Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2024 – 2025

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

1. Compute $\frac{4}{5}$ of 75% of 120% of 675.

2. A geometric sequence is defined as a sequence such that the quotient of 2 consecutive terms is a constant. For $x > 0$, if $a_1 = x - 5$, $a_2 = \sqrt{5x}$, and $a_3 = x + 24$ form a geometric sequence, compute a_3.

1. ☐

2. ☐

Time: 10 minutes **Name: __________________________________**

3. If 378 is divided by an odd number n, the result is a prime number. Compute n.

4. Fifteen racquetballs are placed in a row and are all indistinguishable except for their color. There are 10 black and 5 purple racquetballs. Compute the number of possible row arrangements of all 15 racquetballs if no two purple racquetballs are adjacent to each other.

3.

4.

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5. Compute n^2 if $n = \frac{7\sqrt{5}+5\sqrt{7}}{\sqrt{7}+\sqrt{5}}$.

6. If $x + y = 2$ and $2^x + 2^y = 6$, compute $16^x + 16^y$.

5. [ ]

6. [ ]

Solutions for Contest #1

1. Rewrite the given expression as $\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{6}{5} \cdot 675 = \frac{18}{25} \cdot 675 = 18 \cdot 27 = \mathbf{486}$.

2. The second term in the geometric sequence is the mean proportional between the first term and the third term. Thus, $5x = (x-5)(x+24) \rightarrow 5x = x^2 + 19x - 120 \rightarrow$ $x^2 + 14x - 120 = 0 \rightarrow (x-6)(x+20) = 0 \rightarrow x = 6$ and $a_3 = 6 + 24 = \mathbf{30}$.

3. Since 2 divides 378 resulting in 189, and since 2 is prime and 189 is odd, n must be **189**.

   Alternate Solution: Since $378 = 2 \cdot 3^3 \cdot 7$, 378 cannot be successfully divided by any of the factors that include a 2 because the divisor will not be odd. Therefore, the divisor must be $3^3 \cdot 7 = \mathbf{189}$.

4. Place the 10 black racquetballs in a row. Each purple racquetball must be placed either between 2 black racquetballs or at an end of the row of black racquetballs. Therefore, there are 11 available positions for purple racquetballs. Thus, the requirements of the problem are met in $\binom{11}{5} = \mathbf{462}$ ways.

5. Multiply both numerator and denominator by the conjugate of the denominator.
   $\left(\frac{7\sqrt{5}+5\sqrt{7}}{\sqrt{7}+\sqrt{5}}.\right)\left(\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}\right) = \frac{7\sqrt{35}+35-35-5\sqrt{35}}{2} = \sqrt{35}$. Then, $n^2 = \mathbf{35}$.

6. Square the second given equation: $2^{2x} + 2^{2y} + 2 \cdot 2^x \cdot 2^y = 36 \rightarrow$
   $2^{2x} + 2^{2y} + 2 \cdot 2^{(x+y)} = 36 \rightarrow 2^{2x} + 2^{2y} + 2 \cdot 2^2 = 36 \rightarrow 2^{2x} + 2^{2y} + 8 = 36 \rightarrow$
   $2^{2x} + 2^{2y} = 28$. Now, squaring both sides, $2^{4x} + 2^{4y} + 2 \cdot 2^{2(x+y)} = 784 \rightarrow$
   $2^{4x} + 2^{4y} + 2 \cdot 2^{2 \cdot 2} = 784 \rightarrow 16^x + 16^y + 32 = 784 \rightarrow 16^x + 16^y = \mathbf{752}$.