Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2020 – 2021

Calculators are allowed.

Time: 10 minutes **Name: ________________________________**

7) On eight tests on which grades can vary from 0 to 100, inclusive, Margaret's mean test score is 90. Compute the lowest possible test score Margaret could have achieved on one of these 8 tests.

8) Compute the value of x for which $(x+y-11)^2+(x-y-3)^2=0$.

7. [ ]

8. [ ]

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9) Compute the number of integer solutions to $|4x+5| \leq 5$ and $|3x-2| \geq -8$.

10) The lengths of the sides of an isosceles trapezoid are 10, 10, 10, and 22. If the length of a diagonal of the trapezoid, expressed in simplest radical form, is $p\sqrt{q}$, compute pq.

9. [ ]   10. [ ]

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11) In Math Town a taxi charges \$1 per $\frac{1}{5}$ mile when traveling faster than k miles per hour. The charge is \$0.75 per **minute** when moving less than k miles per hour. At k miles per hour both methods produce the same charge. Compute k.

12) Compute the sum of all values of non-negative integers, $n \leq 10$, for which $n^2 + n + 44$ represents a perfect square.

11. ☐ 12. ☐

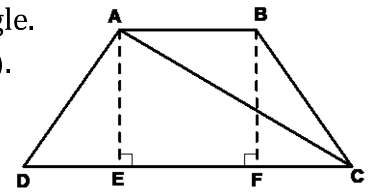
Solutions for Contest #2

7) Margaret's total score on her 8 tests is 720. If she earned 7 100's, then she could have scored **20** as her lowest result.

8) In order for the sum of two squares to be zero, each of the squared terms must be zero. Therefore, $x + y - 11 = 0$ and $x - y - 3 = 0 \rightarrow x + y = 11$ and $x - y = 3 \rightarrow x =$ **7.**

9) $|4x + 5| \leq 5 \rightarrow -5 \leq 4x + 5 \leq 5 \rightarrow -10 \leq 4x \leq 0 \rightarrow -2.5 \leq x \leq 0$ and $|3x - 2| \geq -8 \rightarrow x \in \{\text{reals}\}$. There are only **3** integers, $\{-2, -1, 0\}$, in the intersection of these solutions.

10) Let $AD = AB = BC = 10$, $(\overline{AB} \parallel \overline{DC})$. Quadrilateral $AEFB$ is a rectangle. Therefore, $EF = 10 \rightarrow DE = FC = 6 \rightarrow AE = BF = 8$ (6-8-10 right Δ). Then, use the Pythagorean Theorem in ΔAEC, $(AC)^2 = 16^2 + 8^2 = 320 \rightarrow AC = \sqrt{320} = 8\sqrt{5}$. Thus the required product is **40**.



11) The LCM of \$1.00 and \$0.75 is \$3.00. Note that \$1 per $\frac{1}{5}$ mile = \$3.00 per $\frac{3}{5}$ mile, and \$0.75 per minute = \$3.00 per 4 minutes or $\frac{1}{15}$ hours. So, the charge is the same at

$$\frac{\frac{3}{5}\text{ mile}}{\frac{1}{15}\text{ hour}} = \mathbf{9}\text{ mph.}$$

12) Note that $n^2 + n + 44 > n^2$, because $n \geq 0$. If $n^2 + n + 44$ is a perfect square, then it must be the square of an integer greater than n. Therefore, set $n^2 + n + 44 = (n + c)^2$, where c is some positive integer. Thus, $n + 44 = c^2 + 2cn \rightarrow$
$44 - c^2 = n(2c - 1) \geq 0$. So, c is a positive integer for which $c^2 < 44$. Thus, c is a member of $\{1,2,3,4,5,6\}$. Use $n = \frac{44-c^2}{2c-1}$ to check the six values of c. Note that n is an integer only when c is 1, 3, or 4. The corresponding values of $n \leq 10$ are 7 and 4. For these values of n, the resulting values of $n^2 + n + 44$ are $100 = 10^2$ and $64 = 8^2$, respectively. The required sum is $7 + 4 =$ **11.**
Alternatively, use trial and error to find the two values of n.