Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2023 – 2024

Calculators are allowed.

Time: 10 minutes **Name: _____**

7. When 9 gallons of gasoline are put into the tank of a vehicle, the gasoline indicator moves from $\frac{1}{4}$ of a tankful to $\frac{5}{8}$ of a tankful. Assuming that the gas gauge is accurate, compute the total capacity of the gasoline tank in gallons.

8. Compute the sum of the digits of the integer determined by the following product:

   $384^3 \cdot 5^{20}$.

7. ☐ 8. ☐

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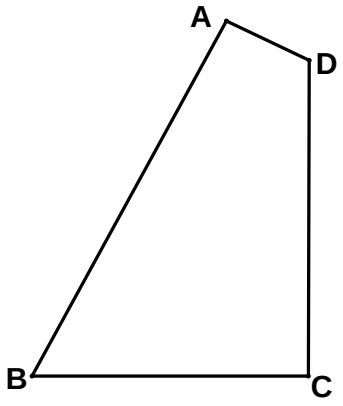
9. Compute the sum of three consecutive positive integers if their product is 29,760.

10. Quadrilateral $ABCD$ has right angles at vertices A and C.
If $m\measuredangle ADC = 120°, AD = 10,\ CD = 40$, and BC, in simplest form, is $p\sqrt{q}$, compute pq^2.



9. 10.

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11. In a club of 36 students, each student has as at least one pet, either a cat, or a dog, or both. Of the students who have cats as pets, $\frac{1}{3}$ also have dogs. Of the students who have dogs as pets, $\frac{1}{4}$ also have cats. Compute the number of students who have cats as pets.

12. Compute the sum of all positive integers x such that $x^3 - 2$ is a multiple of $x - 4$.

11. ☐ 12. ☐

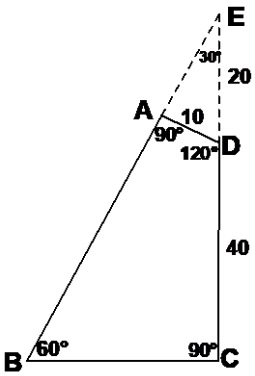
Solutions for Contest #2

7. If x is the tank's capacity, then $\frac{1}{4}x + 9 = \frac{5}{8}x \rightarrow \frac{3}{8}x = 9 \rightarrow x = \mathbf{24}$.
   Alternatively, nine gallons of gasoline fill $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$ of the tank. So, $\frac{1}{8}$ of the tank is filled by 3 gallons and the full tank can hold 24 gallons.

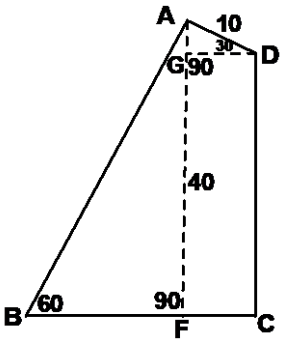
8. Re-write the product in the following manner: $384^3 \cdot 5^{20} = (3 \cdot 128)^3 \cdot 5^{20} = 3^3 \cdot (2^7)^3 \cdot 5^{20} = 3^3 \cdot 2^{21} \cdot 5^{20} = 3^3 \cdot 2 \cdot 2^{20} \cdot 5^{20} = 54 \cdot 10^{20}$. The required sum is **9**.

9. Note that 29,760 is close to 27, 000 or 30 cubed. Trial and error will yield the consecutive integers 30, 31, and 32. Their sum is **93**.
   Alternatively, let $x - 1, x$, and $x + 1$ represent the integers. So, $(x - 1)(x)(x + 1) = 29{,}760 \rightarrow x^3 - x = 29{,}760$. Solving on the calculator yields $x = 31$.

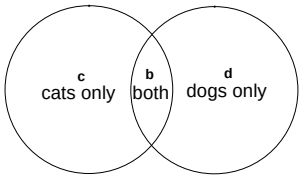
10. Extend $\overrightarrow{BA}$ and $\overrightarrow{CD}$ until they intersect at point E. Then $m\measuredangle ADE = 60° \rightarrow \Delta ADE$ is a 30–60–90 triangle and $DE = 20$. Triangle ECB is also a 30–60–90 triangle and $BC = \frac{60}{\sqrt{3}} = 20\sqrt{3}$. The required product is **180.**



   Alternatively, draw $\overline{AF} \perp \overline{BC}$ and draw $\overline{DG} \perp \overline{AF}$ creating a rectangle and two 30-60-90 triangles. $GD = 5\sqrt{3} = FC, AG = 5$, and $AF = 45$. So, $BF = \frac{45}{\sqrt{3}} = 15\sqrt{3}$. Thus, $BC = 20\sqrt{3}$ and the required product is 180.



11. In a Venn diagram, $c + d + b = 36, b = \frac{1}{3}(b + c),\ b = \frac{1}{4}(b + d) \rightarrow c = 2b$ and $d = 3b \rightarrow 2b + 3b + b = 36 \rightarrow b = 6, d = 18, c = 12 \rightarrow c + b = \mathbf{18}$.



12. Divide the former by the latter to obtain $\frac{x^3-2}{x-4} = x^2 + 4x + 16 + \frac{62}{x-4}$. The remainder must be an integer. So, $x - 4 = \pm 1,\ \pm 2,\ \pm 31$, or $\pm 62 \rightarrow x = 3, 5, 2, 6, 35$, or 66. The required sum is **117.**