NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE
ALL-STAR PROGRAM

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Practice #1: Facts and Formulas, Tricks and Trivia

While it is definitely true that problem solving skill is not solely based on your knowledge of formulas, terminology, and other minutae, it *is* true that should such information be required to solve a problem and you *don't* know it, you are almost certainly going to get it wrong. With that in mind, we have compiled (with the unwitting help of the NYC math team) a collection of terminology, formulas, and generally just…stuff…that will probably come in handy at least some of the time. Enjoy!

Part 1: Terminology

- Answer forms
  - Compute: The word **compute** implies that the answer must be in simplest exact form. If the problem specifies a form for the answer, that form must be used. Otherwise:
    - Decimals must be either terminating or repeating (indicated by an over-bar.)
    - Fractions may be written either as an improper fraction or mixed number, but in either case, the fraction involved must be <u>reduced</u>.
    - Radicals must be expressed in simplest radical form.
    - All denominators of fractions must be rationalized and may not contain either radicals or imaginary numbers.
    - Answers involving π or e must be in terms of the symbols π or e, not replaced by 3.14 or 22/7 or 2.718.
    - Functions must be evaluated: answers such as sin 45° or log 100 are not acceptable.
  - Ordered pair (triple, n-tuple): A problem asking for an **ordered pair** requires that the two values be written, <u>with parentheses</u>, <u>separated by a comma</u>. If the answer is (3, 5), the answers 3, 5 or (3 5) or [3 5], etc. will not be accepted.
- Symbols:
  - $i = \sqrt{-1}$
  - $\pi \approx 3.1415926535\ldots$
  - $e \approx 2.178\ldots$
  - **Sigma notation**: $\sum_{n=a}^{b} f(n) = f(a) + f(a+1) + f(a+2) + \ldots + f(b)$
  - **Pi notation**: $\prod_{n=a}^{b} f(n) = f(a)f(a+1)f(a+2)\ldots f(b)$
- Set theory:
  - A **set** is a collection of **elements**. The order of elements within a set does not matter, and no element is repeated within a set. All that matters for any given item is whether or not it is contained within a given set.
  - A set be listed within **braces**: $\{e_1, e_2, \ldots, e_n\}$. A set containing no elements is still a set, known as the **empty set** or the **null set**, indicated as $\{\ \}$ or $\varnothing$.
  - A set may also be specified by using **set-builder notation**, as follows: {(form)|(conditions)}. For example, the set of all odd numbers can be specified as $\{2n+1 \mid n \in \mathbb{Z}\}$.
  - If x is one of the elements of a set S, we write $x \in S$. Otherwise, we write $x \notin S$.
  - If, given sets S and T, it is true that there are no elements of S that are not in T, then S is a **subset** of T, written $S \subset T$. If $S \neq T$, then S is a **proper subset** of T. Note that $\varnothing$ is a subset of every set.
  - **Union**: $S \cup T = \{x \mid x \in S \text{ or } x \in T\}$
  - **Intersection**: $S \cap T = \{x \mid x \in S \text{ and } x \in T\}$

