Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2022 – 2023

Calculators are allowed.

Time: 10 minutes **Name: ________________________________**

7. The complex fraction $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{\frac{1}{5}-\frac{1}{6}}}$ can be expressed in simplest form as $\frac{p}{q}$, where p and q are integers. Compute $p+q$.

8. Rhombus $WXYZ$ has vertices whose coordinates are $W(5,6)$, $X(a,b)$, $Y(11,12)$, and $Z(c,8)$. Compute $a+b+c$.

7. ______ 8. ______

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9. Your mathletes coach always averages 30 miles per hour for the 10-mile drive to get to school on time. Today, your coach had to go very slowly for the first 5 minutes but then arrived at school on time by averaging 36 miles per hour the rest of the way. Compute your coach's average speed in miles per hour for the first five minutes.

10. A fair coin is tossed five times. The probability that it lands heads up exactly four times, in simplest form, is $\frac{p}{q}$. Compute $p + q$.

9. [ ]   10. [ ]

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11. The areas of 3 faces of a right rectangular prism in square units are 15, 35, and 21. Compute the number of cubic units in the volume of the prism.

12. Let point P be any point on the circle whose equation is $x^2 + y^2 + 4x - 8y + 11 = 0$ and let point Q be any point on the circle whose equation is $(x - 10)^2 + (y + 1)^2 = 16$. Compute the smallest possible distance between points P and Q.

11. [ ] 12. [ ]

<u>Solutions for Contest #2</u>

7. $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{\frac{1}{5}-\frac{1}{6}}} = \frac{\frac{1}{6}}{\frac{1}{\frac{1}{30}}} = \frac{\frac{1}{6}}{30} = \frac{1}{180}$. The required sum is **181**.

8. The diagonals of a rhombus intersect at their midpoint M with coordinates $(8,9)$. Therefore, $\frac{b+8}{2} = 9$ and $\frac{a+c}{2} = 8 \rightarrow b = 10$ and $a + c = 16$. The required sum is **26**.

9. Ordinarily, the 10-mile trip takes 20 minutes or $\frac{1}{3}$ hour at 30 mph. If the coach's speed for the first 5 minutes $\left(\frac{1}{12}\text{ hr}\right)$ is x mph, then $x \cdot \frac{1}{12} + 36 \cdot \left(\frac{1}{3} - \frac{1}{12}\right) = 10 \rightarrow x = \mathbf{12}$ mph.

10. This is a Bernoulli probability problem. Exactly four heads can occur in five tosses in $\binom{5}{4} =$ 5 ways and the probability of tossing heads equals the probability of tossing tails. Each is one-half. The required probability is $5 \cdot \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} = \frac{5}{32}$. The required sum is **37**.
Note: The notation $\binom{n}{r}$ is the number of combinations of n things taken r at a time.

11. Since the areas are easily factored, the dimensions of the faces are 3 by 5, 5 by 7, and 7 by 3. Thus, the volume of the solid is (3)(5)(7) or **105** cubic units. This can be confirmed algebraically.

12. Complete the squares in the first given equation to re-write it as $(x+2)^2 + (y-4)^2 = 3^2$ The centers of the circles are points with coordinates $(-2, 4)$ and $(10, -1)$ and the lengths of their radii are 3 and 4. The distance between the centers using the distance formula is 13 (5-12-13 triangle). In order for the distance between the points P and Q to be a minimum, subtract the sum of the radii from 13 to get the result: $13 - 7 = \mathbf{6.}$