Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2023 – 2024

Calculators are allowed.

Time: 10 minutes **Name: ______________________________**

19. Compute the area of the region bounded by the lines whose equations are $y = \frac{1}{2}x$, $y = 4x$, and $x = 2$.

20. If $\frac{a}{x+2} + \frac{b}{2x-3} = \frac{5x-11}{2x^2+x-6}$, compute $a^2 + b^2$.

19. ☐

20. ☐

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21. Compute x so that $\frac{1}{3-\frac{x}{1-x}} = \frac{3}{15}$.

22. A cabinet contains n socks and 18 of them are striped. Two socks are simultaneously removed from the cabinet in the dark. Compute the largest possible value for n if the probability that both socks are striped is greater than $\frac{1}{2}$.

21. ☐ 22. ☐

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23. If $3^{2025} - 3^{2024} + 3^{2023} - 3^{2022} = x \cdot 3^{2022}$, compute x.

24. The sum of the solutions (in radians) of $\sin x + \cos x = \sqrt{\frac{3}{2}}$ for $0 \le x \le \pi$ is $n\pi$. Compute $10n$.

23.

24.

Solutions for Contest #4

19. Each of the first two lines intersects the third line in the points whose coordinates are (2,1) and (2,8). The first two lines intersect at the origin. The region bounded by these three lines is a triangle. The length of the base of the triangle is 7 and the length of the altitude is 2. Thus, the required area is **7.**

20. $\frac{a}{x+2} + \frac{b}{2x-3} = \frac{5x-11}{(x+2)(2x-3)} \rightarrow a(2x-3) + b(x+2) = 5x - 11 \rightarrow 2ax - 3a + bx + 2b =$ $5x - 11 \rightarrow 2a + b = 5$ and $-3a + 2b = -11 \rightarrow -3a + 2(5 - 2a) = -11 \rightarrow$ $-7a + 10 = -11 \rightarrow a = 3, b = -1 \rightarrow a^2 + b^2 = \mathbf{10}.$ Alternatively, by the Heaviside method starting with $a(2x-3) + b(x+2) = 5x - 11$:
If $x = -2, -7a = -21 \rightarrow a = 3$ and if $x = \frac{3}{2}, \frac{7}{2}b = \frac{15}{2} - 11 \rightarrow 7b = 15 - 22 \rightarrow b = -1.$
So, $a^2 + b^2 = 10.$

21. Since $\frac{1}{3-(-2)} = \frac{1}{5}$ and $\frac{3}{15} = \frac{1}{5}$, then $\frac{x}{1-x} = -2 \rightarrow -2 + 2x = x \rightarrow x = \mathbf{2}.$ Alternatively, cross-multiply to get $9 - \frac{3x}{1-x} = 15 \rightarrow 9 - 9x - 3x = 15 - 15x \rightarrow 3x = 6 \rightarrow x = 2.$

22. The probability that the first sock is striped is $\frac{18}{n}$ and the probability that the second sock is striped is $\frac{17}{n-1}$. Since it is required that both socks be striped, $\frac{18}{n} \cdot \frac{17}{n-1} > \frac{1}{2} \rightarrow$ $2 \cdot 18 \cdot 17 > n(n-1) \rightarrow n(n-1) < 612.$ Realize that the square of 25 is 625 and that the product of 24 and 25 is less than 612. So, the largest possible value for n is **25.**

23. Factor out 3^{2022} and divide by that number to result in $3^3 - 3^2 + 3 - 1 = x \rightarrow x = \mathbf{20}.$

24. Square both sides to obtain $(\sin x)^2 + 2 \sin x \cos x + (\cos x)^2 = \frac{3}{2} \rightarrow$ $1 + \sin 2x = \frac{3}{2} \rightarrow \sin 2x = \frac{1}{2}$ and $0 \leq 2x \leq 2\pi$. So, $2x = \frac{\pi}{6}$ or $\frac{5\pi}{6} \rightarrow x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$.
The sum of these roots is $\frac{\pi}{2}$, so $10n = 10\left(\frac{1}{2}\right) = \mathbf{5}.$