- Logic:
  - **Universal quantifier**: the symbol $\forall_x$ means "for all x". Conditions on x are written in the subscript. Any statement following the quantifier is given to apply to all x that fit the condition. For example, the statement $\forall_{x\in\mathbb{R}} : x^2 \geq 0$ means "for all real x it is true that x squared is at least zero."
  - **Existential quantifier**: the symbol $\exists_y$ means "there exists a y". Any statement following the quantifier must apply to at least one y that fits the condition. The existential quantifier is often used in conjunction with the universal quantifier, as in the postulate: $\forall_{x,y\in\mathbb{R}^+} \exists_{n\in\mathbb{Z}^+} : nx > y$, which states that "for all positive real numbers x and y, there exists a positive integer n such that nx > y."
  - **Statement**: A statement is a sentence which can be evaluated to **true** or **false**. An **open sentence** is a statement which evaluates to true or false depending on the value of some variable or variables.
  - **Logical equivalence**: two statements are logically equivalent if they always have the same truth value. Such statements are interchangeable in logic.
  - **Negation**: the logical negation (represented by "not" or the symbol $\sim$ or $\neg$) of a statement is the logical opposite of the statement. If a statement is true, its negation is false, and vice-versa.
  - **Conjunction**: the conjunction of two statements ("A and B" or "$A \wedge B$") is a compound statement which is only true if the two component statements are true.
  - **Disjunction**: the disjunction of two statements ("A or B" or "$A \vee B$") is a compound statement which is true if either (or both) of the two component statements is true.
  - **Exclusive or (xor)**: the xor of two statements ("A xor B" or "$A \otimes B$") is a compound statement which is true if either (but not both) of the two component statements is true.
  - **Conditional**: a conditional ("A implies B", "If A then B", "B if A", "$A \rightarrow B$") is a compound statement which is true unless A is true and B is false. Given a conditional $A \rightarrow B$, the:
    - **Inverse** is $\sim A \rightarrow \sim B$
    - **Converse** is $B \rightarrow A$
    - **Contrapositive** is $\sim B \rightarrow \sim A$. The contrapositive is logically equivalent to the original conditional.
  - **Biconditional**: a biconditional ("A if and only if B", "A iff B", "$A \leftrightarrow B$") is a compound statement which is true only when A and B have the same truth values. A biconditional is the conjunction of two conditionals: $(A \leftrightarrow B) = (A \rightarrow B) \wedge (B \rightarrow A)$
- Interval notation
  - The interval $[a,b]$ is called a **closed interval** and indicates that the endpoints are to be included in the range of values: $[a,b] = \{x \mid x \in \mathbb{R}, a \leq x \leq b\}$.
  - The interval (a,b) is called an **open interval** and indicates that the endpoints are not to be included in the range of values: $(a,b) = \{x \mid x \in \mathbb{R}, a < x < b\}$
  - **Half-open** (or **half-closed**) intervals are possible: $[a,b) = \{x \mid x \in \mathbb{R}, a \leq x < b\}$
  - An **open-ended interval** has only one endpoint and extends forever in one direction. The open end is indicated by the infinity symbol. The end with the infinity always uses parentheses, never brackets: $(-\infty, a] = \{x \mid x \in \mathbb{R}, x \leq a\}$ and $(a, \infty) = \{x \mid x \in \mathbb{R}, x > a\}$.
- Bounding and rounding functions:
  - The **Greatest Integer Function**, also known as the **Floor Function**, is defined as follows: $\lfloor x \rfloor$ =the greatest integer less than or equal to x.
  - The **Least Integer Function**, also known as the **Ceiling Function**, is defined as follows: $\lceil x \rceil$ =the least integer greater than or equal to x.
  - A set S of values is **bounded above** if there exists an upper bound value M such that $M \geq x$ for all $x \in S$. The **least upper bound** or **supremum** of S is the smallest value which can serve as an upper bound for S.
  - The terms **bounded below**, **greatest lower bound**, and **infimum** of S are defined analagously.
  - The **completeness axiom** of the real numbers states that any set which has an upper bound has a least upper bound, and any set which has a lower bound has a greatest lower bound.

- The **maximum** of a set is the largest value in the set, the **minimum**, the smallest. Any finite set of real numbers has a maximum and a minimum. The **well-ordering principle** states that any set of positive integers (finite or infinite) must have a minimum.

