Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2022 – 2023

No calculators are allowed.

Time: 10 minutes **Name: __________________________________**

13. Compute $\left|\left[\frac{(5-7)^3}{4^2}\right]^{-7}\right| + \frac{\sqrt{300}+\sqrt{108}+\sqrt{48}}{\sqrt{27}+\sqrt{12}}$.

14. Compute the number of diagonals of a convex penta-decagon (15-sided polygon).

13. ☐

14. ☐

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15. Compute x if the common solution of the system $\frac{3}{y} - \frac{1}{x} = \frac{5}{4}$ and $\frac{2}{x} + \frac{1}{y} = 1$ is (x, y).

16. P and Q are the two solutions of the equation: $\frac{a-bi}{2} + \frac{-2+i}{a+bi} = 1$ when written in $a + bi$ form. Compute $|P - Q|$.

15. [ ] 16. [ ]

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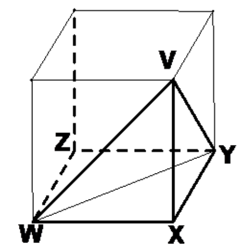
17. The base of a cube has vertices W, X, Y, and Z as shown in the figure. A fifth vertex V of the cube, is directly above vertex X. The surface area of the cube is 216 square inches. Compute the number of cubic inches in the volume of the tetrahedron whose vertices are W, X, Y, and V.



18. If the solutions of $x^2 - 40x + 8 = 0$ are $x = r$ and $x = s$, compute $\left(\sqrt[3]{r} + \sqrt[3]{s}\right)^3$.

17. ______

18. ______

Solutions for Contest #3

13. $\left|\left[\frac{(5-7)^3}{4^2}\right]^{-7}\right| + \frac{\sqrt{300}+\sqrt{108}+\sqrt{48}}{\sqrt{27}+\sqrt{12}} = \left|\left(\frac{-8}{16}\right)^{-7}\right| + \frac{10\sqrt{3}+6\sqrt{3}+4\sqrt{3}}{3\sqrt{3}+2\sqrt{3}} = \left|\left(-\frac{1}{2}\right)^{-7}\right| + \frac{20\sqrt{3}}{5\sqrt{3}} =$

$128 + 4 = \mathbf{132}$.

14. The number of line segments connecting 15 vertices is $\binom{15}{2}$. This number counts the 15 sides of the polygon as well. Therefore, the polygon has $\binom{15}{2} - 15 = \frac{15 \cdot 14}{2} - 15 = 90$ diagonals. Alternatively, the number of diagonals of a convex n-gon is $\frac{n(n-3)}{2}$.
So, $\frac{15(15-3)}{2} = \mathbf{90}$.

15. Multiply the second equation by 3 and subtract the first equation from the resulting equation to get $\frac{7}{x} = \frac{7}{4} \rightarrow x = \mathbf{4}$.

16. From the given equation, $a^2 + b^2 - 4 + 2i = 2a + 2bi = 2a + 2bi \rightarrow$
$b = 1$ and $a^2 - 3 = 2a \rightarrow a^2 - 2a - 3 = 0 \rightarrow (a-3)(a+1) = 0 \rightarrow a = 3$ or $a = -1$.
The solutions are $P = 3 + i$ and $Q = -1 + i$. The required difference is $|P - Q| = \mathbf{4}$.

17. The surface area of a cube whose edge length is x is $6x^2$. If $6x^2 = 216 \rightarrow x = 6$. The volume of a tetrahedron is $\frac{1}{3}Bh$, where B is the area of the base and h is the length of the height. So, $\frac{1}{3}(\text{area of } \Delta WXY) \cdot VX = \frac{1}{3}\left(\frac{1}{2}(6)^2\right) \cdot 6 = \mathbf{36}$.

18. $\left(\sqrt[3]{r} + \sqrt[3]{s}\right)^3 = r + 3\sqrt[3]{r^2 s} + 3\sqrt[3]{rs^2} + s = (r+s) + 3\sqrt[3]{rs}\left(\sqrt[3]{r} + \sqrt[3]{s}\right)$. From the given equation, the sum of the roots, $r + s = 40$, and the product of the roots, $rs = 8$.
Let $a = \sqrt[3]{r} + \sqrt[3]{s}$, so $a^3 = 40 + 3(2)(a) = 40 + 6a$. Since a^3 is an integer, a is an integer.
Consider perfect cubes greater than 40. The next one is $64 = 4^3$ which happens to satisfy the equation $4^3 = 40 + 6(4)$. Hence, $\left(\sqrt[3]{r} + \sqrt[3]{s}\right)^3 = 64$. Alternatively use synthetic division to solve the equation $a^3 - 6a - 40 = 0$, which will yield $a = 4$ and $a^3 = \mathbf{64}$.