

Nassau County Interscholastic Mathematics League

Contest #5      Answers must be integers from 0 to 999, inclusive.      2020 – 2021

No calculators are allowed.

**Time: 10 minutes**

**Name:** \_\_\_\_\_

25) Compute  $499^2 - 498^2$ .

26) Planet X has a year of only 9 days. If 3 inhabitants of planet X are at an event and the probability that at least 2 of them have the same birthday is  $\frac{p}{q}$  in simplest form, compute  $p + q$ .

25.

26.

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**Name:** \_\_\_\_\_

- 27) Compute the number of integers between 2600 and 5000 that are perfect squares.
- 28) Point  $P$  is 12 inches from the center of a circle whose radius is 13 inches long. Compute the number of chords of this circle with integer lengths which contain point  $P$ .

27.



28.



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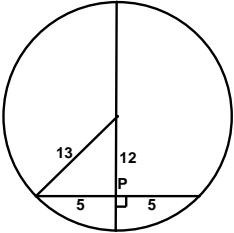
**Name:** \_\_\_\_\_

- 29) Danny's Diner offers a breakfast special that consists of five items selected from: egg, muffin, yogurt, toast. No item may be selected in a special more than twice. Compute the number of possible breakfast specials.
- 30) If  $x = (\log_{25} 81)(\log_{27} 64)(\log_2 625)$ , compute the number of digits in the base ten numerical representation of  $10^x$ .

29.

30.

## Solutions for Contest #5

- 25) Factor using the difference of two squares:  
 $499^2 - 498^2 = (499 + 498)(499 - 498) = 997 \cdot 1 = \mathbf{997}$ .
- 26) The probability that inhabitant number 2 does not have the same birthday as inhabitant number 1 is  $\frac{8}{9}$ . The probability that inhabitant number 3 does not have the same birthday as inhabitant number 2 is  $\frac{7}{9}$ . The probability that no two inhabitants have the same birthday is  $\frac{8}{9} \cdot \frac{7}{9} = \frac{56}{81}$ . Therefore, the probability that at least two inhabitants have the same birthday is  $1 - \frac{56}{81} = \frac{25}{81}$ . The required sum is **106**.
- 27) Note that  $50^2 = 2500 < 2600 < 2601 = 51^2$  and  $70^2 = 4900 < 5000 < 5041 = 71^2$ . The squares in the given interval are  $51^2, 52^2, 53^2, \dots, 70^2$ . There are  $70 - 51 + 1 = \mathbf{20}$  squares in the given interval.
- 28) The longest chord to contain point  $P$  is the diameter through point  $P$ . Its length is 26 inches. The shortest chord to contain point  $P$  is the chord perpendicular to the diameter. Its length is 10 inches using the 5-12-13 right triangle shown in the diagram. In addition, there are two chords each with lengths 11, 12, 13, ..., 25. There are **32** in total.
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- 29) A breakfast special may consist of 2 of exactly one item and exactly one of each remaining item in 4 ways because there are 4 possibilities for the doubled item. Or, the breakfast special may consist of one item once, one item not at all, and two items twice. This can occur in 12 ways – one item 0 times in 4 ways and the next item once 3 ways and the remaining items twice for a total of **16** ways.
- 30) Using the change-of-base property,  $(\log_{25} 81)(\log_{27} 64)(\log_2 625) = \frac{\log 81}{\log 25} \cdot \frac{\log 64}{\log 27} \cdot \frac{\log 625}{\log 2} = \frac{4 \log 3}{2 \log 5} \cdot \frac{6 \log 2}{3 \log 3} \cdot \frac{4 \log 5}{\log 2} = \frac{96}{6} = 16$ . Thus,  $10^x = 10^{16}$  has **17** digits.