

Nassau County Interscholastic Mathematics League

Contest #2 Answers must be integers from 0 to 999, inclusive. 2017 – 2018

Calculators are allowed.

Time: 10 minutes

Name: _____

- 7) Let $A = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^{2018}$ and
let $B = (-1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10)^{2018}$. Compute $A - B$.
- 8) Leon wants to buy a boat that costs \$100,000. He has only \$60,000 to spend on the boat. The store selling the boat insists on the full price if the boat is sold during the first week. After that, the store will reduce the price of the boat 10% per week based on the previous week's price until the boat is sold. Compute the least number of weeks before the boat is within Leon's budget.

7.

8.

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- 9) George Greedy went to the racetrack and on his first bet doubled the amount of money he brought to the track. On his second bet, he lost \$800. On his third bet, he bet all his remaining money and doubled it. On his fourth bet, he again lost \$800. On his fifth bet, he again bet all his remaining money and doubled it. On his sixth and final bet, he lost \$800 again and was broke. Compute the number of dollars George had when he started.
- 10) In parallelogram $WXYZ$, $WX = 8$, $WZ = 3$, and $m\angle ZWX = 30$ degrees. A circle whose center is point W contains point Z , and a circle whose center is point Z contains point W . The circles intersect at points P and Q . Point P is closer to point X than point Q is. If $PY = \sqrt{k}$, compute k .



9.



10.

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- 11) Compute the sum of the roots of $|x - 5|^2 - 6|x - 5| = -8$.
- 12) Point C is the intersection of the three medians (centroid) of $\triangle PQR$. A line through point C and parallel to \overline{PQ} intersects \overline{PR} in point A and \overline{QR} in point B . If the area of $\triangle RAB$ is 80, compute the area of trapezoid $ABQP$.

11.



12.

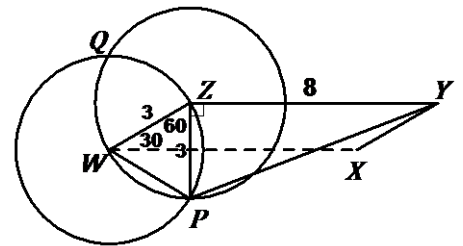


Solutions for Contest #2

- 7) The given expression may be re-written as $55^{2018} - (-55)^{2018} = \mathbf{0}$.
- 8) The price of the boat after n weeks is $\$100000(0.9)^n$, For the boat to be within the budget, $10000(0.9)^n \leq 60000 \rightarrow (0.9)^n \leq 0.6$. A calculator can solve this inequality. Else, $9^n \leq 6 \cdot 10^{n-1}$. Trial and error yields $9^5 = 59049 < 6 \cdot 10^4 = 60000$. Therefore the required number of weeks is **5**.
- 9) If George started with x dollars, then $2[2(2x - 800) - 800] - 800 = 0 \rightarrow x = \mathbf{700}$.

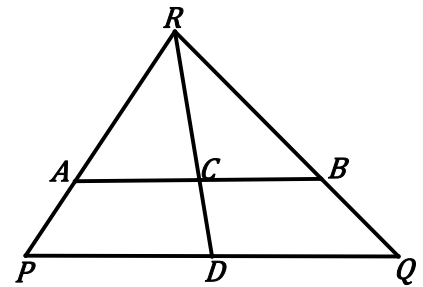
Alternatively, work backwards from 0: $0 \rightarrow 800 \rightarrow 400 \rightarrow 1200 \rightarrow 600 \rightarrow 1400 \rightarrow \mathbf{700}$.

- 10) The two given circles each have a radius of length 3, so ΔWZP is equilateral, and $m\angle WZP = 60^\circ$. Since $m\angle ZWX = 30^\circ$ and $WXYZ$ is a parallelogram, $m\angle WZY = 150^\circ$. So, $\angle PZY$ is a right angle. By the Pythagorean Theorem in ΔPZY , $PY = \sqrt{3^2 + 8^2} = \sqrt{73}$. Thus, $k = \mathbf{73}$.



- 11) Re-write the given equation as $|x - 5|^2 - 6|x - 5| + 8 = 0$. If $a = |x - 5|$, then $a^2 - 6a + 8 = 0 \rightarrow (a - 4)(a - 2) = 0 \rightarrow |x - 5| = 4$ or $|x - 5| = 2$. The solution set is $\{1, 9, 7, 3\}$. The sum of these roots is **20**.

- 12) Since $\overline{AB} \parallel \overline{PQ}$, $\Delta RAB \sim \Delta RPQ$. If C is the centroid of ΔPQR , then \overline{RC} and \overline{RD} are corresponding medians of the similar triangles. Since a centroid divides a triangle's median into segments whose ratio is $2 : 1$, $\frac{RC}{RD} = \frac{2}{3} \rightarrow \frac{[RAB]}{[RPQ]} = \frac{4}{9} \rightarrow \frac{80}{[RPQ]} = \frac{4}{9} \rightarrow [RPQ] = 180$. Thus, $[ABQP] = 180 - 80 = \mathbf{100}$.



Note: The symbol $[RPQ]$ means the area of polygon RPQ .