Nassau County Interscholastic Mathematics League

Contest #3   Answers must be integers from 0 to 999, inclusive.   2020 – 2021

No calculators are allowed.

Time: 10 minutes   **Name: ______________________________**

13) Compute the product of the least common multiple (LCM) of 12 and 15 and the greatest common factor (GCF) of 18 and 45.

14) Compute the only real root of $\sqrt{16x-80}+\sqrt{x-5}-\frac{1}{3}\sqrt{9x-45}=48$.

13. ☐

14. ☐

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15) If $a = 10^{10}$, compute $\sqrt[13]{\left(a \cdot \sqrt[5]{a^8}\right)}$.

16) The area of a triangle is 144. The lengths of its medians equal the lengths of the sides of a second triangle. The lengths of the medians of the second triangle equal the lengths of the sides of a third triangle. Compute the sum of the areas of the three triangles.

15. ☐

16. ☐

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17) Glass A contains only 10 ounces of apple juice. Glass G contains only 10 ounces of grape juice. I pour 1 ounce of juice from glass A into glass G and stir thoroughly.
Then, I pour 1 ounce of juice from glass G into glass A and stir thoroughly. If the fraction of grape juice in glass A in simplest form is $\frac{p}{q}$, compute $p + q$.

18) If $\tan\theta = \frac{(\sin 56)(1-\cos 28)}{(\cos 28)(1-\cos 56)}$, where all angles are measured in degrees,

compute θ in degrees.

Hint: By the double-angle formulas, $\sin 2x = 2\sin x \cos x$ and
$\cos 2x = 2(\cos x)^2 - 1 = 1 - 2(\sin x)^2$

17. ☐

18. ☐

Solutions for Contest #3

13) $\text{LCM}(12,15)\cdot\text{GCF}(18,45) = 60\cdot 9 = \mathbf{540}$.

14) $4\sqrt{x-5}+\sqrt{x-5}-\sqrt{x-5} = 48 \rightarrow 4\sqrt{x-5} = 48 \rightarrow \sqrt{x-5} = 12 \rightarrow x = \mathbf{149.}$

15) $\sqrt[13]{\left(a\cdot\sqrt[5]{a^8}\right)} = \sqrt[13]{a\cdot a^{8/5}} = \sqrt[13]{a^{13/5}} = a^{1/5}$. When $a = 10^{10}$, $(10^{10})^{1/5} = 10^2 = \mathbf{100.}$

16) The ratio of the area of $\frac{A_{k+1}}{A_k}$ will be the same no matter what type of triangle. So, use an equilateral triangle. Note that the median of an equilateral triangle with a side of s is $s\frac{\sqrt{3}}{2}$. So, the area of the median triangle will be $\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$ times the area of its predecessor. Thus, the required sum is, $144\left(1+\frac{3}{4}+\left(\frac{3}{4}\right)^2\right) = \mathbf{333.}$

17) After step 1, glass A contains 9 ounces of apple juice and glass G contains 10 ounces of grape juice and 1 ounce of apple juice. After step 2, glass A contains $9\frac{1}{11}$ ounces of apple juice and $\frac{10}{11}$ ounces of grape juice and glass G contains $9\frac{1}{11}$ ounces of grape juice and $\frac{10}{11}$ ounces of apple juice. The fraction of grape juice in glass A is then $\frac{\frac{10}{11}}{9\frac{1}{11}+\frac{10}{11}} = \frac{\frac{10}{11}}{10} = \frac{1}{11}$. The required sum is **12.**

18) Using the double-angle formulas $\sin 2x = 2\sin x\cos x$ and $\cos 2x = 2(\cos x)^2 - 1$, $\sin 56 = 2\sin 28\cos 28$ and $\cos 56 = 2(\cos 28)^2 - 1$. So, $\frac{(\sin 56)(1-\cos 28)}{(\cos 28)(1-\cos 56)} =$ $\frac{(2\sin 28\cos 28)(1-\cos 28)}{(\cos 28)(1-(2(\cos 28)^2)-1)} = \frac{(\sin 28)(1-\cos 28)}{1-(\cos 28)^2} = \frac{\sin 28}{1+\cos 28}$. Using the double-angle formulas again, $\sin 28 = 2\sin 14\cos 14$ and $\cos 28 = 2(\cos 14)^2 - 1$. So, $\frac{2\sin 14\cos 14}{1+(2(\cos 14)^2)-1} = \frac{\sin 14}{\cos 14} = \tan 14$. Thus, $\theta = 14$.