Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2018 – 2019

No calculators are allowed.

Time: 10 minutes **Name: ______________________________**

25) Compute the units digit of the product of all prime numbers less than or equal to 120.

26) Five men, working together, who each plow at the same uniform rate, can plow a square field whose side is 80 feet in 4 hours. At the same rate of work, compute the number of hours that 10 men could plow a square field whose side is 160 feet.

25. [ ]

26. [ ]

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27) Two numbers have a sum of 9 and a product of 13. If the sum of the reciprocals of the numbers is expressed in simplest form as $\frac{p}{q}$, compute $p + q$.

28) The legs of a right triangle have lengths 9 and 40. The area of the circle inscribed in this triangle may be expressed in the form $k\pi$. Compute k.

27. ☐

28. ☐

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29) Compute the number of pairs of positive integers (x, y) that satisfy $4x + 5y = 600$.

30) Compute the product of the roots of $x^{0.5(\log_7 x - 1)} = 7$.

29. [ ]

30. [ ]

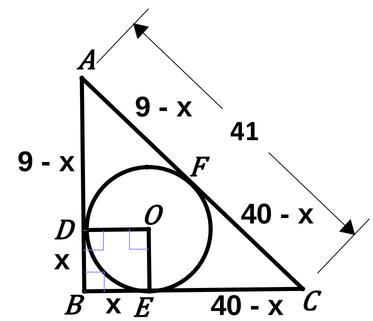
Solutions for Contest #5

25) Since the set of prime numbers less than or equal to 120 contain the numbers 2 and 5, 10 divides the product. So, its units digit is **0**.

26) To plow the field whose area is 6400 square feet requires 20 man-hours of work. Their uniform rate of work is $6400/20 = 320$ square feet per man-hour. If 10 men work at the same rate for h hours, then we require $\frac{160^2}{10h} = 320 \rightarrow \frac{16}{h} = 2 \rightarrow h = \mathbf{8}$.

27) Denote the numbers as x and y. Then, $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{9}{13}$. The required sum is $9 + 13 = \mathbf{22}$.

28) Since $\measuredangle B$ is a right angle, $\overline{OD} \perp \overline{AB}$, $\overline{OE} \perp \overline{BC}$, and radii $OD = OE$, $BEOD$ is a square. If $OD = BD = BE = OE = x$, then $AD = 9 - x$, and $EC = 40 - x$. Since tangent segments drawn to a circle from an external point are congruent, $AF = 9 - x$ and $CF = 40 - x$. Using the Pythagorean triple 9-40-41 in ΔABC, $AC = 41$. So, $9 - x + 40 - x = 41 \longrightarrow 49 - 2x = 41 \longrightarrow x = 4$. The area of circle O is 16π. Thus, $k = \mathbf{16}$. [Note: The pattern for right triangles of given side lengths and the lengths of the radii, r, of their inscribed circles is as follows: 3,4,5: r=1; 5,12,13: r=2; 7,24,25: r=3; 9,40,41: r=4.]



29) From the given equation: $y = \frac{600-4x}{5} = 120 - \frac{4}{5}x$ and y is a positive integer. So, possible values of x are multiples of 5 from 5 to 145, inclusive. Thus, there are $145/5 = \mathbf{29}$ ordered pairs.

30) Take logs of both sides of the given equation using base 7:
$\log_7 x^{0.5(\log_7 x - 1)} = 1 \rightarrow \frac{1}{2}(\log_7 x - 1)\log_7 x = 1$. Let $y = \log_7 x$.
Then, $\frac{1}{2}(y-1)y = 1 \rightarrow y^2 - y - 2 = 0 \rightarrow y = 2$ or $y = -1 \rightarrow x = 49$ or $\frac{1}{7}$.
The required product is $49 \cdot \frac{1}{7} = \mathbf{7}$.