Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2025 – 2026

Calculators are allowed.

Time: 10 minutes **Name: _____**

19. Compute $\sqrt{\frac{3125}{\sqrt{\frac{100}{\sqrt{\frac{160}{\sqrt{\frac{500}{\sqrt{25}}}}}}}}}$.

20. Compute the smallest positive integer x such that $84 \cdot x$ is the cube of an integer.

19. ☐ 20. ☐

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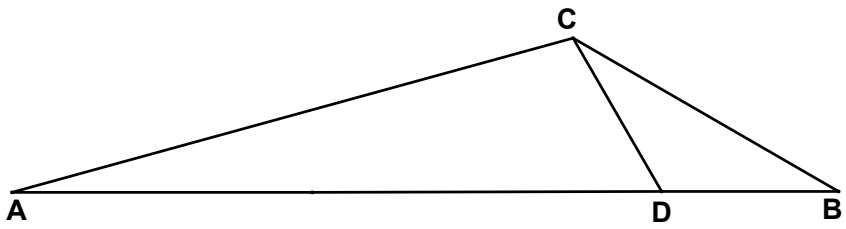
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Time: 10 minutes **Name: _____**

21. Otis Elevators' stock price has its ups and downs. First, the stock price dropped 50% and then went up x% back to the original price. Then, the price dropped 60% and then returned to the original price by going up y%. Finally, the price dropped 80% and returned to the original price by going up z%. Compute $x + y + z$.

22. In the accompanying diagram of ΔABC, point D is on $\overline{AB}$, $m\measuredangle A = 15°$, $m\measuredangle B = 30°$, $m\measuredangle ADC = 60°$, and $DB = 10$. If, in simplest form, $AD = a + b\sqrt{c}$, compute $ab + c$.



21. ☐ 22. ☐

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Time: 10 minutes **Name: _____**

23. If $3^x = 27^y = 81^z$ and $xyz = 144$, compute $x + y + z$.

24. After the math contest, your team is putting away 17 identical pencils into 5 cases. In how many ways can this be done if each case must contain at least two pencils?

23. ☐ 24. ☐

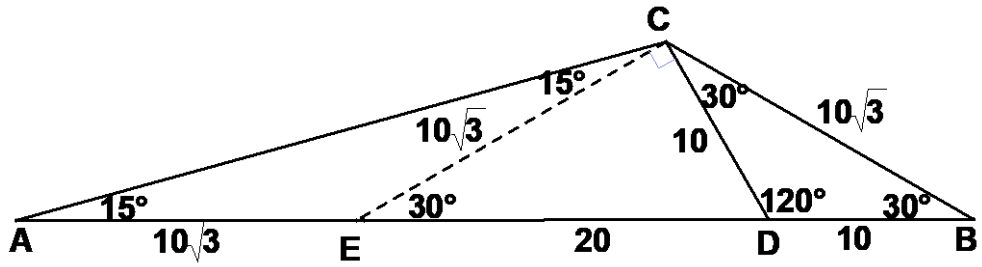
Solutions for Contest #4

19. Start from the innermost part: $\sqrt{\frac{500}{\sqrt{25}}} = 10 \rightarrow \sqrt{\frac{160}{10}} = 4 \rightarrow \sqrt{\frac{100}{4}} = 5 \rightarrow \sqrt{\frac{3125}{5}} = \mathbf{25}.$.

20. For an integer to be a perfect cube, the exponent of each prime in its prime factorization must be a multiple of 3. Since $84 = 2^2 \cdot 3^1 \cdot 7^1$, $x = 2^1 \cdot 3^2 \cdot 7^2 = \mathbf{882}$.

21. Without loss of generality, let the original stock price be \$100. After the 50% price drop, the stock's price is \$50. Then, $50\left(1 + \frac{x}{100}\right) = 100 \rightarrow x = 100$. After the 60% drop, the stock's price is \$40. Then, $40\left(1 + \frac{y}{100}\right) = 100 \rightarrow \frac{2y}{5} = 60 \rightarrow y = 150$. After the 80% drop, the stock's price is \$20. Then $20\left(1 + \frac{z}{100}\right) = 100 \rightarrow \frac{z}{5} = 80 \rightarrow z = 400$.
Thus, $x + y + z = \mathbf{650}$.

22. Note that $m\measuredangle BDC = 120°$, $m\measuredangle BCD = 30°$, and $CD = 10$. Use the law of cosines (or special right triangles) in ΔBCD to determine that $BC = 10\sqrt{3}$. Define point E on $\overline{AD}$ so that $\measuredangle DCE$ is a right angle. Then $m\measuredangle CED = 30° \rightarrow CE = 10\sqrt{3}$ and $DE = 20$. In $\Delta ACE, m\measuredangle ACE = 15° \rightarrow AE = 10\sqrt{3} \rightarrow AD = 20 + 10\sqrt{3}$. The required sum is **203**.



23. Re-write the given as $3^x = 3^{3y} = 3^{4z} \rightarrow y = \frac{x}{3}$ and $z = \frac{x}{4} \rightarrow xyz = x \cdot \frac{x}{3} \cdot \frac{x}{4} = 144 \rightarrow$

$x^3 = 12^3 \rightarrow x = 12, y = 4, z = 3$. The required sum is **19**.

24. Using stars and bars, treat the pencils as stars and the cases (minus 1) as bars. Since each case has to contain at least two pencils, take away 10 pencils to guarantee this requirement. Finding the number of ways to separate them is the combination ${}_nC_r$ where n is the number of remaining pencils + the number of cases minus 1, and r is the number of cases minus 1. Thus, ${}_{11}C_4 = \mathbf{330}$.