Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2019 – 2020

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

25) Compute the least common multiple of all the multiples of 6 between 10 and 35.

26) The roots of $\log_x 4 + \log_4 x = \frac{13}{6},\ x > 1$ are 2^a and 2^b. Compute the product, ab.

25. ☐

26. ☐

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27) Three positive integers are added in pairs yielding sums of 107, 130, and 143. Compute the smallest integer.

28) Inscribe ΔDEF in ΔABC such that $\overline{ADB}$, $\overline{BEC}$, and $\overline{AFC}$. If $AD = BE = CF = 1$, and $DB = EC = FA = 2$, then the area of ΔDEF, in simplest form, is $\frac{a\sqrt{b}}{c}$. Compute the product, abc.

27. [ ]

28. [ ]

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29) Compute the number of even four-digit integers greater than 3000 that can be made using the digits 2, 3, 6, and 7 without repetition.

30) Compute the least positive integer, n, for which $n^2 + 2n + 25$ is a multiple of $3n + 7$.

29. [ ] 30. [ ]

Solutions for Contest #5

25) The LCM of 12, 18, 24, and 30 is $6 \cdot 2 \cdot 3 \cdot 2 \cdot 5 = \mathbf{360.}$

26) Let $u = \log_x 4$ and its reciprocal, $\frac{1}{u} = \log_4 x$. So, $u + \frac{1}{u} = \frac{13}{6} \rightarrow 6u^2 - 13u + 6 = 0 \rightarrow$ $(2u-3)(3u-2) = 0 \rightarrow u = \frac{3}{2}$ or $u = \frac{2}{3}$. So, $\log_x 4 = \frac{3}{2}$ or $\frac{2}{3} \rightarrow x^{\frac{3}{2}} = 4$ or $x^{\frac{2}{3}} = 4 \rightarrow$ $x = 4^{\frac{2}{3}} = 2^{\frac{4}{3}}$ or $x = 8 = 2^3$. Thus, $a = \frac{4}{3}$ and $b = 3 \rightarrow ab = \mathbf{4}$.

27) Call the integers $a, b,$ and c. Then, $a + b = 107$, $b + c = 130$, and $a + c = 143$. Add the three equations and divide by 2 to get $a + b + c = 190$. From this equation, subtract the largest of the pairwise sums, $a + c = 143$. Thus, the smallest of the three integers is $\mathbf{47}$.

28) Since ΔABC is equilateral and equiangular, $\Delta ADF \cong \Delta BED \cong \Delta CFE$ by SAS. Therefore, the sides of ΔDEF are all congruent and its interior angles each measure 60 degrees. Using the Law of Cosines, $DE = EF = FD = \sqrt{2^2 + 1^2 - 2 \cdot 1 \cdot 2 \cdot \cos 60°} = \sqrt{3}$. Therefore, by the formula for the area of an equilateral triangle, $A = \frac{s^2\sqrt{3}}{4}$, the area of $\Delta DEF = \frac{(\sqrt{3})^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$. The required product is **36.**

29) If the units digit is a 2, there are $3 \cdot 2 \cdot 1 = 6$ possible numbers that can be formed. If the units digit is a 6, there are $2 \cdot 2 \cdot 1 = 4$ possible numbers that can be formed. So, together there are $6 + 4 = \mathbf{10}$ such 4-digit numbers.

30) If $n^2 + 2n + 25$ is a multiple of $3n + 7$, then so is $3n^2 + 6n + 75$. Divide $3n^2 + 6n + 75$ by $3n + 7$ to get $n + \frac{-n+75}{3n+7}$. In order for $-n + 75$ to be divisible by $3n + 7$, either $-n + 75 = 3n + 7 \rightarrow n = 17$, or $-n + 75 = 0 \rightarrow n = 75$. For $0 < n < 17$, $\frac{-n+75}{3n+7}$ is non-integral, and when $n > 17$, $3n + 7 > -n + 75$, so divisibility is possible only when $n = 75$ because 0 is divisible by every non-zero number. Thus, of the two possible values of n, $\mathbf{17}$ is the lesser.