Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2020 – 2021

Calculators are allowed.

Time: 10 minutes **Name: ________________________**

19) A college lecture hall has one-fifth of its seats occupied. Another 36 students enter the lecture hall and sit down. Now it has one-half of its seats occupied. Compute the number of seats in the lecture hall.

20) The points A $(1,4)$ and C $(5,10)$ are opposite vertices of rectangle $ABCD$. The four vertices of the rectangle lie on the circle whose equation is $x^2 + y^2 - px - qy + k = 0$. Compute $pq + k$.

19. ☐ 20. ☐

Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2020 – 2021

Calculators are allowed.

Time: 10 minutes **Name:** ______________________________

21) If $19x + 57y = 95$, compute $22x + 66y$.

22) The lengths of the medians to the legs of a right ΔRST are $10\sqrt{3}$ and $2\sqrt{105}$.
Compute the length of the hypotenuse of ΔRST

21. [ ]

22. [ ]

Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2020 – 2021

Calculators are allowed.

Time: 10 minutes **Name: ______________________________**

23) Compute the length of the shortest line segment from the origin to the line whose equation is $4x + 3y = 60$.

24) You toss two standard 6-sided dice and your opponent tosses one die. The probability that at least one of your dice shows a higher number than your opponent's die is, in simplest form, $\frac{p}{q}$. Compute $p + q$.

23. ☐

24. ☐

Solutions for Contest #4

19) There are x seats in the lecture hall. Then, $\frac{x}{5}+36=\frac{x}{2}\rightarrow 36=\frac{3x}{10}\rightarrow x=\mathbf{120.}$

20) The midpoint of diagonal $\overline{AC}$ and the center of the circle is the ordered pair (3,7). Using the distance formula, the radius of the circle is $\sqrt{13}$. Thus, an equation of the circle is $(x-3)^2+(y-7)^2=13\rightarrow x^2+y^2-6x-14y+45=0\rightarrow$ $(p,q,k)=(6,14,45)\rightarrow pq+k=\mathbf{129}.$

21) $19x+57y=19(x+3y)=95\rightarrow x+3y=5\rightarrow 22(x+3y)=22\cdot 5.$
Thus, $22x+66y=\mathbf{110}.$

22) Without loss of generality, let angle S be the right angle, let $ST>RS$, let points M and N be midpoints respectively of legs $\overline{RS}$ and $\overline{ST}$, let $RM=MS=y$, and $SN=NT=x$. Then $x^2+4y^2=300$ and $y^2+4x^2=420\rightarrow 5x^2+5y^2=720\rightarrow x^2+y^2=144\rightarrow$ $4x^2+4y^2=576\rightarrow RT=\mathbf{24}.$

23) The given line is the hypotenuse in the first quadrant of a right triangle whose vertices have coordinates (15, 0), (0, 20), and (0, 0). By the Pythagorean Theorem, the length of the hypotenuse is 25. The shortest line segment from the origin to the given line is the altitude to the hypotenuse of the right triangle. We can calculate the area of the triangle in two ways and set them equal.
$A=\frac{1}{2}bh=\frac{1}{2}\cdot 20\cdot 15=\frac{1}{2}\cdot 25\cdot h\rightarrow h=\mathbf{12}.$ Alternatively, using the formula for the distance from a point to a line, $h=\frac{|4(0)+3(0)-60|}{\sqrt{4^2+3^2}}=\frac{60}{5}=\mathbf{12}.$ Additionally, we can use the theorem that says that the length of the altitude is the mean proportional between the lengths of the segments of the hypotenuse.

24) The probability that your opponent tosses a 1 and does not lose is $\frac{1}{6}\cdot\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$.
The probability that your opponent tosses a 2 and does not lose is $\frac{1}{6}\cdot\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)$.
This reasoning continues so that the probability that your opponent does not lose is

$$\frac{1}{6}\cdot\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\frac{1}{6}\cdot\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)+\frac{1}{6}\cdot\left(\frac{3}{6}\right)\left(\frac{3}{6}\right)+\frac{1}{6}\cdot\left(\frac{4}{6}\right)\left(\frac{4}{6}\right)+\frac{1}{6}\cdot\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)+\frac{1}{6}\cdot\left(\frac{6}{6}\right)\left(\frac{6}{6}\right)=$$

$\frac{1}{6^3}(1+4+9+16+25+36)=\frac{91}{216}$. Therefore, the probability that your opponent loses is $1-\frac{91}{216}=\frac{125}{216}$. The required sum is **341.**