Nassau County Interscholastic Mathematics League

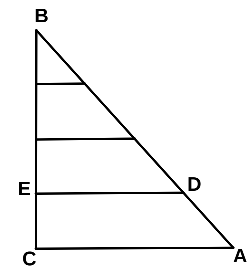
Contest #4 Answers must be integers from 0 to 999, inclusive. 2022 – 2023

Calculators are allowed.

Time: 10 minutes **Name: ______________________________**

19. If the sum of two prime numbers is 91, compute the product of these two prime numbers.

20. The area of right ΔABC is 480 square units. Three equally spaced lines are drawn parallel to $\overline{AC}$ and partition ΔABC into four non-overlapping regions of equal height as shown in the figure. Compute the area of the largest of these four regions.



19.

20.

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21. Compute the units digit of 2023^{2026}.

22. Compute the sum of the roots, in degrees, on the interval $[0°, 180°]$ of

$6\sin(5x) + 7 = 10.$

21. $\boxed{}$ 22. $\boxed{}$

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23. Sidney sells oolong tea for \$1.90 per ounce and Darjeeling tea for \$2.86 per ounce. Sidney's manager wants to create a special blend of oolong tea and Darjeeling tea that will sell for \$2.20 per ounce. How many ounces of Darjeeling tea should be contained in one pound of the special blend?

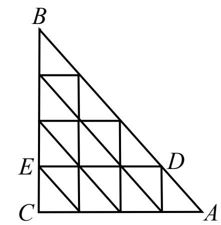
24. Point P is interior to equilateral ΔABC. Point P is 5 inches from $\overline{AC}$, 12 inches from $\overline{AB}$ and 13 inches from $\overline{BC}$. If the perimeter of equilateral ΔABC in inches, expressed in simplest radical form, is $p\sqrt{q}$, compute $p+q$.

23. $\boxed{}$ 24. $\boxed{}$

Solutions for Contest #4

19. All primes other than the number 2 are odd. So, in order for the sum to be odd, one of the primes must be 2 and the other is 89. The required product is **178**.

20. A useful theorem is that the ratio of the areas of similar triangles equals the square of the ratio of similitude. Say that the line segment closest to $\overline{AC}$ is named $\overline{DE}$. Then the area of ΔBDE is $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$ of the area of ΔABC or 270. So, the area of trapezoid $ADEC$ is $480 - 270 = 210$. Alternative solution: A proof without words: $480 \cdot (7/16) = \mathbf{210}$.



21. The units digit of 2023^1 is 3. The units digit of 2023^2 is 9. The units digit of 2023^3 is 7. The units digit of 2023^4 is 1. The cycle of four numbers: 3, 9, 7, 1 repeats forever. Then, 2026 divided by 4 leaves a remainder of 2. So, the units digit of 2023^{2026} is the same as the units digit of 2023^2 or **9**.

22. From the given equation, $\sin(5x) = \frac{1}{2} \rightarrow 5x = 30° + 360°k$ or $150° + 360°k$, where k is an integer. So, $x = 6° + 72°k$ or $30° + 72°k \rightarrow x = 6°, 78°, 150°, 30°, 102°, 174°$. The required sum in degrees is **540**. This can be seen by graphing the equations and finding the 6 intersection points.

23. Sidney sells x ounces of Darjeeling tea in the special blend. Therefore, $190(16 - x) + 286x = 220 \cdot 16 \rightarrow 3040 - 190x + 286x = 3520 \rightarrow 96x = 480 \rightarrow x = \mathbf{5}$.

24. Partition equilateral ΔABC into $\Delta PAB, \Delta PBC$, and ΔPAC. Let $x = AB = BC = CA$. Then the sum of the areas of the interior triangles is the area of ΔABC. So, $\frac{1}{2}x \cdot 5 + \frac{1}{2}x \cdot 12 + \frac{1}{2}x \cdot 13 = x^2\frac{\sqrt{3}}{4} \rightarrow 15x = x^2\frac{\sqrt{3}}{4} \rightarrow x = 20\sqrt{3}$. The perimeter is $3x = 60\sqrt{3}$ and the required sum is **63**. A suggested search for students is Viviani's theorem: The sum of the lengths of the perpendicular segments to the sides of an equilateral triangle is the length of the altitude of the equilateral triangle. That is, $5 + 12 + 13 = \frac{x}{2}\sqrt{3} \rightarrow x = 20\sqrt{3}$ and the perimeter is $60\sqrt{3}$. Ask students to consider what happens if point P is outside the triangle.