Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2024 – 2025

No calculators are allowed.

Time: 10 minutes **Name: __________________________________**

13. Compute $\sqrt{5} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{14} \cdot \sqrt{15} \cdot \sqrt{7}$.

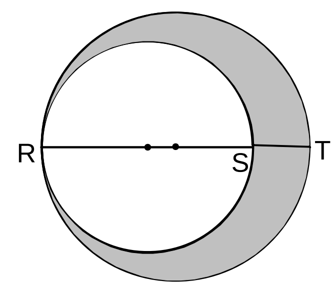
14. A box is a cube with edges whose lengths are each 4 feet. The box is resting on a horizontal surface. Point X is on an edge of the box and is 1 foot above the box's bottom. Point Y is on the edge furthest from point X and is 1 foot below the box's top. Compute the distance XY.

13. ☐ 14. ☐

Time: 10 minutes **Name: ______________________________**

15. If $9{,}991 = pq$ where p and q are prime numbers, compute $p + q$.

16. In the figure, the center of each circle is a point on $\overline{RST}$ and $\frac{RS}{ST} = 4$. If the area of the smaller circle is 144, compute the area of the shaded region.



15. ☐ 16. ☐

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17. The sum of the first 5 terms of an arithmetic sequence equals the sum of the sixth and the seventh term. Compute the value of the third term if the fourth term is 80.

18. The roots of $x^2 + bx + c = 0$ are the squares of the roots of $x^2 + 3x + 5 = 0$. Compute $b + c$.

17. ☐ 18. ☐

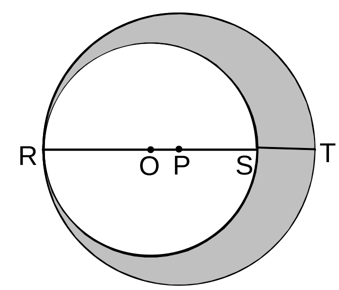
Solutions for Contest #3

13. Re-arrange the terms in the given expression as $\sqrt{5}\cdot\sqrt{3}\cdot\sqrt{15}\cdot\sqrt{2}\cdot\sqrt{7}\cdot\sqrt{14} =$ $\sqrt{15}\cdot\sqrt{15}\cdot\sqrt{14}\cdot\sqrt{14} = 15\cdot 14 = \mathbf{210}$.

14. Points X and Y can be viewed as opposite vertices of a 2 by 4 by 4 box and $XY = \sqrt{2^2+4^2+4^2} = \sqrt{36} = 6$. Alternatively, using 3-dimensional coordinates, if point X is (0,0,1), then point Y is (4,4,3). So, using the distance formula, $XY = \sqrt{(4-0)^2+(4-0)^2+(3-1)^2} = \mathbf{6}$.

15. Since, $9991 = 10000 - 9 = 100^2 - 3^2 = (100+3)(100-3) = 103\cdot 97$, and each number is prime, the required sum is **200**.

16. Let point O be the center of the smaller circle and let point P be the center of the larger circle. Since all circles are similar the ratio of the areas of 2 circles equals the square of the ratios of their radii. $\frac{A_O}{A_P} = \left(\frac{OS}{PT}\right)^2$ so $\frac{144}{A_P} = \left(\frac{2x}{2.5x}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$. Thus the area of circle $P =$ 225 and the area of the shaded region is 225 – 144 = **81**.



Alternate Solution: Let point O be the center of the smaller circle and let point P be the center of the larger circle. Also, let $OR = OS = 2x$. Then $ST = x$ and $OP = \frac{x}{2}$. The area of circle O is $\pi(2x)^2 = 144 \rightarrow 4\pi x^2 = 144 \rightarrow \pi x^2 = 36$. The area of circle P is $\pi\left(\frac{5x}{2}\right)^2 = \frac{25\pi x^2}{4} = \frac{25}{4}\cdot 36 = 225$. The area of the shaded region is $225 - 144 = \mathbf{81}$.

17. If the first term is a_1 and the difference is d, then $a_1 + (a_1+d) + (a_1+2d) + (a_1+3d) + (a_1+4d) = (a_1+5d) + (a_1+6d) \rightarrow$ $5a_1 + 10d = 2a_1 + 11d \rightarrow d = 3a_1$. But, $a_1 + 3d = 80 \rightarrow a_1 + 9a_1 = 80 \rightarrow$ $10a_1 = 80 \rightarrow a_1 = 8$ and $d = 24$. Therefore, the third term is 8 + 24 + 24 = **56.**

18. The roots of $x^2+3x+5=0$ are $\frac{-3\pm\sqrt{-11}}{2}$. Let r_1 and r_2 be the roots of $x^2+bx+c=0$. Then, $r_1 = \left(\frac{-3+\sqrt{-11}}{2}\right)^2 = \frac{-1-3\sqrt{-11}}{2}$ and $r_2 = \left(\frac{-3-\sqrt{-11}}{2}\right)^2 = \frac{-1+3\sqrt{-11}}{2} \rightarrow r_1 + r_2 = -1$ and $r_1 \cdot r_2 = 25 \rightarrow b = 1$ and $c = 25 \rightarrow b + c = \mathbf{26}$.