- The word "base" has many meanings. It can mean:
  - A number which is raised to a power: in 2^5, 2 is the base.
  - In $\log_3 9 = 2$, 3 is the base.
  - In a geometric figure, the side that serves as the "bottom", to which an altitude is drawn, is called the base.
  - In base arithmetic, the base is the number whose powers are used for digit column values. For example, in base four, $2303_4 = 2 \cdot 4^3 + 3 \cdot 4^2 + 0 \cdot 4^1 + 3 \cdot 4^0 = 128 + 48 + 3 = 179_{10}$
- Important types of numbers:
  - **Natural numbers**: a natural number (also called a **counting** number) is a number which can be used to count a discrete number of things: {1, 2, 3, 4, …} The set of natural numbers can be written as $\mathbb{N}$.
  - A **whole number** is either a natural number or zero.
  - An **integer** is either a whole number or the negative of a whole number. The set of integers can be represented as [redacted].
  - A **rational number** is a number which can be written in the form p/q, where p and q are both integers. Rational Numbers can also be written as terminating or repeating decimals. The set of rational numbers is symbolized as $\mathbb{Q}$.
  - A **real number** is defined as the least upper bound of a set of rational numbers, however, you can think of a real number as any number that can be located on the number line. The symbol for the set of all real numbers is $\mathbb{R}$
  - An **irrational number** is a number which is real but not rational.
  - A **complex number** is a number from the set $\mathbb{C} = \{a + bi : a \in \mathbb{R}, b \in \mathbb{R}, i = \sqrt{-1}\}$
  - A **pure imaginary number** is a complex number in the form bi, where b is a real number and $i = \sqrt{-1}$.
  - An **imaginary number** is a complex number which is not real.
- Geometry (see section on sets for some terminology used here.)
  - Undefined terms: **point**, **line**, **plane**. Lines and planes are sets of points.
  - A **line segment** is a continuous finite subset of a line which has two **endpoints**. A line segment is specified by giving its endpoints with an over-bar: $\overline{AB}$. The symbol AB indicates the **distance** between the endpoints of the line segment.
  - The **midpoint** of a line segment $\overline{AB}$ is the (unique) point M such that $AM = MB$. A line or segment that passes through the midpoint of a segment is called a **bisector** of that segment.
  - A **trisection point** is a point on a line segment that creates a smaller line segment that is one-third the length of the segment. A segment has 2 trisection points.
  - Two lines in the same plane that never intersect are called **parallel**. If two line segments can be extended to form parallel lines, then the segments are parallel. Two lines or line segments that intersect to form right angles are called **perpendicular**. Two lines that never intersect but are not parallel (because they are not coplanar) are called **skew** lines.
  - A line (or line segment) that passes through the midpoint of a line segment, and which is perpendicular to that segment, is called a **perpendicular bisector** of the segment.
  - A **ray** is a continuous infinite proper subset of a line which has one **endpoint**. The ray AB has endpoint *A* and the point *B* can be any other point of the ray.
  - An **angle** is the union of two rays (sides of the angle) that share the same endpoint, called the angle's **vertex**. Angles are specified such that $\angle ABC$ is the union of BA and BC. If there is no ambiguity, an angle may be specified by giving only its vertex: $\angle B$.
  - An angle measuring less than 90° is **acute**, exactly 90° is **right**, between 90° and 180° is **obtuse**, exactly 180° is **straight**, and between 180° and 360° is **reflex**. In Euclidean geometry, there is no need for straight and reflex angles, as the former is a line and the latter is a non-reflex angle on the "other side."
  - Two angles whose measures add to 90° are called **complementary**, with each such angle called the **complement** of the other. Two angles that add to 180° are called **supplementary**, and are **supplements** of each other.

