Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2023 – 2024

No calculators are allowed.

Time: 10 minutes **Name: _____**

1. A class is raising money by selling raffles. Each raffle costs \$0.75 and the prize will be \$75.00. The prize money is taken from the money paid for the raffles. If the class wishes its profit to be \$75.00, compute the number of raffles the class needs to sell.

2. Steph's age is more than 9 years old and less than 100 years old. If, from his age, he subtracts the product of the digits of his age, the result is 19. Compute the product of the two different possible ages for Steph.

1. ☐ 2. ☐

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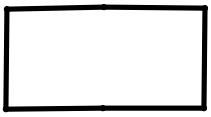
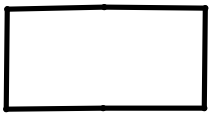
3. Each side of a square sheet of paper has length 100. A square is cut out of the paper reducing the area of the paper by 25%. Compute the length of a side of the cutout.

4. From a table of all five-digit positive integers, a five-digit positive integer is randomly selected. The probability, in simplest form, that the chosen number is divisible by each of 3, 4, and 5 with remainder equal to 0 is $\frac{p}{q}$. Compute $p + q$.

3. 

4. 

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5. Compute the units digit of 2023^{2024}.

6. Compute the number of ordered triples of positive integers (x, y, z) that are solutions to the equation $x + y + z = 9$.

5. ☐ 6. ☐

Solutions for Contest #1

1. The class needs to sell raffles for a total of $\$75 + \$75 = \$150$. So, they need to sell $\$150/\$0.75 = \mathbf{200}$ raffles. Alternatively, If x is the number of raffle tickets to be sold, then $0.75x - 75 = 75 \rightarrow x = 200$.

2. Let Steph's age in years be represented by $10t + u$. Then, $10t + u - tu = 19 \rightarrow u(1 - t) = 19 - 10t \rightarrow u = \frac{10t-19}{t-1}$. Since $10t - 19$ is odd, and u is an integer, $t - 1$ must also be odd. So, t is even. If $t = 2, u = 1$ and if $t = 4, u = 7$. So, the two ages are 21 and 47. The required product is **987**.

3. The area of the given square is 10,000. Therefore, the area of the cutout is 2,500 or 50^2. Thus, the length of a side of the cutout is **50**. Alternatively, for similar polygons, the ratio of their areas is the square of the ratio of their corresponding sides. So, $\frac{A_1}{A_2} = \frac{s_1^2}{s_2^2} = \frac{10000}{2500} = \frac{100^2}{50^2} \rightarrow s_2 = 50$.

4. For a positive integer to be divisible by each of 3, 4, and 5 with remainder equal to 0, it must be divisible by their least common multiple, or 60. There are a total of $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 90{,}000$ five-digit positive integers beginning with 10,000 and ending with 99,999. Since $\frac{90{,}000}{60} = 1500$, the requested probability is $1500/90{,}000 = 1/60$. The required sum is **61.**

5. The units digit of the powers of a number ending in 3 occur in cycles of 4: 3, 9, 7, 1, 3, 9, 7, 1, … . Since 2024 is a multiple of 4, the units digit will be the fourth digit in the cycle, or **1.**

6. Use stars and bars: Draw a row with 9 stars. Place 2 vertical bars in any two spaces between the stars to designate one particular solution.
For example, ∴ | ⋯ | ⋯⋯ represents (2,3,4). Since there are 8 spaces between the stars, choosing any 2 of them yields $\binom{8}{2} = {}_8C_2 = \mathbf{28}$ solutions. Alternatively, the solutions can be listed as follows: (1,1,7), (1,2,6), (1,3,5), (1,4,4), (2,2,5), (2,3,4), and (3,3,3). Each of (1,1,7), (1,4,4), and (2,2,5) has 3 arrangements. Each of (1,2,6), (1,3,5), and (2,3,4) has 6 arrangements. The solution (3,3,3) has only 1 arrangement. The equation has $3 \cdot 3 + 3 \cdot 6 + 1 = 28$ solutions in positive integers.