Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2019 – 2020

No calculators are allowed.

**Time: 10 minutes** **Name: ________________________________**

13) Compute the smallest positive integral factor of 3652 that contains two digits.

14) Compute $(2+2i)^8-(2-2i)^8$, where $i=\sqrt{-1}$.

13. ☐

14. ☐

**Time: 10 minutes** **Name: ___________________________________**

15) Tom has ten cards. On each card, exactly one of these ten numbers is printed: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. He chooses three of the cards and adds the numbers on the cards to yield the sum of 13. Compute the number of different sets of cards that yield a sum of 13.

16) Compute the absolute value of $x$ that satisfies $|x-1|-2|x+3|+x+11=0$.

15. ☐

16. ☐

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17) A rectangular box is 6 inches long, 8 inches wide, and $\sqrt{96}$ inches high. A straight iron rod of negligible thickness can be fit into the box. Compute the largest possible length, in inches, of the rod.

18) Consider all values of positive integers $n$ such that EACH of the following is a prime number: $6n^2 + 5$, $2n^2 + 3$, and $n^2 + 1$. Compute the sum of the primes generated by each of the positive integers $n$ that makes EACH of the above binomials a prime.

17. [ ]

18. [ ]

<u>**Solutions for Contest #3**</u>

13) Factor 3652 into primes: $3652 = 2^2 \cdot 11 \cdot 83$. Thus the smallest positive integral factor of 3652 that contains two digits is **11.**

14) $(2+2i)^2 = 4 + 8i + 4i^2 = 8i$ and $(2-2i)^2 = 4 - 8i + 4i^2 = -8i \rightarrow$
$(2+2i)^8 - (2-2i)^8 = ((2+2i)^2)^4 - ((2-2i)^2)^4 = (8i)^4 - (-8i)^4 = \mathbf{0}.$
Alternatively, by DeMoivre's Theorem: $\left(\sqrt{8}\text{cis}45°\right)^8 - \left(\sqrt{8}\text{cis}225°\right)^8 =$
$\left(\sqrt{8}\right)^8\text{cis}360° - \left(\sqrt{8}\right)^8\text{cis}1800° = 0.$

15) We can organize the selections as follows:
$\{10,2,1\}, \{9,3,1\}, \{8,4,1\}, \{8,3,2\}, \{7,5,1\}, \{7,4,2\}, \{6,5,2\}, \{6,4,3\}$. The answer is **8.**

16) If $x \geq 1$, $x - 1 - 2(x+3) + x + 11 = 0$. This equation has no solution.
If $-3 \leq x < 1$, $1 - x - 2(x+3) + x + 11 = 0 \rightarrow -2x + 6 = 0 \rightarrow x = 3$. This value of $x$ is not in the interval. If $x < -3$, $1 - x - 2(-x-3) + x + 11 = 0 \rightarrow 2x + 18 = 0 \rightarrow$ $x = -9$. The required answer is **9.**

17) The bottom of the box is a rectangle whose diagonal is 10 inches. In order to find the required length, we need to find the length of a diagonal that goes from the bottom in one corner to the top in the opposite corner. Use the Pythagorean Theorem on the right triangle whose legs are the box's bottom and the box's height:
$10^2 + \sqrt{96}^2 = x^2 \rightarrow x = \mathbf{14}$. Alternatively, by the Extended Pythagorean Theorem,
$d^2 = a^2 + b^2 + c^2$, $d^2 = 6^2 + 8^2 + \left(\sqrt{96}\right)^2 \rightarrow d = 14.$

18) Using a bit of trial and error: If $n = 1$, then the three numbers in the order stated are 11, 5, and 2. If $n = 2$, then the three numbers in the order stated are 29, 11, and 5. If $n = 3$, then the three numbers in the order stated are 59, 21, and 10. If $n = 4$, then the three numbers in the order stated are 101, 35, and 17. If $n = 5$, then the three numbers in the order stated are 155, 53, and 26. Notice that in each set exactly one number is a multiple of 5. Is this always the case? If $n \equiv 0 \bmod 5$, then $6n^2 + 5 \equiv 0 \bmod 5$. If $n \equiv \pm 1 \bmod 5$, then $2n^2 + 3 \equiv 0 \bmod 5$. If $n \equiv \pm 2 \bmod 5$, then $n^2 + 1 \equiv 0 \bmod 5$. Therefore, for any positive integer $n$, one of the three binomials is a multiple of 5. Thus, for a binomial to be prime, its value must be 5. For any positive integer $n$, $6n^2 + 5$ must always be greater than 5. Note that $2n^2 + 3 = 5$ only for $n = 1$ and $n^2 + 1 = 5$ only for $n = 2$. Therefore, the required sum is 11+5+2+29+11+5=**63.**