- A **polygon** is the union of line segments, called **sides**: $\overline{V_1V_2} \cup \overline{V_2V_3} \cup \ldots \cup \overline{V_{n-1}V_n} \cup \overline{V_nV_1}$ such that no two segments intersect anywhere other than their endpoints. The set $\{V_1, V_2, \ldots, V_n\}$ are called the **vertices** (singular, vertex) of the polygon. Two vertices that are connected by a side of the polygon are called consecutive vertices of the polygon. If the number of sides of a polygon is unknown or unspecified, it is called an **n-gon**, if *n* is the number of vertices. By convention, all n-gons are nondegenerate, that is, no three consecutive vertices of a polygon are collinear.
- The following terms apply to all polygons:
  - **Equilateral**: all sides are congruent (have the same length)
  - **Equiangular**: all angles are congruent.
  - **Regular**: both equilateral and equiangular.
  - **Diagonal**: a line segment whose endpoints are non-consecutive vertices of the polygon.
- The following types of polygons must be known (n = # of sides = # of vertices).
  - Triangle (n = 3)
    - **Scalene**: no two sides have the same length.
    - **Isosceles**: at least two sides have the same length. If only two sides are congruent, these two sides are called **legs** and the third side the **base**.
    - **Right**: contains one right angle. The sides adjoining the right angle are called **legs** and the third side the **hypotenuse.**
    - **Acute**: all three angles measure less than 90°.
    - The terms equilateral and equiangular are synonymous for triangles only.
  - Quadrilateral (tetragon) (n = 4)
    - **Trapezoid**: exactly two (necessarily nonconsecutive) sides are parallel. The two parallel sides are called **bases**, the other sides **legs**.
    - **Parallelogram**: both pairs of opposite sides are both parallel and congruent. Both pairs of opposite angles are also congruent.
    - **Rhombus**: an equilateral quadrilateral. All rhombi are parallelograms.
    - **Rectangle**: an equiangular quadrilateral. All rectangles are parallelograms.
    - **Square**: a regular quadrilateral. All squares are rectangles, rhombi, and parallelograms.
  - Pentagon (n = 5)
  - Hexagon (n = 6)
  - Heptagon (septagon) (n = 7)
  - Octagon (n = 8)
  - Nonagon (enneagon) (n = 9)
  - Decagon (n = 10)
  - Dodecagon (n = 12)
- An **inscribed** circle is a circle inside of a polygon which is tangent to all the sides of the polygon. A **circumscribed** circle is a circle outside of a polygon which passes through all the vertices of the polygon.
- **Convex**: a polygon is considered convex if all of its diagonals lie on the interior of the polygon. In general, a closed subset S of the plane is considered convex if all line segments whose endpoints lie within S are subsets of S.
- **Concave**: a polygon or closed subset of the plane which is not convex.
- **Cevian**: a line segment connecting one vertex of a triangle to a point somewhere on the opposite side. Special cevians include:
  - **Median**: connects a vertex to the midpoint of the opposite side.
  - **Angle bisector**: bisects the vertex angle that it's drawn from.
  - **Altitude**: is drawn perpendicular to the opposite side. Since this may not be possible without extending the opposite side, an altitude might not be in the interior of the triangle.
- A **center** of a triangle is a special point associated with a triangle. Some centers:
  - **Incenter**: the center of the inscribed circle, which occurs where the angle bisectors coincide.
  - **Circumcenter**: the center of the circumscribed circle, which occurs where the perpendicular bisectors of the sides coincide.
  - **Centroid**: the "center of mass" of the triangle, which occurs where the medians coincide.
  - **Orthocenter**: the point where the altitudes of the triangle intersect.

- Coordinate Geometry
  - **Abscissa**: the x-coordinate of a point.
  - **Ordinate**: the y-coordinate of a point.
  - **Intercepts**: places where a curve crosses or touches an axis.
  - **Lattice point**: a point all of whose coordinates are integers. A complex number that graphs as a lattice point in the complex plane is called a **Gaussian Integer**.
- Trigonometry

- The **circular functions** are the six basic trigonometric functions: sin, cos, tan, sec, csc, cot.
- The circular functions can be grouped in pairs of **cofunctions**: sin and cos, tan and cot, sec and csc. Each function of an angle is equal to the cofunction of the complement of that angle.
- The **inverse trigonometric functions** are the (function) inverses of the six circular functions. Since none of the circular functions is one-to-one, the inverse trig functions are limited to specific ranges, known as the **principal values** of those functions.

- Means
  - The **arithmetic mean** or **average** of a set of values is defined as $AM = \dfrac{a_1 + a_2 + \ldots + a_n}{n}$
  - The **geometric mean** of a set of values is defined as $GM = \sqrt[n]{a_1 a_2 \ldots a_n}$
  - The **harmonic mean** of a set of values is defined as the reciprocal of the average of their reciprocals:

$$HM = \frac{1}{\frac{1}{n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}}$$

