Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2022 – 2023

No calculators are allowed.

Time: 10 minutes **Name: _____**

25. If $\sqrt{\sqrt{\sqrt{249+x}}} = 2$, compute x.

26. If $x = 2^k$, compute k for $\log_2 x + \log_8 x + \log_{32} x = 23$.

25. ☐ 26. ☐

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27. Compute: $\frac{4^2+8^2+12^2+\cdots+100^2}{1^2+2^2+3^2+\cdots+25^2}$.

28. Ava leaves home and walks toward school at a constant speed. Her sister Charlotte leaves home 4 minutes later than Ava and rides her bike at a constant speed. Charlotte passes Ava $\frac{2}{3}$ of the way from home to school and arrives at school 4 minutes later. Compute the number of minutes it takes Ava to walk from home to school.

27. ☐ 28. ☐

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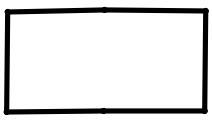
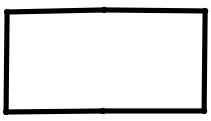
29. In square $WXYZ, WX = 2$. From a point P, not in the plane of the square, four line segments, each of length 2 are drawn, each one ending in a vertex of the square. Compute the degree measure of $\measuredangle WPX$.

30. When $\frac{(21!)^2-(20!)^2}{(22!)^2}$ is expressed in simplest $\frac{p}{q}$ form, compute $q - 400p$.

29. 

30. 

Solutions for Contest #5

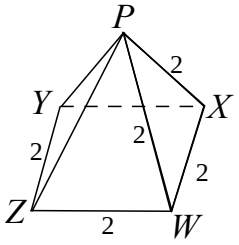
25. $\sqrt{\sqrt{\sqrt{249+x}}} = \sqrt[8]{249+x} = 2 \to 249 + x = 256 \to x = \mathbf{7}$.

26. Use the base change law on the given equation and the result is
$\frac{\log x}{\log 2} + \frac{\log x}{\log 8} + \frac{\log x}{\log 32} = 23 \to \frac{\log x}{\log 2} + \frac{\log x}{3\log 2} + \frac{\log x}{5\log 2} = 23 \to \frac{\log x}{\log 2}\left(1 + \frac{1}{3} + \frac{1}{5}\right) = 23 \to$
$\frac{\log x}{\log 2} \cdot \frac{23}{15} = 23 \to \log x = 15 \log 2 \to x = 2^{15} \to k = 15$
OR: $\log_2 2^k + \log_8 2^k + \log_{32} 2^k = 23 \to k + \frac{k}{3} + \frac{k}{5} = 23 \to \frac{23k}{15} = 23 \to k = \mathbf{15}$.

27. A common factor in the numerator is 4^2. When this is factored out, the remaining factor in the numerator cancels with the denominator and the result is **16**.

28. Since Charlotte bikes $\frac{1}{3}$ of the distance from home to school in 4 minutes, it takes her 8 minutes to bike from home to the overtake spot. So, it takes Ava $4 + 8 = 12$ minutes to walk to the overtake spot. At the same rate, it takes Ava another 6 minutes to get to school, for a total of **18** minutes.

29. Since ΔWPX is equilateral and equiangular, each of its angles has a degree measure of **60**.



30. $\frac{(21!)^2-(20!)^2}{(22!)^2} = \frac{(21)^2(20!)^2-(20!)^2}{(22)^2(21)^2(20!)^2} = \frac{(20!)^2(21^2-1)}{(20!)^2(22)^2(21)^2} = \frac{(21-1)(21+1)}{(22)^2(21)^2} = \frac{(20)(22)}{(22)^2(21)^2} =$
$\frac{20}{(22)(21)^2} = \frac{10}{4851}$. Thus, $q - 400p = 4851 - 4000 = 851$.
Alternatively, $\frac{(21!)^2-(20!)^2}{(22!)^2} = \frac{(21!-20!)(21!+20!)}{(22!)(22!)} = \left(\frac{1}{22} - \frac{1}{21\cdot 22}\right)\left(\frac{1}{22} + \frac{1}{21\cdot 22}\right) =$
$\frac{1}{22}\left(1 - \frac{1}{21}\right)\left(\frac{1}{22}\right)\left(1 + \frac{1}{21}\right) = \frac{1}{22^2} \cdot \frac{20}{21} \cdot \frac{22}{21} = \frac{10}{4851} = \frac{p}{q}$.
Thus, $q - 400p = 4851 - 4000 = \mathbf{851}$.