Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2019 – 2020

Calculators are allowed.

Time: 10 minutes **Name: ________________________________**

7) Two dozen hard boiled eggs have the integers: 1, 2, 3, … , 24 written on them, with exactly one integer on each egg. Kevin separated the eggs into two groups. The sums of the integers on the eggs in each group are equal. Compute that sum.

8) If r and s are the roots of the equation $4x^2 - 5x + 3 = 0$ and $rs^3 + sr^3$ is written in simplest $\frac{p}{q}$ form, compute $p + q$.

7. [ ]

8. [ ]

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9) If $f(x) = \sqrt{x+1}$, $g(x) = x^2 + a$, and $f\left(g(1)\right) = 17$, compute a.

10) A cube whose surface area is $1152\ \text{in}^2$ is inscribed in a sphere. Compute, in inches, the length of a radius of the sphere.

9.

10.

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11) Compute the number of 4-digit positive integers using exactly 4 distinct numbers from the set $\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

12) Consider all ordered pairs of positive integers (x, y) such that $x^2 + y$ exceeds $x + y^2$ by exactly 14. Compute the sum of the abscissas of all such ordered pairs.

11. ☐

12. ☐

Solutions for Contest #2

7) Let $S = 1 + 2 + 3 + \cdots + 24 = \frac{24 \cdot 25}{2} = 300$. Therefore, each sum is **150.**

8) Let $k = rs^3 + sr^3 = rs(s^2 + r^2) = rs((r+s)^2 - 2rs)$. Since the sum and product of the roots are $r + s = \frac{5}{4}$ and $rs = \frac{3}{4}$, respectively, $k = \frac{3}{4}\left(\left(\frac{5}{4}\right)^2 - 2 \cdot \frac{3}{4}\right) = \frac{3}{64}$ and the required sum is **67**.

9) $f(g(x)) = \sqrt{x^2 + a + 1} \rightarrow f(g(1)) = \sqrt{a + 2} = 17 \rightarrow a + 2 = 289 \rightarrow a = \mathbf{287}$.

10) Each square face of the cube has an area of $\frac{1152}{6} = 192 \text{ in}^2$. The length of each edge is $\sqrt{192} = 8\sqrt{3}$ inches. The main diagonal of the cube is $8\sqrt{3} \cdot \sqrt{3} = 24$ inches, and this diagonal is also a diameter of the sphere. Thus, the length of a radius of the sphere is **12** inches.

11) We can choose four digits out of the six digits in ${}_6C_4 = \frac{6!}{4! \cdot 2!} = 15$ ways. Each of these four-digit sets can be arranged to form a number in $4! = 24$ ways. The product of 15 and 24 is **360**. Alternatively, the number of permutations of 6 digits taken 4 at a time is ${}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360$.

12) Solve $(x^2 + y) - (x + y^2) = 14$ in positive integers $\rightarrow (x^2 - y^2) - (x - y) = 14$ $\rightarrow (x + y)(x - y) - (x - y) = 14 \rightarrow (x - y)(x + y - 1) = 14$. Since x and y are positive integers, $x + y - 1 > 0$, so $x - y > 0$. Hence, both factors of 14 are positive and $x + y - 1 > x - y$. Therefore, either $x - y = 1$ and $x + y - 1 = 14$ or $x - y = 2$ and $x + y - 1 = 7$. The solution of the first system is $(8, 7)$ and the solution of the second is $(5, 3)$. The sum of the abscissas is **13**.