Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2019 – 2020

No calculators are allowed.

Time: 10 minutes **Name: ____________________________**

1) The quadratic function, $f(x) = ax^2 + bx + c$ satisfies $f(0) = 3$, $f\left(\frac{1}{2}\right) = 0$, and $f(2) = 21$. Compute a.

2) Compute: $2021 - \left(\frac{1}{2020} + \frac{2021 \cdot 2019}{2020}\right)$.

1.

2.

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3) Assume all coders write code at the same rate. If 6 coders can write 8 programs in 3 days, compute the number of coders that will be required to write 20 programs in 15 days.

4) In right ΔXYZ, $\measuredangle Z$ is a right angle, $XZ = 30$, and $YZ = 30\sqrt{3}$. Points A, B, and C are chosen so that $\overline{XAZ}$, $\overline{YBZ}$, and $\overline{XCY}$, $\overline{AB} \parallel \overline{XY}$, and ΔABC is equilateral. Compute AB.

3. $\boxed{}$

4. $\boxed{}$

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5) In Ms. Fermat's mathematics class, 25 students took a test. If the arithmetic mean of the passing grades was 84, and the arithmetic mean of the failing grades was 59, and the arithmetic mean of all the grades was 80, compute the number of students who passed the test.

6) If the only positive real solution of $\sqrt[3]{x+5} - \sqrt[3]{x} = 1$ is written in simplest $\frac{a+b\sqrt{c}}{d}$ form, compute $a+b+c+d$.

5. $\boxed{}$

6. $\boxed{}$

Solutions for Contest #1

1) From the first function value, $c = 3$. From the second function value, $\frac{a}{4} + \frac{b}{2} + 3 = 0$. From the third function value, $4a + 2b + 3 = 21$. Solve the latter two equations simultaneously to yield $b = -11$ and $a = \mathbf{10}$.

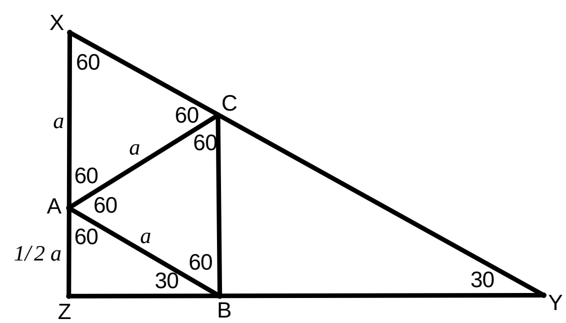
2) $2021 - \left(\frac{1}{2020} + \frac{2021 \cdot 2019}{2020}\right) = \frac{2021 \cdot 2020 - 1 - 2021 \cdot 2019}{2020} = \frac{2021(2020-2019)-1}{2020} = \frac{2021 \cdot 1 - 1}{2020}$

$= \frac{2020}{2020} = \mathbf{1}$. Note: This problem is an application of the identity ($a \neq 0$):

$a + 1 - \left(\frac{1}{a} + \frac{(a+1)(a-1)}{a}\right) = a + 1 - \left(\frac{1}{a} + \frac{a^2-1}{a}\right) = a + 1 - \frac{a^2}{a} = a + 1 - a = 1.$

3) Let x coders $= \frac{20 \text{ programs}}{15 \text{ days}}$. It is given that 6 coders $= \frac{8 \text{ programs}}{3 \text{ days}}$. Divide these two equations to yield $\frac{x}{6} = \frac{20}{15} \div \frac{8}{3} = \frac{60}{120} = \frac{1}{2}$. Thus, $x = \mathbf{3}$.

4) Since $XZ = 30$ and $ZY = 30\sqrt{3}$ are the legs of a right triangle, ΔXYZ is a 30-60-90 triangle. Since $\Delta ABZ \sim \Delta XYZ$, ΔABZ is also a 30-60-90 triangle. Since ΔABC is equilateral, ΔAXC is also equilateral. Let $a = AB = AC = AX$. So, $AZ = \frac{1}{2}a$ and $a + \frac{1}{2}a = 30$. Thus, $a = \mathbf{20}$.



5) If x students passed the test, then $84x + 59(25 - x) = 80 \cdot 25 \rightarrow$

$25x + 1475 = 2000 \rightarrow x = \mathbf{21}$.

6) The given equation yields $\sqrt[3]{x+5} = 1 + \sqrt[3]{x} \rightarrow x + 5 = 1 + 3\sqrt[3]{x} + 3\left(\sqrt[3]{x}\right)^2 + x$

$\rightarrow 3\left(\sqrt[3]{x}\right)^2 + 3\sqrt[3]{x} - 4 = 0 \rightarrow \sqrt[3]{x} = \frac{-3 \pm \sqrt{57}}{6}$. Only the positive answer is required, so

$6\sqrt[3]{x} = \sqrt{57} - 3 \rightarrow 216x = 57\sqrt{57} - 3(57)(3) + 3\left(\sqrt{57}\right)(9) - 27 \rightarrow$

$216x = -540 + 84\sqrt{57} \rightarrow x = \frac{-540 + 84\sqrt{57}}{216} = \frac{-45 + 7\sqrt{57}}{18}$. The required sum is **37.**