Nassau County Interscholastic Mathematics League

Contest #1   Answers must be integers from 0 to 999, inclusive.   2018 – 2019

No calculators are allowed.

Time: 10 minutes   **Name: ______________________________**

1) Compute the product of the positive square root of 1764 and the largest prime factor of 1764.

2) Compute the product of the roots of $\left|2-|x|\right|=1$.

1. ☐

2. ☐

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3) Compute the number of cubic yards of topsoil needed to elevate a 27 foot by 27 foot garden by 4 inches.

4) Compute the smallest positive integer x such that $\sqrt{x^2 + 12x + 35} > 20$.

3. [ ]

4. [ ]

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5) If $3^{50} + 3^{50} + 3^{50} = 3^{x}$, compute x.

6) Diameter $\overline{AC}$ of circle O is drawn. Chord $\overline{AB}$ is extended through point B to external point D so that $AB : BD = 3 : 1$ and $\overline{DC}$ is tangent to the circle . If the ratio of the area of ΔACD to the area of ΔAOB is $\frac{p}{q}$, where $\frac{p}{q}$ is in simplest form, compute $p + q$.

5. ☐

6. ☐

Solutions for Contest #1

1) Since $1764 = 42^2 = (2 \cdot 3 \cdot 7)^2$, the required product is $42 \cdot 7 = \mathbf{294}$.

2) Either $2 - |x| = 1$ or $2 - |x| = -1$. In the first case, $x = \pm 1$. In the second case, $x = \pm 3$. The required product is **9.**

3) $27 \text{ ft} \cdot 27 \text{ ft} \cdot \frac{1}{3} \text{ ft} = 243 \text{ cu ft} \cdot \left(\frac{1 \text{ cu yd}}{27 \text{ cu ft}}\right) = \mathbf{9}$ cu yds.

4) $\sqrt{x^2 + 12x + 35} = \sqrt{(x+6)^2 - 1} > 20 \rightarrow (x+6)^2 - 1 > 400 \rightarrow x + 6 > 20 \rightarrow$ $x > 14$. Therefore the smallest positive integer to satisfy the given inequality is **15.**

5) $3^{50} + 3^{50} + 3^{50} = 3 \cdot 3^{50} = 3^{51}$. Thus, $x = \mathbf{51}$**.**

6) In a circle, if tangent and secant segments are drawn from the same external point, the square of the length of the tangent segment equals the product of the lengths of the secant and its external segment. So, $(CD)^2 = (4x)(x) \longrightarrow CD = 2x$. Since diameter $\overline{AC}$ is perpendicular to tangent $\overline{CD}$, and $CD = \frac{1}{2}(AD)$, ΔADC is a 30-60-90 triangle with $m\measuredangle A = 30°$ and $m\measuredangle D = 60°$. So, $AC = 2x\sqrt{3}$ and radii $AO = OB = x\sqrt{3}$. In isosceles ΔAOB, draw altitude $\overline{OE}$ whose length is $\frac{1}{2}(AO) = \frac{x\sqrt{3}}{2}$. The area of $\Delta AOB =$ $\frac{1}{2}(3x)\left(\frac{x\sqrt{3}}{2}\right) = \frac{3x^2\sqrt{3}}{4}$ The area of $\Delta ADC = \frac{1}{2}(2x)(2x\sqrt{3}) = 2x^2\sqrt{3}$. Thus, the required ratio is $\frac{8}{3}$ and $8 + 3 = \mathbf{11}$