Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2023 – 2024

No calculators are allowed.

Time: 10 minutes **Name: ____________________________________**

13. Compute x if $\sqrt{4x^3 + 4x^3 + 4x^3 + 4x^3} = 256$.

14. The fifth term of a geometric sequence is 30 and the eleventh term is 12. If the eighth term, in simplest radical form, is expressed as $p\sqrt{q}$, compute p^2q.

13. ☐ 14. ☐

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15. Andy picked $\frac{1}{5}$ of the apples in an orchard. Barbara then picked $\frac{1}{6}$ of the remaining apples. She left 250 apples in the orchard. Compute the number of apples in the orchard originally.

16. Compute the number of square units in the area of a rhombus if the sum of the lengths of its diagonals is 56 units and the absolute value of the difference of their lengths is 8 units.

15. ☐ 16. ☐

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17. Compute k if the number of minutes in the month of February when it contains exactly 28 days is expressed as $k!$ (k factorial).

18. Kevin rolls a fair die and notes the result. He challenges Tom to roll the die until he matches Kevin's result. If the probability that it will take Tom at least 4 tosses until he succeeds may be expressed in simplest form as $\frac{p}{q}$, compute $p + q$.

17. [ ]

18. [ ]

Solutions for Contest #3

13. Re-write the given equation as $\sqrt{4 \cdot 4x^3} = 4^4 \rightarrow 4\sqrt{x^3} = 4^4 \rightarrow \sqrt{x^3} = 4^3 \rightarrow x^3 = 4^6 \rightarrow$ $x = 4^2 = \mathbf{16}$. Alternatively, square both sides so that $4(4x^3) = 256^2 = 16^4$. So, $x^3 = 16^3 \rightarrow x = 16$.

14. In a geometric sequence, the eighth term is the geometric mean between the fifth term and the eleventh term. Therefore, the eighth term is $\sqrt{30 \cdot 12} = \sqrt{360} = 6\sqrt{10} \rightarrow p^2q = \mathbf{360.}$ Alternatively, using the formula $a_n = ar^{n-1}$ where a is the first term and r is the common ratio, solve the system $12 = ar^{10}$ and $30 = ar^4$ for a and r. Then find $a_8 = ar^7$.

15. The 250 remaining apples are $\frac{5}{6}$ of the apples that Barbara found in the orchard. She found $\frac{6}{5} \cdot 250 = 300$ apples. This represents $\frac{4}{5}$ of the number that Andy found originally. So, there were $\frac{5}{4} \cdot 300 = \mathbf{375}$ apples originally. Alternatively, let x be the number of apples originally, let $\frac{1}{5}x$ be the number of apples Andy picked, and let $\frac{1}{6}\left(\frac{4}{5}x\right)$ be the number of apples Barbara picked. So, $x - \frac{1}{5}x - \frac{4}{30}x = 250 \rightarrow \frac{2}{3}x = 250 \rightarrow x = 375$. Another solution: Let $30x$ be the number of apples originally. Let $6x$ be the number of apples Andy picked, and let $\frac{1}{6}(24x) = 4x$ be the number of apples Barbara picked. So, $30x - 6x - 4x = 250 \rightarrow x = 12.5$. Thus, $30x = 375$.

16. Let the lengths of the diagonals of the rhombus be d_1 and d_2 with $d_1 > d_2$. Then $d_1 + d_2 = 56$ and $d_1 - d_2 = 8 \rightarrow d_1 = 32$ and $d_2 = 24$. The formula for the area of a rhombus is $K = \frac{d_1 d_2}{2} = \frac{32 \cdot 24}{2} = \mathbf{384.}$

17. February can have 28 days times 24 hours per day times 60 minutes per hour = $28 \cdot 24 \cdot 60$ minutes. Then $28 \cdot 24 \cdot 60 = 7 \cdot 4 \cdot 3 \cdot 8 \cdot 2 \cdot 5 \cdot 6 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$. Therefore $k = \mathbf{8}$.

18. The problem is equivalent to requiring 3 consecutive failures on the first 3 rolls or $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$. The required sum is $\mathbf{341.}$