2^{x-9} \rightarrow$ $2y = x - 9 \rightarrow x = 2y + 9$. Then, $3y + 3 = 2y + 9 \rightarrow y = 6$ and $x = 21$. The required sum is **27**.

26. After Fred worked for 2 hours, three-fifths of the job remained. Each hour after that, Fred can do one-fifth of the job and Wilma can do one-fourth of the job. Together, they do $\frac{9}{20}$ of the job each hour. Then, $\frac{3/5}{9/20} = \frac{4}{3}$ hours or **80** minutes. Alternatively, if x represents the number of hours Fred and Wilma work together, then $\frac{x}{5} + \frac{x}{4} = \frac{3}{5} \rightarrow 4x + 5x = 12 \rightarrow 9x = 12 \rightarrow x = \frac{4}{3}$ hours or 80 minutes. Another solution: Use the equation $\frac{t+2}{5} + \frac{t}{4} = 1$, where t represents the number of hours Fred and Wilma work together.

27. There is at least one factor of 5 in each of 5, 10, 15, ..., 120, 125 (count: 25). There is a second factor of 5 in each of 25, 50, 75, 100, 125 (count: 5). There is a third factor of 5 in 125 (count: 1). The answer is $25 + 5 + 1 =$ **31**.

28. Call the point of tangency point P and call the center of the circle point O. The coordinates of point O are $(a, a - 1)$. Then, since the slopes of the radius and tangent line are opposite reciprocals, $\frac{a-1-8}{a-10} = \frac{4}{5} \rightarrow 5a - 45 = 4a - 40 \rightarrow a = 5, a - 1 = 4$. The center of the circle is $(5, 4)$ and $OP = \sqrt{(10-5)^2 + (8-4)^2} = \sqrt{41} \rightarrow (OP)^2 =$ **41.**

29. If the length of a side of the square is x and the length of a diagonal is $x + 1$, then by the Pythagorean Theorem, $x^2 + x^2 = (x+1)^2 \rightarrow 2x^2 = x^2 + 2x + 1 \rightarrow$ $x^2 - 2x - 1 = 0 \rightarrow x = 1 + \sqrt{2} \rightarrow x^2 = 3 + 2\sqrt{2}$. This is the square's area and the required product is **12.**

30. Two people arrive respectively x minutes and y minutes after 7 PM. To satisfy the problem, $|x - y| \leq \frac{1}{4} \rightarrow x - \frac{1}{4} \leq y \leq x + \frac{1}{4}$. Generally, this means that a random point is to be chosen within the unit square on $[0,1] \times [0,1]$ and in the region between the lines $y = x - \frac{1}{4}$ and $y = x + \frac{1}{4}$. The area of the region between the lines and in the square is $1 - 2 \cdot \frac{1}{2} \cdot \left(\frac{3}{4}\right)^2 = \frac{7}{16}$ and the required sum is **23.**