Nassau County Interscholastic Mathematics League

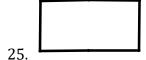
Contest #5 Answers must be integers from 0 to 999, inclusive. 2023 - 2024

No calculators are allowed.

| Time: 10 minutes | Name: |
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25. Compute the value of the sum x + y if $3^x = 27^{y+1}$ and $4^y = 2^{x-9}$.

26. Fred, working alone, can build a dinosaur nest in 5 hours. Wilma, working alone, can build the same dinosaur nest in 4 hours. If Fred works alone for 2 hours and then Wilma and Fred work together, compute the number of **minutes** it will take for them to complete the job working together.





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27. Given that n and k are positive integers, and that $125! = 5^n(k)$. Compute the maximum possible value of n. (Note: 125! is 125 factorial).

28. A circle is tangent to a line whose equation is y = -1.25x + 20.5 at the point whose coordinates are (10,8). The center of the circle is on the line whose equation is y = x - 1. Compute the square of the radius of the circle.





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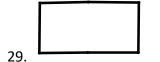
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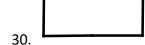
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| Time: 10 minutes | Name: |

29. The area of a square in which the length of its diagonal is one inch longer than the length of its side can be expressed as $a + b\sqrt{c}$ square inches in simplest radical form. Compute the product: abc.

30. Two people will meet for dinner. Each of them arrives at a random time between 7:00 PM and 8:00 PM. If the probability that they arrive within 15 minutes of each other, in simplest form, is $\frac{p}{q}$, compute the sum: p+q.





Solutions for Contest #5

- 25. $3^x = 27^{y+1} \rightarrow 3^x = (3^3)^{y+1} \rightarrow x = 3y + 3$ and $4^y = 2^{x-9} \rightarrow 2^{2y} = 2^{x-9} \rightarrow 2y = x 9 \rightarrow x = 2y + 9$. Then, $3y + 3 = 2y + 9 \rightarrow y = 6$ and x = 21. The required sum is **27**.
- 26. After Fred worked for 2 hours, three-fifths of the job remained. Each hour after that, Fred can do one-fifth of the job and Wilma can do one-fourth of the job. Together, they do $\frac{9}{20}$ of the job each hour. Then, $\frac{3}{5} = \frac{4}{3}$ hours or **80** minutes. Alternatively, if x represents the number of hours Fred and Wilma work together, then $\frac{x}{5} + \frac{x}{4} = \frac{3}{5} \to 4x + 5x = 12 \to 9x = 12 \to x = \frac{4}{3}$ hours or 80 minutes. Another solution: Use the equation $\frac{t+2}{5} + \frac{t}{4} = 1$, where t represents the number of hours Fred and Wilma work together.
- 27. There is at least one factor of 5 in each of 5, 10, 15, ..., 120, 125 (count: 25). There is a second factor of 5 in each of 25, 50, 75, 100, 125 (count: 5). There is a third factor of 5 in 125 (count: 1). The answer is 25 + 5 + 1 = 31.
- 28. Call the point of tangency point P and call the center of the circle point O. The coordinates of point O are (a, a 1). Then, since the slopes of the radius and tangent line are opposite reciprocals, $\frac{a-1-8}{a-10} = \frac{4}{5} \rightarrow 5a 45 = 4a 40 \rightarrow a = 5, a 1 = 4$. The center of the circle is (5,4) and $OP = \sqrt{(10-5)^2 + (8-4)^2} = \sqrt{41} \rightarrow (OP)^2 = 41$.
- 29. If the length of a side of the square is x and the length of a diagonal is x+1, then by the Pythagorean Theorem, $x^2+x^2=(x+1)^2 \to 2x^2=x^2+2x+1 \to x^2-2x-1=0 \to x=1+\sqrt{2} \to x^2=3+2\sqrt{2}$. This is the square's area and the required product is **12**.
- 30. Two people arrive respectively x minutes and y minutes after 7 PM. To satisfy the problem, $|x-y| \le \frac{1}{4} \to x \frac{1}{4} \le y \le x + \frac{1}{4}$. Generally, this means that a random point is to be chosen within the unit square on $[0,1] \times [0,1]$ and in the region between the lines $y = x \frac{1}{4}$ and $y = x + \frac{1}{4}$. The area of the region between the lines and in the square is $1 2 \cdot \frac{1}{2} \cdot \left(\frac{3}{4}\right)^2 = \frac{7}{16}$ and the required sum is **23.**