  - The **AM-GM-HM** inequality states that, for any set of numbers, $AM \geq GM \geq HM$, with the equality holding if and only if all the numbers in the set are equal.
- Sequences and Series
  - **Sequence**: an ordered list $a_1, a_2, a_3, \ldots$ of values. Can be written $\left\{a_k\right\}_{k=1}^{\infty}$
  - **Series**: the sum of a sequence $a_1 + a_2 + a_3 + \ldots$. Can be written $\sum_{k=1}^{\infty} a_k$
  - Special sequences. Each of these can be made into a series by replacing the commas by plus signs.
    - **Arithmetic**: pronounced arithMETic, this sequence has a constant difference between consecutive terms: $a, a+d, a+2d, a+3d, \ldots$
    - **Geometric**: this sequence has a constant ratio between consective terms: $a, ar, ar^2, ar^3, \ldots$
    - **Harmonic**: $\left\{1/n\right\}_{n=1}^{\infty} = 1, 1/2, 1/3, 1/4, \ldots$
    - **Fibonacci**: $a_1 = a_2 = 1, a_n = a_{n-1} + a_{n-2}$; $\left\{a_n\right\}_{n=1}^{\infty} = 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$
    - **Lucas**: the generalization of the Fibonacci sequence with any two starting numbers. The most famous Lucas sequence starts 1, 3, 4, 7, 11, 18, 29, …
- Number Theory
  - A **divisor** or **factor** of an integer n is an integer d such that $dk = n$ for some integer k. A **proper divisor** of a number is a divisor that is less than the number itself. Note that every number is a divisor of 0.
  - A **prime number** is a positive integer which has exactly 2 distinct divisors.
  - A **composite number** is a positive integer which has more than 2 distinct divisors.
  - The number 1 is neither prime nor composite. The number 1 is called a **unit**.
  - A **perfect number** is a number which is the sum of its proper divisors. A number whose proper divisors add up to less than the number is called **deficient**, more than the number, **abundant**.
- Probability
  - The **probability** $P(E)$ of an event E is the fraction of the time that the event will occur, given a large number of trials. The **law of large numbers** states that as the number of trials increases, the fraction of the time that the event occurs will get closer and closer to the probability of the event.
  - **Observed probability** is the fraction of the time that an event actually occurs over a large number of trials. The **theoretical probability** is the predicted probability of an event, based on calculation. When a theoretical probability is not known, it can be estimated by performing a large number of trials and computing the observed probability. This is known as the **Monte Carlo method**.
  - The set of all possible outcomes of an event is known as the event's **sample space**.
  - Two events are **mutually exclusive** if they cannot both occur at the same time, that is, the probability of both events occurring is zero.
  - **Conditional probability** is the probability of an event B occurring, given that event A has already occurred. Such probability is written $P(A \mid B)$, meaning "probability of A, given B."

- Two events are **independent** if the outcome of one event does not affect the other probability, in other words, $P(A \mid B) = P(A)$

- Combinatorics
  - The **factorial** function is the product of all the positive integers from 1 to the given number, inclusive. $n! = \prod_{k=1}^{n} k$. By definition, $0! = 1$.
  - A **permutation** is an arrangement of some or all of the items of a set. Two permutations are considered different if they contain the same items in different orders: (1, 2, 3, 4) is not the same permuatation as (1, 4, 2, 3). The <u>number</u> of possible permutations you can make by selecting r items from a set of n items is indicated by ${}_nP_r$.
  - A **combination** is a selection of some or all of the items of a set. Two combinations are considered the same if they contain the same items in different orders: {1, 2, 3, 4} is the same combination as {1, 4, 2, 3}. The number of possible combinations you can make by selecting r items from a set of n items is indicated by ${}_nC_r$ or $\binom{n}{r}$.

Part 2: Formulas

There are a considerable number of formulas on the NYC math team formula sheet (provided), but here are some more:

- Probability
  - $P(A \wedge B) = P(A) \cdot P(B \mid A)$
  - Thus, conditional probability is $P(B \mid A) = \dfrac{P(A \wedge B)}{P(A)}$
  - If A and B are independent events, $P(A \wedge B) = P(A)P(B)$
  - If A and B are mutually exclusive, then $P(A \wedge B) = 0$ and $P(A \vee B) = P(A) + P(B)$.
  - $P(A) + P(\sim A) = 1$
  - Inclusion/exclusion principle:
    - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
    - $P(A \vee B \vee C) = P(A) + P(B) + P(C) - P(A \wedge B) - P(A \wedge C) - P(B \wedge C) + P(A \wedge B \wedge C)$
  - Bernoulli experiment: If p is the probability that event E will occur, the probability that event E will occur exactly r times out of n trials is ${}_nC_r \cdot p^r \cdot (1-p)^{n-r}$
- Combinatorics
  - ${}_nP_r = \dfrac{n!}{(n-r)!}$
  - Permutations with repetition: given a collection of items such that in the collection, item k occurs r_k times, the number of arrangements of the collection is $\dfrac{(r_1 + r_2 + \ldots + r_n)!}{r_1! r_2! \ldots r_n!}$
  - ${}_nC_r = \dfrac{n!}{r!(n-r)!}$
  - ${}_nC_r = {}_nC_{n-r}$
  - ${}_nC_0 = {}_nC_n = 1$
  - ${}_nC_1 = {}_nC_{n-1} = n$

