Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2024 – 2025

Calculators are allowed.

Time: 10 minutes **Name: ________________________________**

19. Kevin has a 20-ounce drink of coffee and milk that is 60% coffee. He adds milk until his drink is only 40% coffee. Compute the number of ounces of milk that Kevin added to his drink.

20. Compute the radius of the circle with the following equation:
$x^2 + y^2 + 14x - 12y + 60 = 0$.

19. ☐ 20. ☐

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21. In trapezoid $WXYZ$, sides $\overline{WX}$ and $\overline{ZY}$ are parallel, $WZ = ZY$, and $WY = WX$. If $m\measuredangle WXY = 70°$, compute the number of degrees in $m\measuredangle WZY$.

22. Compute the sum of the positive integer values of x such that $x^2 + 75$ is a perfect square.

21. [ ]

22. [ ]

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23. If $\sqrt{1+\sqrt{1+\sqrt{x}}} = 2$, compute x.

24. In ΔABC, $AB = 12$, $BC = 4\sqrt{13}$, and $AC = 8$. The angle bisector from point A intersects side $\overline{BC}$ at Q and the median from point C intersects side $\overline{AB}$ at P. The intersection of the angle bisector and the median is R. Let PR be the fraction $\frac{a}{b}$, in simplest terms. Compute ab.

23. 24.

Solutions for Contest #4

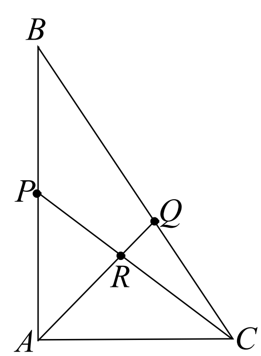
19. If Kevin adds x ounces of milk, then $\frac{12}{20+x} = \frac{2}{5} \rightarrow 60 = 40 + 2x \rightarrow x = \mathbf{10}$.

20. Rearrange the given equation: $x^2 + 14x + y^2 - 12y = -60 \rightarrow$ $x^2 + 14x + 49 + y^2 - 12y + 36 = -60 + 49 + 36 = 25 \rightarrow (x+7)^2 + (y-6)^2 = 5^2$. So, the radius of the circle is **5.**

21. Since ΔWXY is isosceles, $m\measuredangle WYX = 70°$ and $m\measuredangle YWX = 40°$. Since $\overline{WX}$ and $\overline{ZY}$ are parallel, $m\measuredangle ZYW = 40°$ and because ΔZWY is isosceles, $m\measuredangle ZWY = 40°$. Thus, $m\measuredangle WZY = \mathbf{100}°$.

22. If one positive integer that meets the condition of the problem is y, then $y^2 = x^2 + 75 \rightarrow y^2 - x^2 = 75 \rightarrow (y+x)(y-x) = 75$. This can be separated into several pairs of equations: $y + x = 75,\ y - x = 1 \rightarrow (x,y) = (37{,}38)$; $y + x = 25$, $y - x = 3 \rightarrow (x,y) = (11{,}14)$; $y + x = 15,\ y - x = 5 \rightarrow (x,y) = (5{,}10)$. The sum of the x values is $37 + 11 + 5 =$ **53.**

23. $\sqrt{1+\sqrt{1+\sqrt{x}}} = 2 \rightarrow 1 + \sqrt{1+\sqrt{x}} = 4 \rightarrow \sqrt{1+\sqrt{x}} = 3 \rightarrow 1 + \sqrt{x} = 9 \rightarrow \sqrt{x} = 8 \rightarrow$ $x = \mathbf{64}$.



24. Since $12^2 + 8^2 = \left(4\sqrt{13}\right)^2$ the given triangle is a right triangle with hypotenuse BC. The median divides AB into 2 equal parts so $AP = 6$. Apply the Pythagorean theorem in right ΔAPC to find that $PC = 10$. Use the angle bisector theorem to find PR: $\frac{PA}{AC} = \frac{6}{8} = \frac{3}{4}$ so PR is $\frac{3}{7}$ of PC and RC is $\frac{4}{7}$ of PC. It follows that $\frac{a}{b} = \frac{30}{7}$ so $ab = 30 \times 7 = \mathbf{210}$.