Nassau County Interscholastic Mathematics League

Contest #3 Answers must be integers from 0 to 999, inclusive. 2017 – 2018

No calculators are allowed.

Time: 10 minutes **Name: __________________________________**

13) Compute the sum of the prime factors of 399.

14) Two candles that appear to be identical are lit simultaneously. One burns completely uniformly in 3 hours and the other burns completely uniformly in 2 hours. Compute the number of **minutes** it will take for one candle to be exactly twice as long as the other.

13. ☐ 14. ☐

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15) If Mr. Skinflint pays for a one-dollar item with 48 coins and receives no change, compute the maximum number of nickels he could use for the purchase.

16) The solution set of $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ is $\{x : A \leq x \leq B\}$. Compute $A+B$.

15. $\boxed{}$

16. $\boxed{}$

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17) The number 555,555 is expressed as a product of two positive three-digit numbers, T and D. If $T < D$, compute T.

18) In parallelogram $ABCD$, $AB = 13$, $AD = 21$, angle A is acute, and the length of altitude $\overline{BP}$ to side $\overline{AD}$ is 12. The semicircles drawn whose diameters are $\overline{AB}$ and $\overline{AD}$ intersect in points A and X. If $AX = \frac{p}{q}$ is expressed in simplest form, compute $p + q$.

17. $\boxed{}$ 18. $\boxed{}$

Solutions for Contest #3

13) Since $399 = 400 - 1 = 20^2 - 1^2 = (20+1)(20-1) = 21 \cdot 19 = 3 \cdot 7 \cdot 19$, the required sum is **29.**

14) Each candle has an initial height of h inches and burns for t minutes until one candle is twice as high as the other. The flame moves down the first candle at a rate of $\frac{h}{3}$ inches per hour and the flame moves down the second candle at a rate of $\frac{h}{2}$ inches per hour. In order for the height of the slower burning candle to be twice the height of the faster burning candle, $h - \frac{h}{3}t = 2\left(h - \frac{h}{2}t\right) \rightarrow 1 - \frac{t}{3} = 2 - t \rightarrow \frac{2t}{3} = 1 \rightarrow t = 1.5$ hours or **90** minutes.

15) To maximize the number of nickels, Mr. Skinflint should not use any dimes or quarters. Otherwise he could use two nickels for each dime and five nickels for each quarter. Suppose he uses N nickels and P pennies. Then $N + P = 48$ and $5N + P = 100$. So, $4N = 52$ and $N =$ **13.**

16) Let $y = \sqrt{x-1}$. Then $y^2 = x - 1$. The original equation becomes $\sqrt{y^2 - 4y + 4} + \sqrt{y^2 - 6y + 9} = 1 \rightarrow \sqrt{(y-2)^2} + \sqrt{(y-3)^2} = 1 \rightarrow |y-2| + |y-3| = 1$. If $y \geq 3$, $y - 2 + y - 3 = 1 \rightarrow y = 3$. If $2 < y < 3$, $y - 2 + 3 - y = 1 \rightarrow 1 = 1$. If $y \leq 2$, $2 - y + 3 - y = 1 \rightarrow y = 2$. So, $2 \leq y \leq 3 \longrightarrow 2 \leq \sqrt{x-1} \leq 3 \longrightarrow 4 \leq x - 1 \leq 9 \longrightarrow 5 \leq x \leq 10$. The required sum is **15.**

17) Since $555{,}555 = 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 37$, let $T = 5 \cdot 11 \cdot 13 = 715$, and let $D = 3 \cdot 7 \cdot 37 = 777$. It follows that $555{,}555 = 715 \cdot 777$. So, $T =$ **715.**

18) Because point X is on semicircles AXB and AXD, $\measuredangle AXB$ and $\measuredangle AXD$ are right angles and points X, B, and D are collinear. Applying the Pythagorean triple 5-12-13 in ΔABP, $AP = 5$, so $PD = 16$. Applying the Pythagorean triple 12-16-20 in ΔBPD, $BD = 20$.
Since $\Delta BPD \sim \Delta AXD$, $\frac{AX}{AD} = \frac{BP}{BD} \rightarrow \frac{AX}{21} = \frac{12}{20} \rightarrow AX = \frac{63}{5}$.
The required sum is **68.**