Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2025 – 2026

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

1. Compute $64^0 + 64^{\frac{1}{3}} + 64^{\frac{1}{2}} + 64^1 + 16^{\frac{1}{4}} + 16^{\frac{1}{2}}$.

2. Point B is on $\overline{AC}$ and $AB : BC = 2 : 3$. If the coordinates of the points A, B, and C are respectively $(-17, 92)$, (x, y), and $(43, 2)$, compute the product, xy.

1.

2.

Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2025 – 2026

No calculators are allowed.

Time: 10 minutes **Name: ________________________________**

3. Compute the positive value of x such that $(4x)\%$ of $5x$ is x.

4. In ΔABC, point M is the midpoint of $\overline{BC}$ and point N is the midpoint of $\overline{AC}$. If $AM = 12, BN = 18$, and $AC = 20$, compute the area of ΔABC.

3. $\boxed{}$ 4. $\boxed{}$

Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2025 – 2026

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

5. Compute x if $2^{x-3} - 2^{x-4} + 2^{x+5} = 1026$.

6. If $f(x) + f(x-1) = x^2$ and $f(11) = 50$, compute $f(41)$.

5.

6.

Solutions for Contest #1

1. $64^0 + 64^{\frac{1}{3}} + 64^{\frac{1}{2}} + 64^1 + 16^{\frac{1}{4}} + 16^{\frac{1}{2}} = 1 + 4 + 8 + 64 + 2 + 4 = \mathbf{83}$.

2. Since point B is $\frac{2}{5}$ of the distance from point A to point C on $\overline{AC}$:
$x = -17 + \frac{2}{5}(43 - (-17)) = -17 + \frac{2}{5}(60) = 7$ and $y = 92 + \frac{2}{5}(2 - 92) =$
$92 + \frac{2}{5}(-90) = 56$. The required product is **392.**

3. From the problem statement, $\frac{4x}{100} \cdot 5x = x \rightarrow 20x^2 = 100x \rightarrow x^2 = 5x \rightarrow x = \mathbf{5}$.

   Reject $x = 0$ because the problem asks for the positive value only.

4. Let the intersection of $\overline{AM}$ and $\overline{BN}$ be point P. Because of the midpoints, $AN = NC = 10$ and $BM = MC$. Because medians of a triangle intersect and separate in a 2:1 ratio, $AP = 8, PM = 4, BP = 12$, and $PN = 6$. Because the sides of ΔAPN are 6-8-10, it is a right triangle with right $\measuredangle P \rightarrow |\Delta APN| = 24 = |\Delta BPM|$ and $|\Delta APB| = 48 \rightarrow |\Delta AMB| = 48 + 24 = 72$, Then, since $\overline{AM}$ is a median in $\Delta ABC, |\Delta ABC| = 2 \cdot 72 = \mathbf{144}$.
Note: A median of a triangle divides it into two triangles of equal area.
Alternatively, if all three medians are drawn, they divide ΔABC into 6 triangles of equal area. So, if $|\Delta APN|$=24, then $|\Delta ABC| = 6 \cdot 24 = 144$.

5. From the given equation: $2^{x-4}(2 - 1 + 2^9) = 1026 \rightarrow 2^{x-4}(513) = 1026 \rightarrow$
$2^{x-4} = 2^1 \rightarrow x - 4 = 1 \rightarrow x = \mathbf{5}$. Alternatively, $2^x\left(\frac{1}{8} - \frac{1}{16} + 32\right) = 1026$. Multiplying by 16,
$2^x(2 - 1 + 512) = 1026 \cdot 16 \rightarrow 2^x(513) = 1026 \cdot 16 \rightarrow 2^x = 32 \rightarrow x = 5$.

6. Start with $f(41) + f(40) = 41^2, f(40) + f(39) = 40^2$,
$f(39) + f(38) = 39^2, \ldots\ f(13) + f(12) = 13^2,\ f(12) + f(11) = 12^2$. Then, subtract these equations pairwise: $f(41) - f(11) = 41^2 - 40^2 + 39^2 - 38^2 + 38^2 - 37^2 + \cdots +$
$13^2 - 12^2 = (41 + 40)(41 - 40) + (39 + 38)(39 - 38) + (37 + 36)(37 - 36) + \cdots +$
$(13 + 12)(13 - 12) = 81 + 77 + 73 + \cdots + 25 = \frac{15}{2}(81 + 25) = 795$.
So, $f(41) = 795 + f(11) = 795 + 50 = \mathbf{845}$.