Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2019 – 2020

Calculators are allowed.

Time: 10 minutes **Name: ________________________________**

19) If $2x + y + z = 2020$ and $x + 2y + z = 1021$, compute $x - y$.

20) The two-digit number, x, has unequal digits with neither digit equal to zero. The digits of x are interchanged to form a new two-digit number, y. Compute the number other than 1 which **must** be a positive integral factor of $x + y$.

19. ☐

20. ☐

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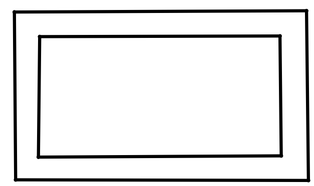
21) Aden and Briana are planning a wedding party. There is a charge to rent the room and there is an additional charge per guest. If they have 400 guests, the total cost would be \$160,000. If they have 520 guests, the total cost will be \$196,000. If they have 700 guests, compute the quotient, in dollars, of the total cost divided by 1000.

22) The area of a 100-foot by 50-foot rectangle is tripled by surrounding the rectangle with a strip of uniform width on its perimeter as in the given diagram. Compute the number of feet in the uniform width (Note: The diagram is not drawn to scale).



21.

22.

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23) Compute the remainder when 2^{2019} is divided by 7.

24) In ΔABC, $AB = 10$, $AC = 10$, and $BC = 4\sqrt{5}$. Point E is on $\overline{AC}$ with $AE = 6$. Circles with centers O_1 and O_2 circumscribe ΔABE and ΔCBE, respectively. Compute the length of $\overline{O_1O_2}$.

23. ☐

24. ☐

__Solutions for Contest #4__

19) Note that $(2x + y + z) - (x + 2y + z) = x - y = 2020 - 1021 = \mathbf{999.}$

20) If $x = 10t + u$ with $t \neq u$ and $t \neq 0 \neq u$, then $y = 10u + t$ and $x + y = 11t + 11u$ $= 11(t + u)$. Since there are many possibilities for $t + u$, the number which must be a positive integral factor of $x + y$ is **11.**

21) Let $x =$ the room charge, and let $y =$ the charge per guest. So, $x + 400y = 160{,}000$ and $x + 520y = 196{,}000$. Solving this system of equations simultaneously yields $x = 40{,}000$ and $y = 300$. So, for 700 guests, $40{,}000 + (300)(700) = 250{,}000$ and the required quotient is **250.**

22) Let w be the uniform width of the additional strip around the rectangle. Then the new area can be expressed as $(50 + 2w)(100 + 2w) = 3 \cdot 50 \cdot 100 \rightarrow (25 + w)(50 + w) = 150 \cdot 25 \rightarrow w^2 + 75w + 1250 = 3750 \rightarrow w^2 + 75w - 2500 = 0 \rightarrow$ $(w + 100)(w - 25) = 0 \rightarrow w = 25$. The number of feet in the uniform width is **25.**

23) When 2^3 is divided by 7, the remainder is 1. Note that 2019 is a multiple of 3 and $2^{2019} = (2^3)^{673}$. The remainder is 1. Alternatively, one can use modular arithmetic: $2^{2019} = x \bmod 7 \equiv (2^3)^{673} \bmod 7 \equiv 8^{673} \bmod 7 \equiv 1^{673} \bmod 7 \rightarrow x = \mathbf{1.}$

24) By the Law of Cosines in ΔABC, $(4\sqrt{5})^2 = 10^2 + 10^2 - 2(10)(10)\cos A \rightarrow \cos A = 0.6$.
By the Law of Cosines in ΔABE, $(BE)^2 = 10^2 + 6^2 - 2(10)(6)(0.6) \rightarrow BE = 8$.
Since 6-8-10 is a Pythagorean triple, ΔABE is a right triangle with hypotenuse $\overline{AB}$.
And, since $\overline{BE} \perp \overline{AC}$, ΔBEC is also a right triangle with hypotenuse $\overline{BC}$. Therefore, points O_1 and O_2 are midpoints of hypotenuses $\overline{AB}$ and $\overline{BC}$, respectively. So, by the Midsegment Theorem, $O_1O_2 = \frac{1}{2}AC = \mathbf{5.}$