Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2018 – 2019

Calculators are allowed.

Time: 10 minutes **Name: ______________________**

7) Compute the maximum number of Fridays in any calendar year.

8) Compute the sum of the series: $\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\cdots+\left(\frac{1}{60}+\frac{2}{60}+\frac{3}{60}+\cdots+\frac{59}{60}\right)$.

7. [ ] 8. [ ]

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9) Compute the sum of two consecutive positive integers whose squares differ by 95.

10) In square $WXYZ$, point M is the midpoint of $\overline{WX}$. If $WX = 13$ and point K is the intersection of $\overline{WY}$ and $\overline{ZM}$, compute $\frac{ZK}{KM}$.

9. ☐

10. ☐

Time: 10 minutes **Name: ______________________________**

11) Compute the number of three-digit numbers such that the units digit is larger than the tens digit.

12) In a rectangular coordinate system, a line contains the point whose coordinates are (8,10). The line's x-intercept is twice its y-intercept. The area of the circle that circumscribes the triangle formed by the line and the coordinate axes is $k\pi$. Compute k.

11. ☐

12. ☐

Solutions for Contest #2

7) A year contains either $365 = 7 \cdot 52 + 1$ or $366 = 7 \cdot 52 + 2$ days. Therefore, a year must contain at least 52 Fridays. Of the remaining 1 or 2 days, no more than 1 may be a Friday. Therefore, there are at most **53** Fridays in a year.

8) Use the fact that $1 + 2 + 3 + \cdots + 58 + 59 = \frac{59 \cdot 60}{2} = 1770$ to rewrite the given sum as $\frac{1}{2} + \frac{3}{3} + \frac{6}{4} + \frac{10}{5} + \cdots + \frac{1770}{60} = \frac{1}{2} + 1 + \frac{3}{2} + 2 + \cdots + \frac{59}{2} = \frac{1}{2}(1 + 2 + 3 + \cdots + 59) = \mathbf{885.}$

9) The integers are x and $x + 1$. Their sum is $2x + 1$. Then, $(x + 1)^2 - x^2 = 95 \rightarrow 2x + 1 = \mathbf{95.}$

10) Notice that $\Delta ZYK \sim \Delta MWK \rightarrow \frac{ZY}{MW} = \frac{ZK}{KM} = \frac{2}{1} = \mathbf{2.}$

11) There are 100 2-digit numbers (00, 01, 02, … , 99) that may be appended to a hundreds digit to form a 3-digit number. Ten of these 2-digit numbers have the same digits (00, 11, 22, … , 99), so of the remaining 90 2-digit numbers, 45 (half) of them have their units digit greater than their tens digit. Since there are 9 possible hundreds digits, $9 \cdot 45 = \mathbf{405}$.

12) As in the diagram, let the coordinates of O be $(0,0)$, A be $(2a, 0)$, B be $(0, a)$, and C be $(8, 10)$. Since the slope of $\overline{AB} = \frac{a-0}{0-2a} = -\frac{1}{2}$, and points A, B, and C are collinear, the slope of $\overline{BC} = \frac{10-a}{8-0} = -\frac{1}{2} \longrightarrow a = 14$. For a right triangle, the center of its circumscribed circle, the circumcenter, is the midpoint of its hypotenuse. The coordinates of midpoint M are $(14, 7)$ and by the distance formula, the length of radius $\overline{OM}$ is $\sqrt{14^2 + 7^2} = \sqrt{245}$. Thus, the area of the circle is 245π and $k = \mathbf{245}$.