Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2018 – 2019

No calculators are allowed.

Time: 10 minutes **Name: ________________________________**

13) Compute $\left[\sqrt[3]{\sqrt{130}-\sqrt{5}}\right]\cdot\left[\sqrt[3]{\sqrt{130}+\sqrt{5}}\right]$.

14) Compute the middle term of an arithmetic sequence of 57 terms whose sum is 4845.

13.

14.

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15) Compute the number of integers between 5000 and 8000 that are squares of integers.

16) In rectangle $ABCD$, $AB = 4$ and $AD = 2$. Point E is on $\overline{CD}$ such that $AE = AB$. When BE is expressed in the form $\sqrt{x} - \sqrt{y}$, compute $x - y$.

15. ☐

16. ☐

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17) Three cuts are made through a large cube to create 8 identical smaller cubes. The surface area of the large cube is 96 cm^2. Compute the number of cm^2 in the total surface area of the 8 smaller cubes.

18) A fourth degree polynomial of the form $x^4 + cx^2 + d$, where c and d are integers, has $\sqrt{3} + i$ as a zero. Compute $c + d$.

17. ☐

18. ☐

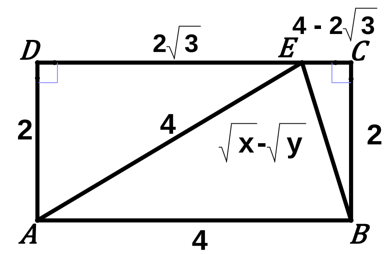
<u>Solutions for Contest #3</u>

13) $\left[\sqrt[3]{\sqrt{130}-\sqrt{5}}\right]\cdot\left[\sqrt[3]{\sqrt{130}+\sqrt{5}}\right]=\sqrt[3]{130-5}=\sqrt[3]{125}=\mathbf{5}.$

14) If x is the middle term and d is the common difference, then the given sequence can be expressed as $x-28d, x-27d, \ldots, x-d, x, x+d, \ldots, x+27d, x+28d$. Using the formula $S_n=\frac{n}{2}(a_1+a_n)$, $4845=\frac{57}{2}(x-28d+x+28d) \longrightarrow 4845=\frac{57}{2}(2x) \longrightarrow$ $4845=57x \longrightarrow x=\mathbf{85}$. Note that in an arithmetic sequence with an odd number of terms, the product of the number of terms and the middle term equals its sum.

15) Start with $70^2=4900$ and $71^2=5041$. Also, $90^2=8100$ and $89^2=7921$. The squares of the integers in the specified interval are the squares of the integers from 71 through 89. There are **19** of them.

16) Use the Pythagorean Theorem in ΔADE (or note that ΔADE is a 30-60-90 triangle) to get $DE=2\sqrt{3}$. Then $CE=4-2\sqrt{3}$. Let $BE=\sqrt{x}-\sqrt{y}$ and use the Pythagorean Theorem again in ΔBCE to get $\left(\sqrt{x}-\sqrt{y}\right)^2=4+\left(4-2\sqrt{3}\right)^2 \longrightarrow x+y-2\sqrt{xy}=$ $32-16\sqrt{3} \longrightarrow x+y-2\sqrt{xy}=32-2\sqrt{192}$. This yields the system of equations: $x+y=32$ and $xy=192 \longrightarrow$ $x(32-x)=192 \longrightarrow x^2-32x+192=0 \longrightarrow$ $(x-24)(x-8)=0$. Since $x>y$, $(x,y)=(24,8)$ and the required difference is 1**6**.



17) Since the ratio of the volumes of a small cube to the large cube is 1 : 8, the ratio of their edges is 1 : 2. Therefore, the surface area of each small cube is one-fourth the surface area of the large cube. Since there are 8 small cubes, their total surface area is $8\cdot\frac{1}{4}=2$ times the surface area of the large cube. That total surface area is **192** cm^2.

18) Since complex zeros of polynomials with integral coefficients occur in conjugate pairs, $\sqrt{3}-i$ is also a zero. Using the sum and product of these zeros, the quadratic function with these two zeros is $x^2-2\sqrt{3}x+4$. Since we are looking for a fourth degree polynomial, we try squaring this quadratic. Doing so, however, will not make all of the coefficients integral because the cubic and linear terms are not eliminated. By some trial and error, noting that all four zeros have a sum of 0, and/or deducing that $-\sqrt{3}+i$ and $-\sqrt{3}-i$ are also zeros, we discover that multiplying the quadratic by $x^2+2\sqrt{3}x+4$ gives us x^4-4x^2+16. Thus, $c+d=-4+16=\mathbf{12}$.