Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2017 – 2018

No calculators are allowed.

Time: 10 minutes **Name: __________________________________**

25) Compute the only two-digit number that equals twice the sum of its digits.

26) A club of 8 students must send a representative group to the student government council. The representative group must contain at least one club member and may contain as many more club members as the club members desire to send to the council. Compute the number of different representative groups the club may select.

25. ☐

26. ☐

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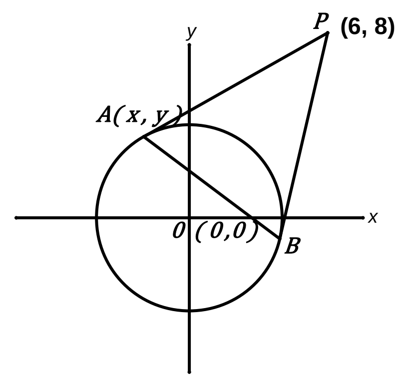
27) Two points whose coordinates are $(2,9)$ and $(c,6)$ are on a line parallel to the line whose equation is $3x + 4y = 12$. Compute c.

28) Through point P, with coordinates $(6,8)$ and exterior to circle O, two tangent lines are drawn to the circle. These lines intersect the circle at points A and B. The equation of the circle is $x^2 + y^2 = 16$. The equation of $\overleftrightarrow{AB}$ can be written in the form $ax + by = c$, where a, b, and c are positive constants, and there is no positive factor other than 1 common to all of a, b, and c. Compute $a + b + c$.



27.

28.

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29) In an arithmetic sequence, the first term is 45 and the fifth term is 81. Compute the sum of the second, third, and fourth terms of the arithmetic sequence.

30) If $0 \leq x < 4\pi$, compute the number of real roots of

$$\log\left(\sqrt{10}\sin x\right) + \log\left(-1 + \sqrt{10}\sin x\right) = \log 6.$$

29. [ ]

30. [ ]

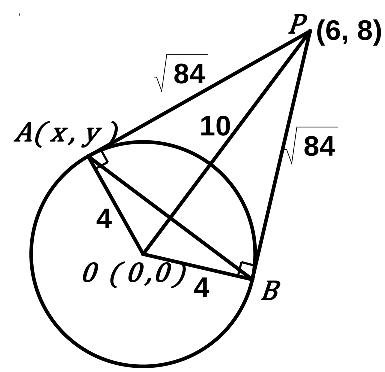
Solutions for Contest #5

25) If we use trial and error, we need not try any odd numbers. We quickly see that **18** works. Alternatively, if the number is $10t+u$, then $10t+u=2(t+u) \rightarrow u=8t \rightarrow$ $t=1$ and $u=8$. Note that if $t=2$, $u>9$, which fails.

26) $\sum_{n=1}^{8} {}_8C_n = 8+28+56+70+56+28+8+1 = \mathbf{255}.$
Alternatively, the number of subsets for a set containing 8 elements is 2^8. Since we must exclude the empty set, the answer is $2^8-1=\mathbf{255}$.

27) The equation of the given line may be transformed into $y=-\frac{3}{4}x+3$. The slope of the given line is $-\frac{3}{4}$. Since the two lines are parallel, their slopes are equal. Therefore, $\frac{6-9}{c-2}=-\frac{3}{4} \rightarrow c=\mathbf{6}.$

28) Since $\overline{OA}$ and $\overline{OB}$ are radii of length 4 of circle O, each forms a right angle with the appropriate tangent line to create right triangles OBP and OAP. Since $OP=\sqrt{6^2+8^2}=10$, we use the Pythagorean Theorem to show $PA=PB=\sqrt{10^2-4^2}=\sqrt{84}$. If the coordinates of point A are (x,y), then by the distance formula, $(x-6)^2+(y-8)^2=84$ and $x^2+y^2=16$. After expanding the binomial squares in the former equation and then subtracting the latter equation from the former, the result is $-12x+36-16y+64=68 \rightarrow -12x-16y=-32 \rightarrow$ $3x+4y=8$. The required sum is $3+4+8=\mathbf{15}$. Note: We can assert that this is an equation of $\overleftrightarrow{AB}$ because (x,y) is a point on the circle that is a distance of $\sqrt{84}$ from point P. There are only two such points, A and B.



29) Let the common difference be d. Then, $81=45+4d \rightarrow d=9$. Therefore, the second term is 54. The third term is 63. The fourth term is 72. The sum of those three terms is **189.**

30) Rewrite the given equation using the logarithm of a product law:
$\log[(\sqrt{10}\sin x)(-1+\sqrt{10}\sin x)]=\log 6$. Then, set the arguments equal:
$-\sqrt{10}\sin x+10\sin^2 x=6 \rightarrow 10\sin^2 x-\sqrt{10}\sin x-6=0$. By the Quadratic Formula, $\sin x=\frac{3\sqrt{10}}{10}$ or $\frac{-\sqrt{10}}{5}$. We reject $\sin x=\frac{-\sqrt{10}}{5}$ because in the given equation, the argument of the first logarithm, $\sqrt{10}\sin x=-2<0$. On the interval $[0,2\pi]$, $\sin x=\frac{3\sqrt{10}}{10}$ has 2 roots, so on the interval $[0,4\pi]$, $\sin x=\frac{3\sqrt{10}}{10}$ has **4** roots.