Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2025 – 2026

No calculators are allowed.

Time: 10 minutes **Name: _____**

13. Compute $\sqrt[3]{\sqrt[4]{31^{24}}}$.

14. At Problem Solving High School, each of 218 students voted on two issues in a school election. There were 159 votes in favor of the first issue and 129 votes in favor of the second issue. Exactly 39 students voted against both issues. Compute the number of students who voted in favor of both issues.

13. ☐ 14. ☐

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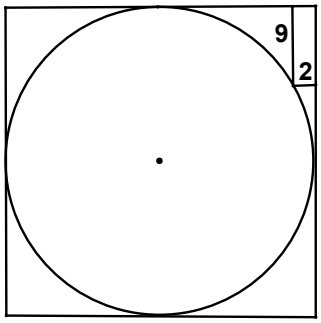
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Time: 10 minutes **Name: _____**

15. Compute $497 \cdot 499 - 496 \cdot 498$.

16. A circle is inscribed in a square as depicted in the diagram. A point is chosen on the circle that is 2 inches from the right side of the square and 9 inches from the top side of the square. Compute the length of the radius of the circle.



15. ☐ 16. ☐

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17. The sum of the first five terms of an arithmetic sequence is 150 less than the sum of the next five terms of the arithmetic sequence. Compute the positive difference between any two consecutive terms of the arithmetic sequence.

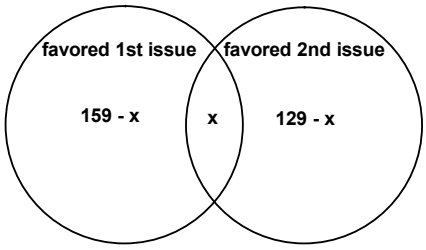
18. If $S = \frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \cdots + \frac{21}{10^2 \cdot 11^2}$ is expressed in simplest $\frac{p}{q}$ form, compute $p + q$.

17. ☐ 18. ☐

Solutions for Contest #3

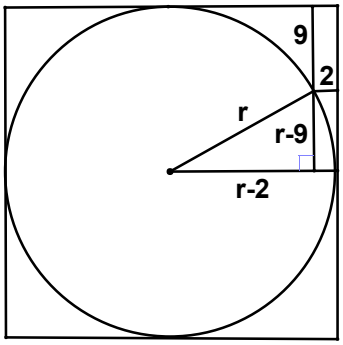
13. $\sqrt[3]{\sqrt[4]{31^{24}}} = \sqrt[3]{31^6} = 31^2 = \mathbf{961}$.

14. There were $218 - 39 = 179$ students who voted in favor of at least one issue. Among them, 159 favored the first issue and 129 favored the second issue. Thus, there were $159 + 129 = 288$ students who were in favor of at least one issue, counting twice those students who were in favor of both issues. Therefore, $288 - 179 = 109$ were in favor of both issues. Alternatively, using a Venn diagram, if x represents the number of students in favor of both issues, then $159 - x + x + 129 - x = 218 - 39 \rightarrow x = \mathbf{109}$.



15. Let $x = 497$, then $x + 2 = 499, x + 1 = 498$, and $x - 1 = 496$. Re-write the given expression as $x(x+2) - (x-1)(x+1) = x^2 + 2x - (x^2 - 1) = 2x + 1 \rightarrow 2 \cdot 497 + 1 = \mathbf{995}$.

16. Draw radii to the given point and to the right side of the square. Then, draw a perpendicular from the given point to the horizontal radius. From the Pythagorean Theorem in that right triangle: $r^2 = (r-2)^2 + (r-9)^2 \rightarrow r^2 = r^2 - 4r + 4 + r^2 - 18r + 81 \rightarrow 0 = r^2 - 22r + 85 \rightarrow (r-17)(r-5) = 0 \rightarrow r = \mathbf{17}$, only.



17. If the first term is a and the positive difference is d, then $a + (a+d) + (a+2d) + (a+3d) + (a+4d) = (a+5d) + (a+6d) + (a+7d) + (a+8d) + (a+9d) - 150 \rightarrow 5a + 10d = 5a + 35d - 150 \rightarrow 25d = 150 \rightarrow d = \mathbf{6}$.
Alternatively, without loss of generality, let $a = 0$, so $10d = 35d - 150 \rightarrow d = 6$.

18. Note that $\sum_{n=1}^{10} \frac{2n+1}{n^2 \cdot (n+1)^2} = \sum_{n=1}^{10} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \frac{1}{10^2} - \frac{1}{11^2} = 1 - \frac{1}{11^2} = \frac{120}{121}$. The required sum is $\mathbf{241}$.