- "Pascal's Triangle" property: ${}_{n+1}C_{r+1} = {}_nC_r + {}_nC_{r+1}$
- "Hockey Stick" property: ${}_nC_0 + {}_{n+1}C_1 + {}_{n+2}C_2 + \ldots + {}_{n+k}C_k = {}_{n+k+1}C_k$
- ${}_nC_0 + {}_nC_1 + \ldots + {}_nC_n = 2^n$
- ${}_nC_r = 0$ if n < 0 or n > r

- Logic
  - Order of operations: negation first, then conjunction, disjunction, and finally conditionals
  - The following statements are **tautologies**, which are statements that are always true:
    - $p \rightarrow (p \vee q)$
    - $(p \wedge q) \rightarrow p$
    - $\sim (p \wedge \sim p)$
    - $\sim (\sim p) \leftrightarrow p$
    - Law of the excluded middle: $p \vee \sim p$
    - Disjunctive inference: $(p \vee q) \wedge \sim p \rightarrow q$
    - Modus ponens/syllogism: $\left[(p \rightarrow q) \wedge p\right] \rightarrow q$
    - Modus tollens: $\left[(p \rightarrow q) \wedge \sim q\right] \rightarrow \sim p$
    - Law of the contrapositive: $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
    - DeMorgan's Laws:
      - $\sim (p \wedge q) \leftrightarrow (\sim p \vee \sim q)$
      - $\sim (p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
    - Distributive properties:
      - $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
      - $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$

- Pythagorean triples:
  - 3-4-5, 5-12-13, 7-24-25, 9-40-41, 11-60-61, …
  - 4-3-5, 6-8-10, 8-15-17, 10-24-26, 12-35-37, …
  - 20-21-29
  - $n^2 - m^2, 2mn, n^2 + m^2$

Warm-ups:

1. Compute the smallest positive integer factor of 2005 that has at least two digits.

2. Bobby wants to fill a bag with a total of 10 candies, choosing from four flavors (apple, banana, cherry, grape). If he can use any number (including zero) of each candy, in how many distinct ways can he fill the bag?

3. Find the harmonic mean of the solutions to $z^4 - 4z^3 + 8z^2 + 28z + 1 = 0$.

4. Find $\sin 1^\circ \cos 89^\circ + \sin 2^\circ \cos 88^\circ + ... + \sin 89^\circ \cos 1^\circ$

Practice problems:

1. The number 2008 is a four-digit number consisting of four even digits whose sum is 10. Compute the number of integers in {1000, 1001, 1002, …, 9999} that consist of four even digits whose sum is 10. [NYSML 2008 I2]

2. Compute the sum of the reciprocals of the <u>real</u> solutions to $x^5 - 4x^4 + 3x^3 - 8x^2 + 32x - 24 = 0$. [NYSML 2008 I4]

3. Compute $\sin(2008^\circ)\cos(28^\circ) - \sin(28^\circ)\cos(2008^\circ)$. [NYSML 2008 I5]

4. Compute the ordered pair of positive integers (a,b) for which $\sqrt{151+28\sqrt{3}} = a + b\sqrt{3}$. [NYSML 2007 I2]

5. Compute the sum of all the positive integer factors of 2006. [NYSML 2006 I2]

6. Compute the sum of the reciprocals of the solutions to $x^5 + 5x^4 - 5x^3 - 25x^2 + 4x + 20 = 0$. [NYSML 2006 I5]

7. Compute all real roots of $x^2 + 2x - 25 = \sqrt{4x^2 + 8x - 40}$. [NYSML 2006 I7]

8. For each of the four positions in a 2 by 2 determinant, flip a fair coin and insert 1 in the position if the result is HEADS and -1 in the position if the result is TAILS. Let M be the maximum possible value of the determinant, and let p be the probability that the determinant will take on that maximum value. Compute the ordered pair (M, p).