Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2024 – 2025

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

25. Compute $\sqrt{123^2 + 2 \cdot 123 \cdot 678 + 678^2}$

26. The area of a triangle whose perimeter is 8 and each of whose side lengths is an integer can be expressed in simplest form as $p\sqrt{q}$. Compute $(pq)^3$.

25. ☐

26. ☐

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27. Point P, whose coordinates are $(k, 0)$ is equidistant from the origin and from point Q, whose coordinates are $(2,4)$. Compute k.

28. If $f(x) = (x+1)^3 - x^3$ and $f(0) + f(1) + f(2) + \cdots + f(99) = a \cdot 10^6$, compute a.

27. [ ]

28. [ ]

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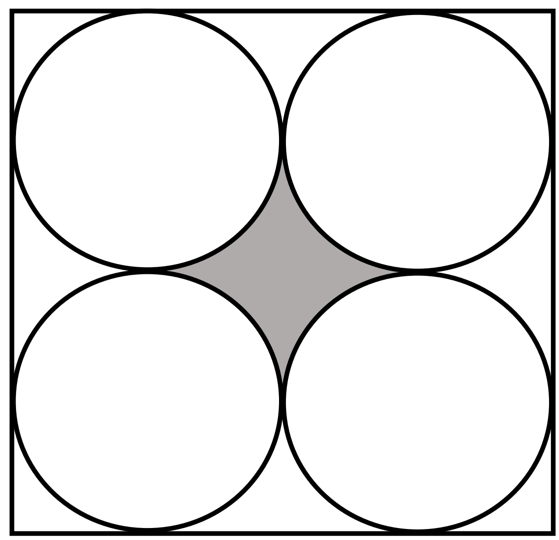
29. Compute the sum of the members of the set of all positive two-digit numbers such that if the digits of each number are reversed, the result is another two-digit number that is 75% greater than the original.

30. In the accompanying diagram, the area of the shaded region is $64 - 16\pi$. Each of the congruent circles in the diagram is tangent to two sides of the square and to two adjacent circles. Compute the area of the square.



29.

30.

Solutions for Contest #5

25. The given expression is an expanded form of $\sqrt{(123+678)^2} = \sqrt{801^2} = \mathbf{801}$.

26. Since the perimeter of the triangle with integer side lengths is 8, no side length can exceed 3 because then the sum of the remaining side lengths would be 4 or less, contradicting the triangle inequality. Also, not all three sides can be less than 3 because that would make the perimeter less than 6. So, one side-length is 3 and the others sum to 5. Therefore, the triangle is isosceles with side-lengths of 3, 3, and 2. The altitude to the side whose length is 2 must be $\sqrt{8}$ from the Pythagorean Theorem. The area of the triangle is one-half the product of 2 and $\sqrt{8}$ or $2\sqrt{2} \rightarrow p = 2$ and $q = 2$. The required response is **64**.

27. From the distance formula, $\sqrt{(k-2)^2 + (0-4)^2} = k \rightarrow k^2 - 4k + 4 + 16 = k^2 \rightarrow$ $4k = 20 \rightarrow k = \mathbf{5}$.

Alternate Solution: Let point R be the origin. The equation of line RQ is $y = 2x$. The midpoint of segment $\overline{RQ}$ is $(1, 2)$. The equation of the line perpendicular to $\overleftrightarrow{RQ}$ is:

$y - 2 = -\frac{1}{2}(x-1)$. To find k, replace y with 0 and x with $k \rightarrow k = 5$.

28. Start with $f(0) + f(1) + f(2) + \cdots + f(99) = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \cdots +$ $(100^3 - 99^3)$. After appropriate cancellation, the sum is $100^3 = 10^6 = 1 \cdot 10^6$ and $a = \mathbf{1}$.

29. Each original two-digit number is represented by $10t + u$, where t is the tens digit and u is the units digit. Then, $10u + t = 1.75(10t + u) \rightarrow 40u + 4t = 70t + 7u \rightarrow$ $33u = 66t \rightarrow u = 2t$. Thus, the original set is $\{12, 24, 36, 48\}$ and the required sum is **120.**

30. Let the radius of each circle be r. The area of the square is the sum of the areas of the four circles and 4 times the area of the shaded region. Therefore, $(4r)^2 = 4\pi r^2 + 4(64 - 16\pi) \rightarrow 16r^2 = 4\pi r^2 + 256 - 64\pi \rightarrow$ $16r^2 - 4\pi r^2 = 256 - 64\pi \rightarrow r^2 = \frac{256-64\pi}{16-4\pi} = 16 \rightarrow r = 4$. Then the side of the square is 16 and its area is **256**.